Presbyterian Ladies' College

MELBOURNE

VCE
Year 11

2019 Semester 1
Mathematical Methods Examination

PAPER 1

Teachers: **Lang** **Lewis** **Mendan** **Smith** **Taylor**
(Please circle)

Name: ANSWERS **Form:** ______

READING TIME: (15 minutes for Paper 1 and 2)
WRITING TIME: (40 minutes)

Section	Number of Questions	Number of Questions to be answered	Number of Marks
A	9	9	43
			Total Marks =43

No. of Pages 10

Instructions

1. No calculators are permitted.
2. No notes are permitted.
3. No white out is permitted.
4. Exact answers must be given unless otherwise specified.
5. In questions where more than one mark is available, appropriate working must be shown.

SECTION A: Short Answer: All questions should be answered in the spaces provided.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

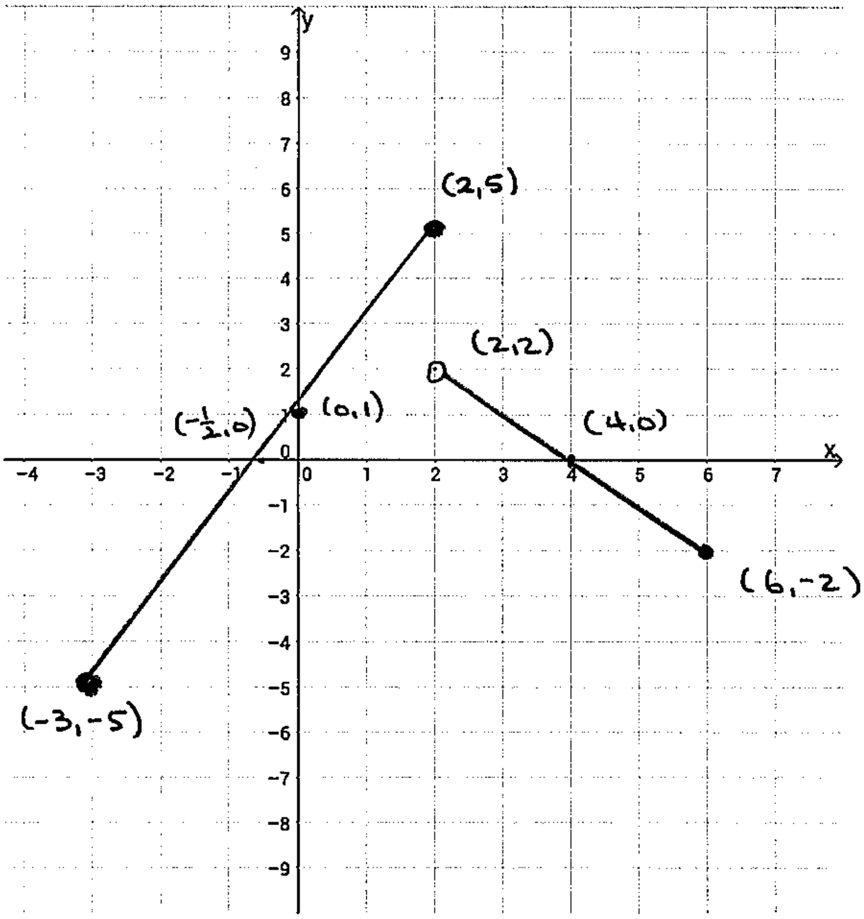
5/11

Section A: Short Answer: No calculator allowed.

Question 1 (5 marks)

a. Sketch the graph of $f(x) = \begin{cases} 2x+1, & -3 \le x \le 2 \\ 4-x, & 2 < x \le 6 \end{cases}$,

marking the axial intercepts and end points with their coordinates.



b. State the range of f.

[-5, 5]

(4+1 = 5 marks)

Question 2 (4 marks)

Find the equation of the line that passes through the point with coordinates $(1, -3)$ and is:

a parallel to the y-axis

$x = 1$

b perpendicular to the line with equation $y = 1 - \frac{x}{3}$

$m = 3$

$y + 3 = 3(x - 1)$

$y = 3x - 6$

(1 + 3 = 4 marks)

Question 3 (4 marks)

For $f(x) = x^3 - ax^2 + bx + 9$, find the values of a and b if $x + 3$ is a factor and $f(-2) = 15$

$f(-3) = 0$ $\quad -27 - 9a - 3b + 9 = 0$

$9a + 3b = -18$

$3a + b = -6$. Eq 1

$f(-2) = -8 - 4a - 2b + 9 = 15$

$4a + 2b = -14$

$2a + b = -7$ Eq 2

Eq 1 − Eq 2 $\quad a = 1$

$b = -9$

(4 marks)

Question 4 (7 marks)

If A and B are events such that $Pr(A) = 0.50$, $Pr(B) = 0.3$ *and* $Pr(A \cap B') = 0.25$.

a Complete the following Karnaugh diagram.

