MM12 Assignment 2 - Polynomials and Quadratics

Name:

Skills (Section A and B) **Analysis (Section C)**

Section A: Short Answer **No Calculators Allowed**

NOTE: Exact answers are required unless instructed otherwise within the question.

1. Given $P(x) = x^3 - 2x^2 + 1$ and $Q(x) = 3x^2 + x - 5$

a) State the degree of P(x)

3

(1 mark)

b) Find each of the following, simplifying where appropriate.

i) $P(\frac{1}{2})$

$= \frac{5}{8}$

ii) $Q(-1)$

$= -3$

iii) $Q(2x + 1)$

$= 3(2x+1)^2 + (2x+1) - 5$

$= 3(4x^2+4x+1) + 2x - 4$

$= 12x^2 + 14x - 1$

(1+1+2 = 4 marks)

2. i) Write down the first 5 rows of Pascal's triangle.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

ii) Hence expand $(2x - 1)^4$.

$(2x)^4 - 4(2x)^3 + 6(2x)^2 - 4(2x) + 1$

$= 16x^4 - 32x^3 + 24x^2 - 8x + 1$

(1+2 = 3 marks)

3. Factorise over Q the expression $1 + 27x^3$

$$1 + (3x)^3$$

$$= (1+3x)(1-3x+9x^2)$$

(2 marks)

4. **a)** Show without division that $x - 2$ is a factor of $x^3 - 2x^2 - 25x + 50$

$$P(2) = (2)^3 - 2(2)^2 - 25(2) + 50$$

$$= 8 - 8 - 50 + 50$$

$$= 0$$

Since $P(2) = 0$, $x-2$ is a factor

(1 mark)

b) Hence fully factorise $x^3 - 2x^2 - 25x + 50$.

$$\begin{array}{r|l} & x^2 \quad\quad - 25 \\ \hline x-2 & x^3 - 2x^2 - 25x + 50 \\ & \underline{x^3 - 2x^2} \\ & \quad\quad\quad -25x + 50 \\ & \quad\quad\quad \underline{-25x + 50} \\ & \quad\quad\quad\quad\quad 0 \end{array}$$

Factors $(x-2)(x+5)(x-5)$

(3 marks)

5. Given $P(x) = x^3 - kx^2 - 1$ has a remainder of -1 when divided by $x - 3$ find k.

$$P(3) = -1$$

$$27 - 9k - 1 = -1$$

$$9k = 27$$

$$k = 3$$

(2 marks)

6. Let $y = 3x^2 - 12x + 6$

a) Find the coordinates of the y-intercept of the graph of $y = 3x^2 - 12x + 6$

y intercept, $x = 0$

$(0, 6)$

(1 mark)

b) Use completing the square to write the equation in the form $y = a(x + b)^2 + c$

$$y = 3(x^2 - 4x + 2)$$
$$= 3\left[(x^2 - 4x + 4) - 4 + 2\right]$$
$$= 3\left[(x-2)^2 - 2\right]$$
$$y = 3(x-2)^2 - 6$$

(3 marks)

c) Hence find the exact coordinates of:

i) the turning point $y = 3x^2 - 12x + 6$

$$(2, -6)$$

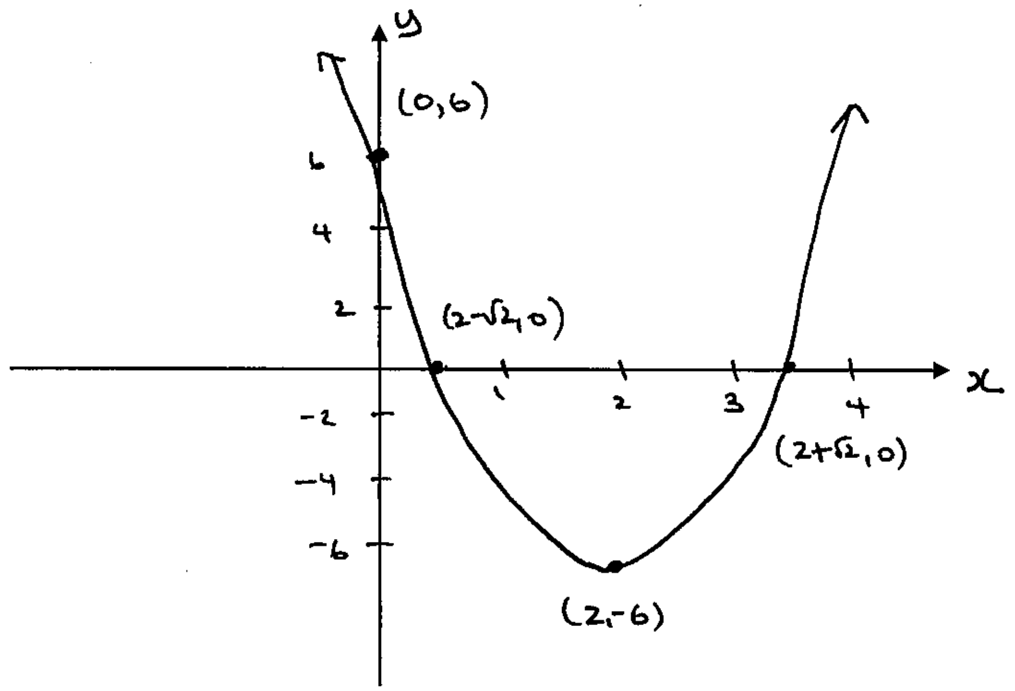
ii) the x-intercepts of $y = 3x^2 - 12x + 6$

$$3(x-2)^2 - 6 = 0$$
$$(x-2)^2 - 2 = 0$$
$$(x - 2 + \sqrt{2})(x - 2 - \sqrt{2}) = 0$$

x intercepts.
$(2 - \sqrt{2}, 0)$ $(2 + \sqrt{2}, 0)$.

(1 + 3 = 4 marks).

d) Sketch the graph of $y = 3x^2 - 12x + 6$, labelling the coordinates of all key points.



(3 marks)

End of Short Answer

Section B: Multiple Choice Questions **Calculators Are Allowed**

<u>**Circle Correct Response**</u>

1. Which one of the following functions is a polynomial.

A $y = 3x^4 - \frac{1}{x}$

B $y = \sqrt{x}$

C $y = \frac{1}{2}x^3 - 1$

D $y = x^{-2} + 3x + 2$

E $y = \frac{1}{x^2}$

2. Correct to 2 decimal places the graph of $y = x^2 - 38x + 260$ has x-intercept(s) at:

A only (8.95, 0)

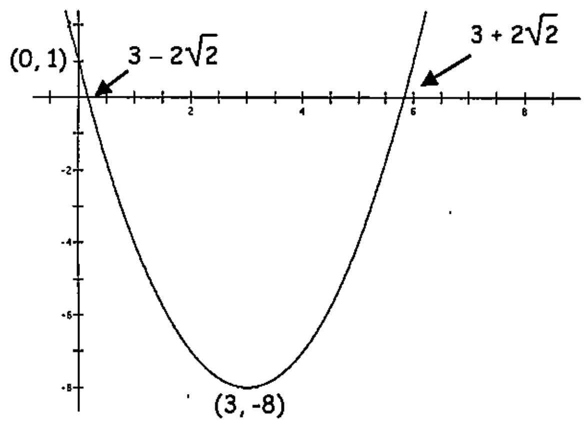
B only (-8.95, 0)

C (8.95, 0) and (29.04, 0)

D (8.95, 0) and (29.05, 0)

E (8.95, 0) and (-8.95, 0)