	A	A'	
B	0.25	0.05	0.30
B'	0.25	0.45	0.70
	0.5	0.5	1

b Find

i $\Pr(A \cap B)$

0.25

ii $\Pr(A' \cap B')$

0.45

iii $\Pr(A' \cup B)$

$= Pr(A') + Pr(B) - Pr(A' \cap B)$

$= 0.5 + 0.3 - 0.05 = 0.75$

iv $\Pr(A' | B)$ $= \dfrac{Pr(A' \cap B)}{Pr(B)}$

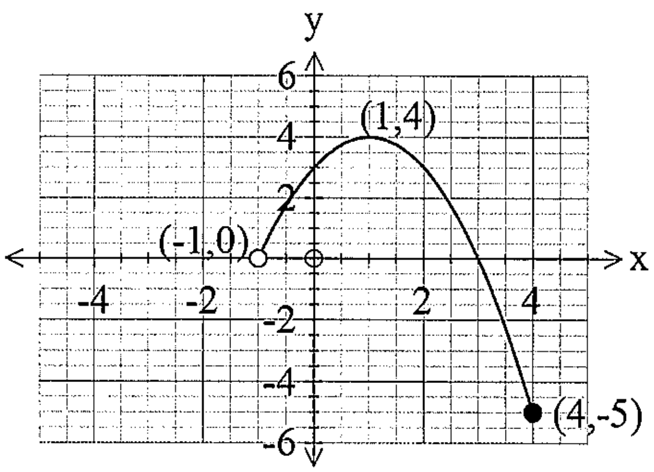
$= \dfrac{0.05}{0.30}$

$= \frac{1}{6}$ Must be simplified

(1 + 6 = 7 marks)

Question 5 (3 marks)

Consider the graph shown below



a What is the domain?

$(-1, 4]$

b What is the range?

$[-5, 4]$

c Is this graph a function?

Yes

(1 +1 + 1 = 3 marks)

Question 6 (5 marks)
Consider the family of quadratic graphs described by $y = 2kx^2 - kx + 1 - k$ where $k \in R/\{0\}$.

a Find the discriminant in terms of k

$$\Delta = (-k)^2 - 4(2k)(1-k)$$
$$= k^2 - 8k(1-k)$$
$$= 9k^2 - 8k$$

b Hence find the values of k where the graphs of the quadratics have:

i one x-intercept

$9k^2 - 8k = 0$ $\quad$ $k = 0$ or $8/9$

$k(9k-8) = 0$ $\quad$ $k = 8/9$ since $k \neq 0$

ii two x-intercepts

$k(9k-8) > 0$ $\quad$ $k < 0$ or $k > 8/9$

iii no x-intercepts

$k(9k-8) < 0$ $\quad$ $0 < k < \frac{8}{9}$

(2 + 1 + 1 + 1 = 5 marks)

Question 7 (7 marks)

A circle has the equation $x^2 + 4x + y^2 - 6y + 3 = 0$.

a Find the coordinates of the x-intercepts of the circle

$y = 0 \qquad x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = -3, -1$

$(-3, 0) \qquad (-1, 0)$

b Find the coordinates of the centre of the circle

$(x^2 + 4x + 4) + (y^2 - 6y + 9) = 10$

$(x+2)^2 + (y-3)^2 = 10$

Centre $(-2, 3)$ M

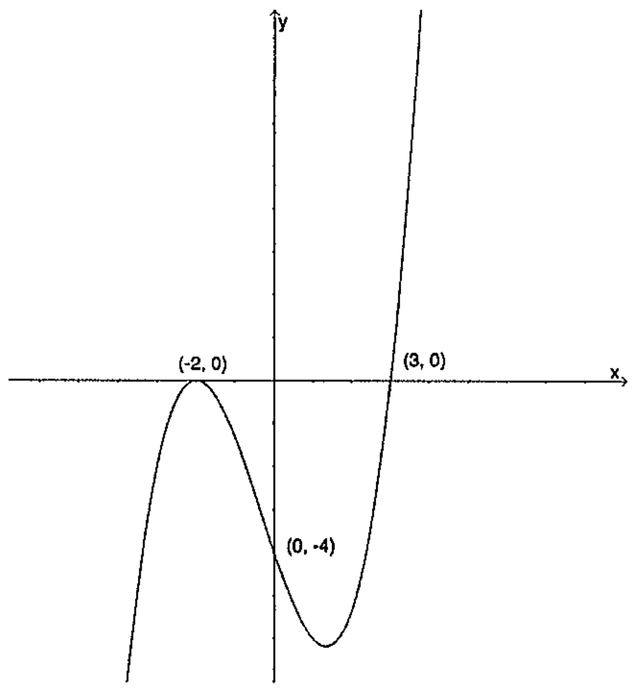
c Find the radius of the circle

$\sqrt{10}$

(3 + 3 + 1 = 7 marks)

Question 8 (3 marks)

(a) Find the equation of the cubic graph, $y = f(x)$ shown below.



$y = a(x+2)^2(x-3)$

$x = 0 \quad y = -4$

$-4 = a(4)(-3)$

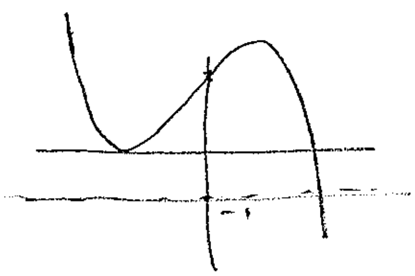
$a = \frac{1}{3}$

$y = \frac{1}{3}(x+2)^2(x-3)$

(2 marks)

(b) If $g(x) = -f(x)$, how many solutions will there be to the equation $g(x) = -1$?

one



(1 mark)

Question 9 (5 marks)

(a) Express the equations $px + 5y = q$ and $3x - qy = 5q$, $(q \neq 0)$, in the form $y = mx + c$

$y = -\frac{p}{5}x + \frac{q}{5}$

$y = \frac{3}{q}x - 5$

(2 marks)

(b) Hence, determine the values of p and q so the system of equations

$$px + 5y = q$$
$$3x - qy = 5q$$

will have infinitely many solutions.

$M_1 = M_2$ $\qquad$ $C_1 = C_2$

$-\frac{p}{5} = \frac{3}{q}$ $\qquad$ $\frac{q}{5} = -5$

$-p = \frac{15}{q}$ $\qquad$ $q = -25$

$p = \frac{-15}{q}$

$= -\frac{15}{-25}$

$p = \frac{3}{5}$

(3 marks)

End of Section A

Presbyterian Ladies' College

MELBOURNE

VCE
Year 11

2019 Semester 1
Mathematical Methods Examination
PAPER 2

Teachers: **Lang** **Lewis** **Mendan** **Smith** **Taylor**
(Please circle)

Name: ANSWERS. **Form:** ______

READING TIME: (15 minutes for both Paper 1&2)
WRITING TIME: (80 minutes)

Section	Number of Questions	Number of Questions to be answered	Number of Marks
B	18	18	18
C	5	5	46
			Total Marks =64

No. of Pages 21

Instructions

1. Calculators can be used in both sections of this paper.
2. You are permitted to refer to 8 sides of notes, whilst completing this paper.
3. Whiteout is not permitted.
4. Exact answers must be given unless otherwise specified.
5. In questions where more than one mark is available, appropriate working must be shown.

SECTION B: Multiple Choice: Use the answer sheet provided.

SECTION C: Analysis: Answer the questions in the spaces provided.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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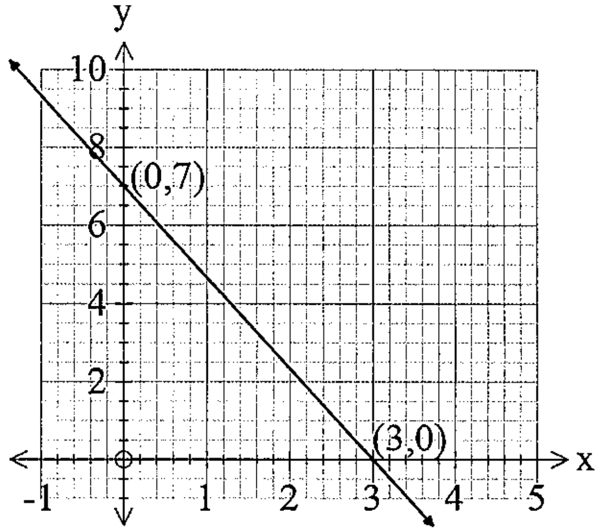
Section B: Multiple Choice **Calculators and notes allowed**

THESE QUESTIONS MUST BE ANSWERED ON THE ANSWER SHEET PROVIDED

Question 1



$m = -\frac{7}{3}$

$c = 7$

A straight line has a Y- intercept of $(0,7)$ and a X-intercept of $(3,0)$. The equation of the line is:

A. $y = \frac{7}{3}x + 7$

B. $y = \frac{-7}{3}x - 7$

C. $7y - 3x - 49 = 0$

D. $7y + 3x - 49 = 0$

E. $3y + 7x - 21 = 0$

The following information refers to Question 2 and 3.

A fair die is thrown and a fair coin is tossed. The sample space is shown below.

H1 ✓	**H2** ✓	**H3** ✓	**H4** ✓	**H5** ✓	**H6** ✓
T1 ✓	**T2** ✓	**T3**	**T4**	**T5**	**T6**

Question 2
Find the probability of obtaining a head *or* a number less than 3.

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

E. $\frac{5}{6}$

$\frac{8}{12} = \frac{2}{3}$

Question 3
If a number less than 3 has fallen, find the probability a head was tossed.

A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. $\frac{1}{6}$

E. $\frac{1}{5}$

$Pr(H \mid \text{less than } 3)$

$= \frac{Pr(H \cap \text{less than } 3)}{Pr(\text{less than } 3)}$

$= \frac{2}{4}$

$= \frac{1}{2}$

Question 4

When $5x^3 - x^2 - 5x + 1$ is divided by $x - 2$ the remainder is:

- **A.** 0
- **B.** 27
- **C.** -33
- **D.** 5
- **E.** -27

$P(2) = 40 - 4 - 10 + 1$

$= 27$

Question 5

The polynomial $4x^3 - 12x^2 + 5x + 6$ expressed as a product of three linear factors is equal to:

- **A.** $(2x + 3)(2x - 1)(x + 2)$
- **B.** $(2x + 3)(2x + 2)(x - 1)$
- **C.** $(x - 1)(2x - 3)(2x - 3)$
- **D.** $(2x - 1)(x - 2)(2x + 3)$
- **E.** $(2x + 1)(x - 2)(2x - 3)$

Question 6

The midpoint of the interval PQ, where P has coordinates $(-2, b)$ and Q has coordinates $(b, \frac{-b}{2})$ is:

- **A.** $\left(b-2, \frac{-b}{4}\right)$
- **B.** $\left(\frac{b}{2}-1, \frac{b}{4}\right)$
- **C.** $\left(\frac{b+2}{2}, \frac{3b}{2}\right)$
- **D.** $\left(\frac{b+2}{2}, -\frac{3b}{2}\right)$
- **E.** $\left(b, \frac{-b}{4}-1\right)$

$\left(\frac{-2+b}{2}, \frac{b+\frac{-b}{2}}{2}\right)$

$\left(\frac{b}{2}-1, \frac{b}{4}\right)$

Question 7

For $f:(-1,3] \rightarrow R, f(x)=1-x^2$, the range of f is:

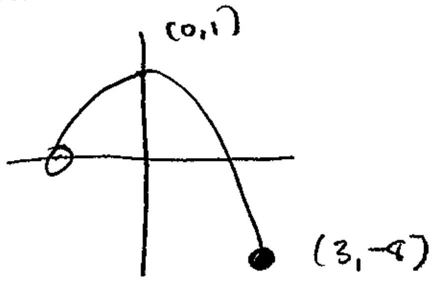
A. $[-8,0)$

B. $[-8,0]$

C. $(-8,1)$

D. $[-8,1]$

E. $[-8,1)$



Question 8

If $\Pr(A \cup B)=0.85$, $\Pr(A)=0.3$ and $\Pr(A \cap B)=0.1$

Find $\Pr(A' \cap B)$

$0.85 = 0.3 + Pr(B) - 0.1$

$Pr(B) = 0.65$

A. 0.15

B. 0.55

C. 0.7

D. 0.9

E. 0.1

	A	A'	
B	0.1	0.55	0.65
B'	0.2	0.15	0.35
	0.3	0.7	

Question 9

For $f(x)=3-x-x^2, f(b)-f(2b)$ is equal to:

A. $3+b-b^2$

B. $3+b+b^2$

C. $3b^2+b$

D. $3b^2-b$

E. $3b^2-3b$

$f(b) = 3 - b - b^2$

$f(2b) = 3 - 2b - 4b^2$

$f(b) - f(2b) = \cancel{3} - b - b^2 - \cancel{3} + 2b + 4b^2$

$= b + 3b^2$

Question 10
The parabola with vertex $(-1,2)$ and y-intercept $(0,1)$ has equation:

A. $y=2+(x+1)^2$

B. $y=3-2x-x^2$

C. $y=2-(x-1)^2$

D. $y=(x+1)^2+1$

E. $y=1-2x-x^2$

$y = a(x+1)^2 + 2$

sub. $(0,1)$

$1 = a(1) + 2$

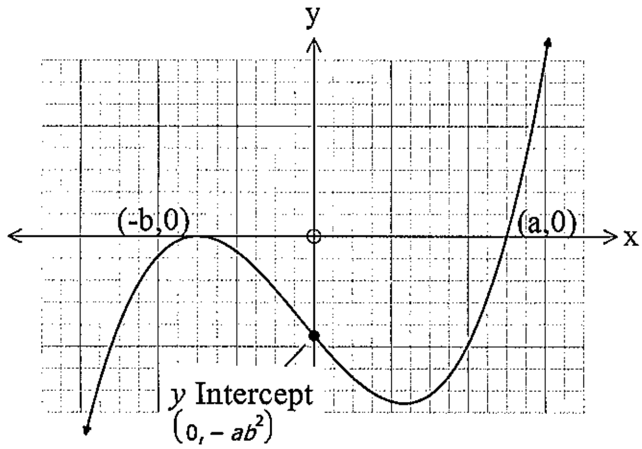
$a = -1$

$y = -(x+1)^2 + 2$

$= -x^2 - 2x + 1$

Question 11

The rule for the cubic function shown below with rule $y = f(x)$ where $a > 0, b > 0$ is:



A. $y=(x-a)^2(x+b)$ ×

B. $y=(x-a)(x-b)^2$ ×

C. $y=-(x-a)(x+b)^2$ ×

D. $y=(x-a)(x+b)^2$ ✓

E. $y=(x-a)^2(x+b)^2$ ×

Question 12
The equation $3x^2 - x - k = 0$ has one solution when k equals:

A. $\frac{-1}{12}$

B. $\frac{1}{12}$

C. $\frac{1}{4}$

D. -12

E. 12

$\Delta = (-1)^2 - 4(3)(-k)$

$0 = 1 + 12k$

$k = -\frac{1}{12}$

Question 13
For a biased coin, the $\Pr(H) = p$.
The coin is tossed twice. Find the probability of obtaining two tails.

A. $p^2 - 2p$

B. $1 - p^2$

C. $1 - 2p + p^2$

D. $2p(1-p)$

E. $(1-p)^2 - p^2$

$\Pr(T) = 1 - p$

$\Pr(2T) = (1-p)(1-p)$

$= 1 - 2p + p^2$

Question 14
The maximal domain of the function $y = \sqrt{x+2}$ is:

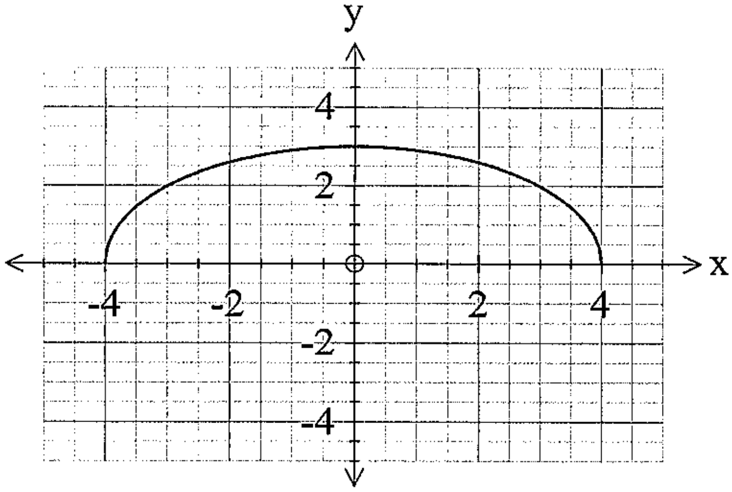
A. $x \in [-2, \infty)$
B. $x \in (-2, \infty)$
C. $x \in [2, \infty)$
D. $x \in (2, \infty)$
E. $x \in R$

$x + 2 \geq 0$

$x \geq -2$

$[-2, \infty)$

Question 15



The graph shown above can be described as a:

A. function, one to one correspondence

B. upper branch of a circle

C. relation, many to many correspondence

D. function, many to one correspondence

E. relation, one to many correspondence.

Question 16

The factors of $x^3 - 64$ are:

A. $(x-4)^3$

B. $(x-4)(x^2-4x+16)$

C. $(x-4)(x^2+8x-16)$

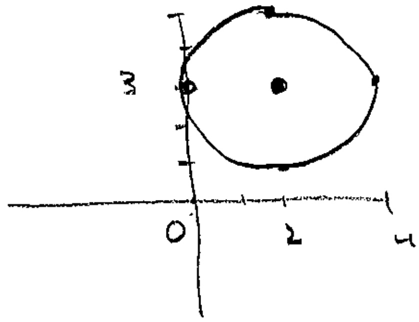
D. $(x-4)(x^2+4x+16)$

E. $(x-4)(x^2-4x-16)$

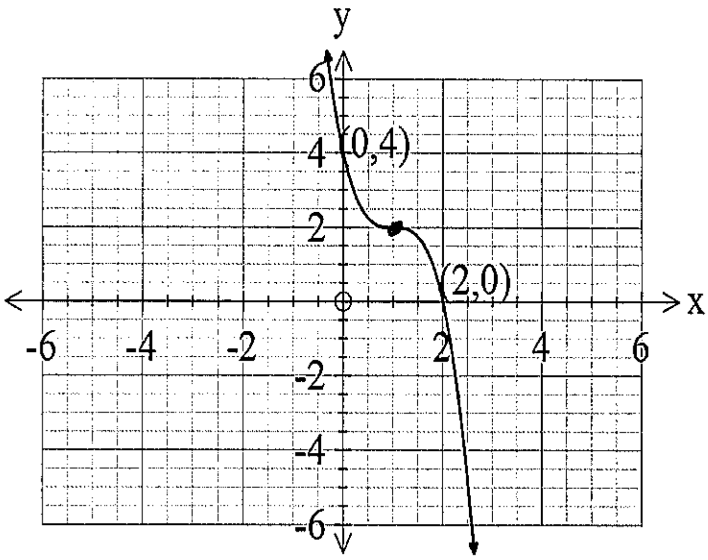
Question 17

The domain and range of the relation $(x-2)^2 + (y-3)^2 = 4$ are:

A. dom = [0, 4] and ran = [1, 5]
B. dom = [0, 4] and ran = [-1, -5]
C. dom = [-4, 0] and ran = [-1, -5]
D. dom = [-4, 0] and ran = [1, 5]
E. dom = [2, 3] and ran = [0, 4]



Question 18



The equation of the graph shown above is:

A. $y=2(x-1)^3+2$

B. $y=(x-1)^3+2$

C. $y=2-2(x-1)^3$

D. $y=2-2(x+1)^3$

E. $y=2(x+1)^3+2$

$y = a(x-1)^3 + 2$

Sub. $(2,0)$

$0 = a(1)^3 + 2$

$a = -2$

$y = -2(x-1)^3 + 2$

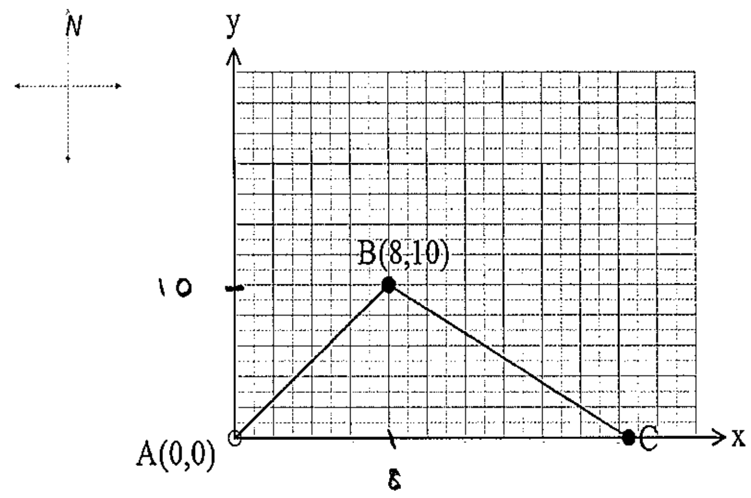
END OF SECTION B

SECTION C: Analysis

Calculators and notes allowed. Exact answers should be given unless instructed otherwise.

Question 1 (9 marks)

A yacht sails in a straight line from $A(0,0)$ to $B(8,10)$. It then turns 90° and sails in a straight line to C which is directly east of A and finally completes the triangular course by sailing directly back to A *(all units in km).*



(a) Find an equation of the straight line AB.

$m = \frac{5}{4}$ $\quad c = 0$

$y = \frac{5}{4}x$

(2 marks)

(b) Find an equation of the straight line BC and the coordinates of point C.

$m = -\frac{4}{5}$

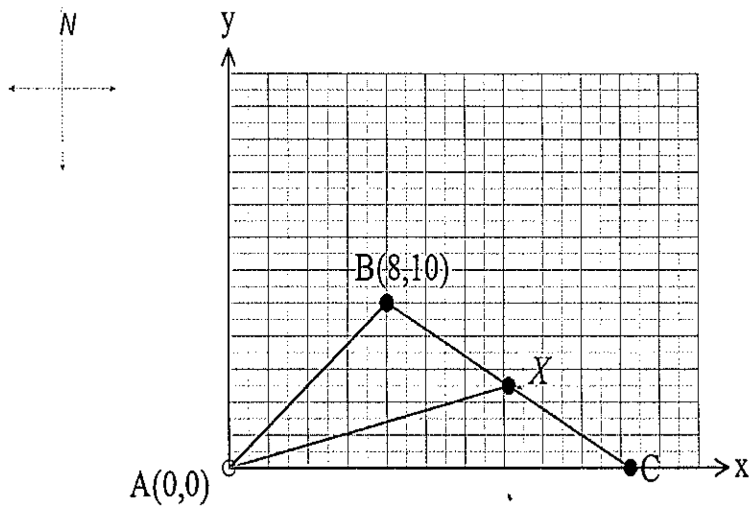
$y - 10 = -\frac{4}{5}(x - 8)$

$y = -\frac{4}{5}x + \frac{82}{5}$

$C \quad (\frac{41}{2}, 0)$

(3 marks)

The yacht breaks down at X, the halfway point of the second leg of the triangular course. A rescue boat located at A sails directly to the yacht.



(c) Find the exact coordinates of X and an equation of AX.

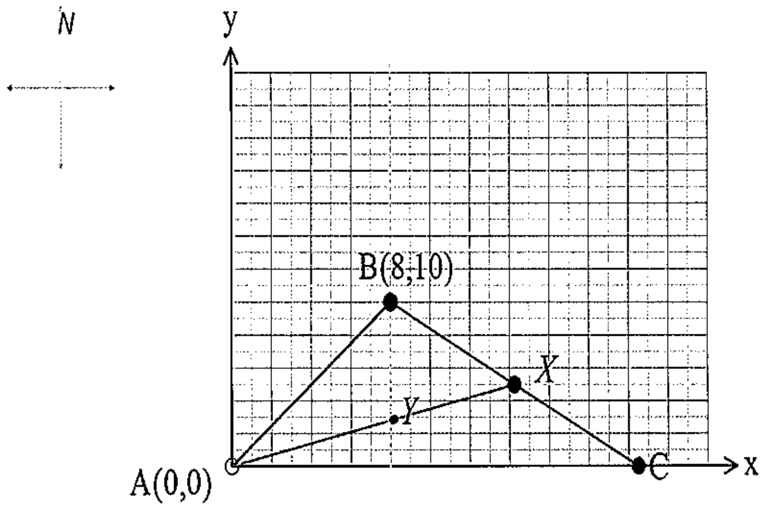
$X \left(\frac{8 + 41/2}{2}, \frac{10+0}{2} \right)$

$\left(57/4, 5 \right)$

$y = \frac{20}{57} x$

(2 marks)

The rescue boat also breaks down at Y which is located due south of B.



(d) Find the exact coordinates of Y.

$y = \frac{20}{57}x$

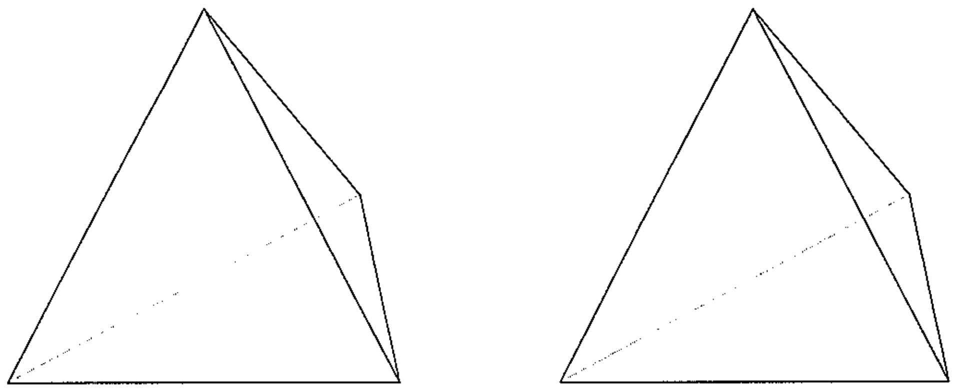
sub $x = 8$

$y = \frac{160}{57}$

$(8, \frac{160}{57})$

(2 marks)

Question 2 (11 marks)
A pair of identical 4-sided biased dice in the shape of tetrahedrons have sides labelled 1, 2, 3 and 4.



When each die is thrown $Pr(1) = Pr(2) = Pr(3) = 0.2$ and $Pr(4) = 0.4$.

(a) Both dice are thrown. Complete the table showing the sixteen outcomes in the sample space and their probability of each outcome.

Die 1 / **Die 2**	**1 (0.2)**	**2 (0.2)**	**3 (0.2)**	**4 (0.4)**
1 (0.2)	0.04	0.04	0.04	0.08
2 (0.2)	0.04	0.04	0.04	0.08
3 (0.2)	0.04	0.04	0.04	0.08
4 (0.4)	0.08	0.08	0.08	0.16

(2 marks)

Let event:
A be an odd number with first die
B be the same number with both dice
C be a total greater than six from both dice
D be a different number on both dice

(b) Find
i) $Pr(A')$

$3 \times 0.04 + 4 \times 0.08 + 0.16$

$= 0.6$

(1 mark)

ii) $\Pr(A \cap B)$ $(1,1)$ $(3,3)$

$\Pr(\text{odd first} \cap \text{same}) = 0.08$

(1 mark)

iii) $\Pr(C \cup D)$

$\Pr(C) + \Pr(D) - \Pr(C \cap D) = 0.32 + 0.72 - 0.16$

$= 0.88$

(1 mark)

iv) $\Pr(A | B)$ $\dfrac{\Pr(A \cap B)}{\Pr(B)} = \dfrac{0.08}{0.28}$

$= 2/7$

(2 marks)

v) $\Pr(C | D)$ $\dfrac{\Pr(C \cap D)}{\Pr(D)} = \dfrac{0.16}{0.72}$

$= 2/9$

(2 marks)

(c) State with reason whether events A and B are independent.

$\Pr(A) \times \Pr(B) = 0.4 \times 0.28 = 0.112$

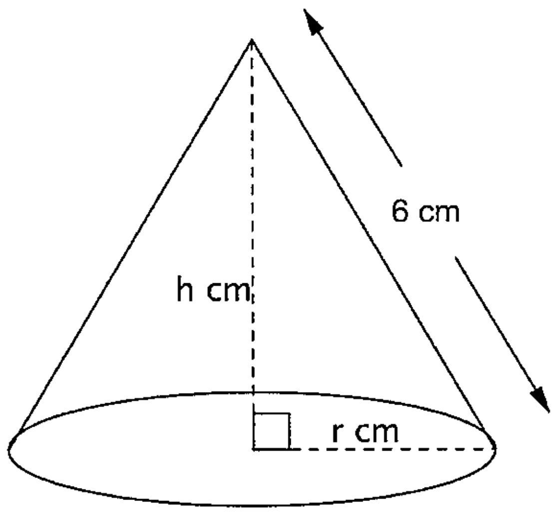
$\Pr(A \cap B) = 0.08$

Not Independent $\Pr(A) \times \Pr(B) \neq \Pr(A \cap B)$

(2 marks)

Question 3 (6 marks)

The slant edge of a right cone measures 6 cm, the height is h cm and the radius is r cm.



(a) Express r in terms of h.

$r = \sqrt{36 - h^2}$

(1 mark)

(b) Given that the formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$ show that

$V = \frac{1}{3}\pi h(6+h)(6-h)$

$V = \frac{1}{3}\pi\left(\sqrt{36-h^2}\right)^2 h$

$= \frac{1}{3}\pi\left(36-h^2\right)h$

$= \frac{1}{3}\pi h(6-h)(6+h)$

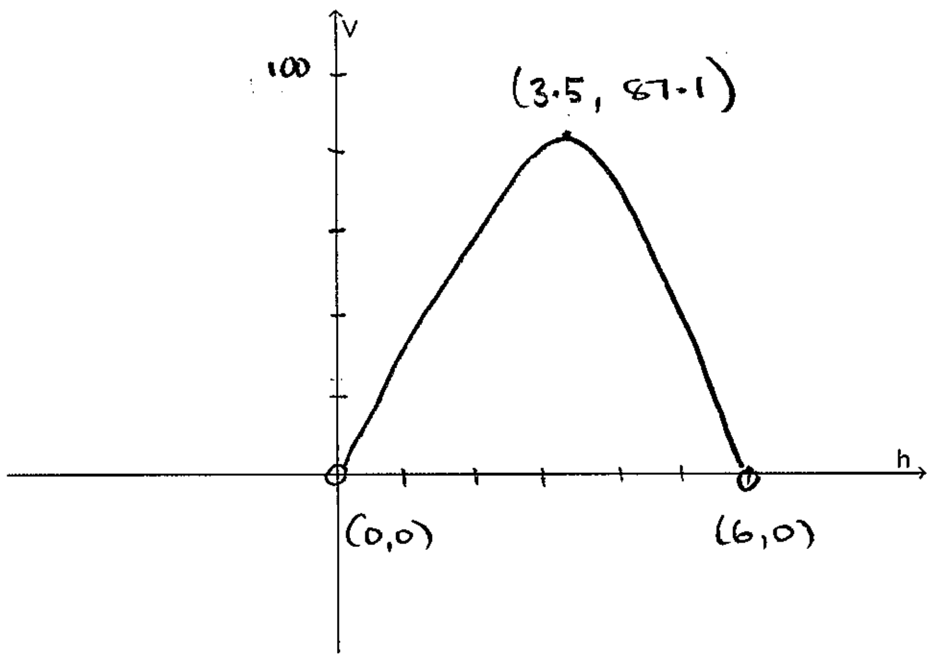
(1 mark)

(c) State an appropriate domain for the function . $V = \frac{1}{3}\pi h(6+h)(6-h)$ in the context of this problem.

$0 < h < 6$

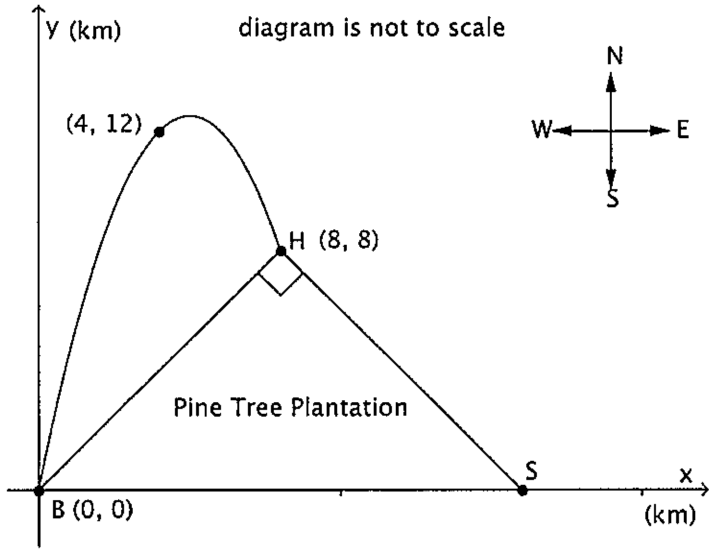
(2 marks)

(d) On the axes provided sketch the graph of . $V = \frac{1}{3}\pi h(6+h)(6-h)$ over an appropriate domain.
Label the coordinates of all important points, including any turning point. Any approximate values should be given correct to one decimal place.



(2 marks)

Question4 (15 marks)



A group of PLC students are to do a hike as part of their Duke of Edinburgh. On the first day they will walk from base camp (B) to Smith's Hut (H). The path they walk has a parabolic shape and is depicted on the diagram above. The y-axis points due north, the x-axis points due East and all units are in kilometres.

The rule for the parabola is of the form $y = ax^2 + bx$.

(a) Given that the parabolic path passes through (4, 12) and (8, 8) show algebraically that the equation of this path is given by $y = -\frac{1}{2}x^2 + 5x$.

$12 = 16a + 4b \quad -(1)$

$8 = 64a + 8b \quad -(2)$

$(2) - (1) \times 2$

$-16 = 32a$

$a = -\frac{1}{2}$

$12 = -8 + 4b$

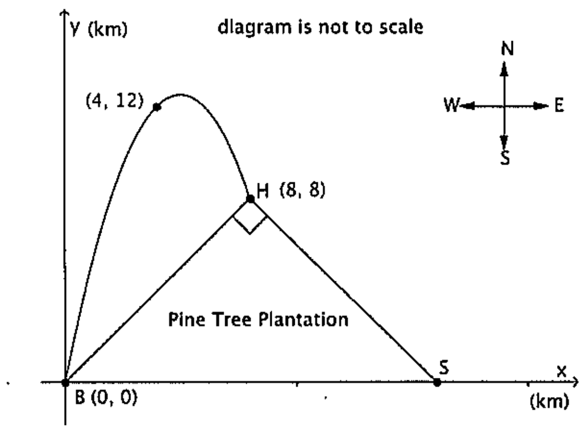
$4b = 20$

$b = 5$

(4 marks)

$y = -\frac{1}{2}x^2 + 5x$

(b) As an alternative to hiking along the parabolic path the girls could walk in a straight line (BH) from the base camp to Smith's Hut beside a Pine Tree Plantation. If they were to take this route find:



i) the exact distance they would walk.

$d = \sqrt{8^2 + 8^2}$

$= 8\sqrt{2}$ km

(2 marks)

ii) the angle the path BH makes with the positive direction of the x-axis.

$\tan\theta = 1$

$\theta = 45^\circ$

(1 mark)

After camping the night at Smith's Hut the girls leave the next morning via another path which runs in a straight line from the Hut to Sealer's Cove (S) due East of Base camp.

(c) Given the line BH is **<u>perpendicular</u>** to HS find

i) the gradient of the line HS.

$m = -1$

(1 mark)

ii) an equation of the line HS.

$y - 8 = -(x - 8)$

$y = -x + 16$

(2 marks)

iii) the **coordinates** of Sealer's Cove (S).

$y = -x + 16$

$y = 0 \quad x = 16 \quad (16, 0)$

(2 marks)

(d) The land enclosed by the triangle BHS is a pine tree plantation. Find the area of triangle BHS.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 16 \times 8$

$= 64$ km^2

(2 marks)

A group of girls from MLC are also hiking in the same area. However the MLC girls start their hike at Goddard's Point (G) which is located at (0, 16). They walk on a path with the equation $y = -\frac{1}{4}x^3 + \frac{3}{2}x^2 - 3x + 16$ to the point where it intersects with the parabolic path $y = -\frac{1}{2}x^2 + 5x$.

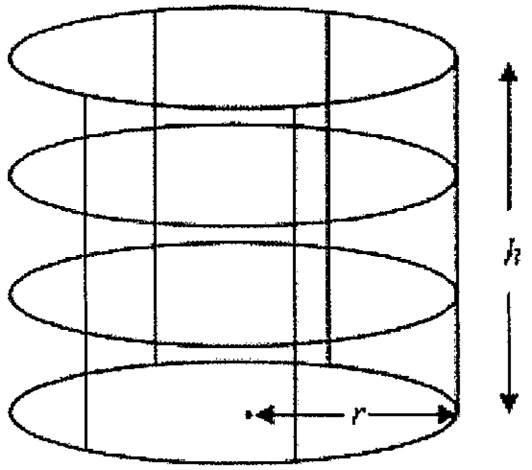
(e) Find the **coordinates** of the point where the MLC girls join the parabolic path.

$(4, 12)$

(1 mark)

Question 5 (5 marks)

A plant trellis in the shape of a cylinder consists of four circles of wire and 5 straight pieces of wire as shown in the diagram below.



The radius of the cylinder is r metres and the height is h metres. The volume of the cylinder must equal 0.8 m^3.

(a) Find an expression for h in terms of r.

$0.8 = \pi r^2 h$

$h = \frac{4}{5\pi r^2}$ (accept $\frac{0.8}{\pi r^2}$)

(1 mark)

(b) The total length of wire is L metres. Show that $L = 8\pi r + \frac{4}{\pi r^2}$.

$L = 4 \times 2\pi r + 5 \times \left(\frac{4}{5\pi r^2}\right)$

$= 8\pi r + \frac{4}{\pi r^2}$

(2 marks)

(c) Find the minimum length of wire, correct to 1 decimal place, and the radius that corresponds with this minimum length, correct to two decimal places.

$L = 17.6$ m

$r = 0.47$ m

(2 marks)

END OF SECTION C