3. Given the graph of $y = x^2 - 6x + 1$ shown below the solution for $x^2 - 6x + 1 > 0$ is:



A $x \in (-\infty, 3 - 2\sqrt{2}\,] \cup [3 + 2\sqrt{2}, \infty)$

B $x \in (-\infty, 3 - 2\sqrt{2}\,) \cup (3 + 2\sqrt{2}, \infty)$

C $x \in (3 - 2\sqrt{2}, 3 + 2\sqrt{2}\,)$

D $x \in [3 - 2\sqrt{2}, 3 + 2\sqrt{2}\,]$

E $x \in [-8, \infty)$

end points are <u>not</u> included

4. Which of the following linear expressions could not possibly be a factor of $x^3 + kx^2 - x + 12$

A $x - 1$

B $x + 2$

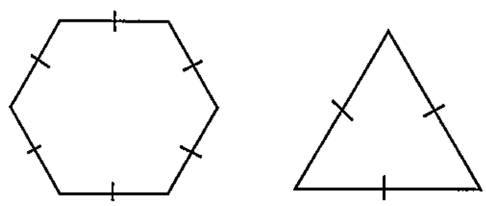
C $x - 3$

D $x + 4$

E $x - 5$

5 is not a factor of 12

1. A regular polygon is a closed shape with straight edges all of the same length.
A couple of examples are shown beside.



If a regular polygon has n sides then the size of each interior angle, in degrees, is given by the formula $\frac{180(n-2)}{n}$.

a) Write in terms of n, the size of each interior angle of a regular polygon with n + 5 sides.

$$\frac{180(n+5-2)}{n+5}$$

$$= \frac{180(n+3)}{n+5}$$

(1 mark)

b) Consider two regular polygons, named polygon A and polygon B. Polygon A has n sides and polygon B has n + 5 sides. If each interior angle of polygon A is 1° less than each interior angle of polygon B, show that n satisfies the equation $n^2 + 5n - 1800 = 0$.

$$B - A = 1$$

$$\frac{180(n+3)}{n+5} - \frac{180(n-2)}{n} = 1$$

$$180n(n+3) - 180(n-2)(n+5) = n(n+5)$$

$$180n^2 + 540n - 180(n^2+3n-10) = n^2+5n$$

$$\cancel{180n^2} + \cancel{540n} - \cancel{180n^2} - \cancel{540n} + 1800 = n^2 + 5n$$

$$n^2 + 5n - 1800 = 0$$

(3 marks)

c) Solve $n^2 + 5n - 1800 = 0$ and hence find the number of sides of polygon B.

$n^2 + 5n - 1800 = 0$

$n = -45$ or 40

since $n > 0$, $n = 40$

Polygon B has 45 sides

(3 marks)

2. Consider the following two functions, where k is an unknown constant.

$y = kx^2 - 3x + 1$

$y = kx + 2$

a) Show that at the point(s) where the graphs of these functions intersect x must satisfy the equation $kx^2 - (3 + k)x - 1 = 0$.

$kx^2 - 3x + 1 = kx + 2$

$kx^2 - 3x - kx + 1 - 2 = 0$

$kx^2 - (3+k)x - 1 = 0$

(2 marks)

b) Show that the discriminant of the quadratic $kx^2 - (3 + k)x - 1 = 0$ is equal to $k^2 + 10k + 9$.

$\Delta = b^2 - 4ac$

$= [-(3+k)]^2 - 4 \times k \times -1$

$= k^2 + 6k + 9 + 4k$

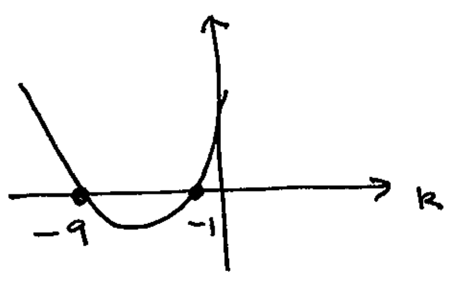
$= k^2 + 10k + 9$

(2 mark)

c) Hence, find the values of **k** for which the graphs of the two functions given at the start of this problem intersect at least once.

$\Delta \geqslant 0$

$k^2 + 10k + 9 \geqslant 0$



$(-\infty, -9] \cup [-1, \infty)$

(4 marks)

3. Delta works as a stunt double on television shows. For one stunt she has to jump from the top of a tall building. Her height, h metres, above the ground after t seconds from when she jumps is given by $h = 21 + t - 5t^2$.

a) Find the height of the building.

$t = 0 \quad h = 21$

Building is 21m high

(1 mark)

b) Find Delta's height above the ground after

i) 1 second

$t = 1$

$h = 17$ m

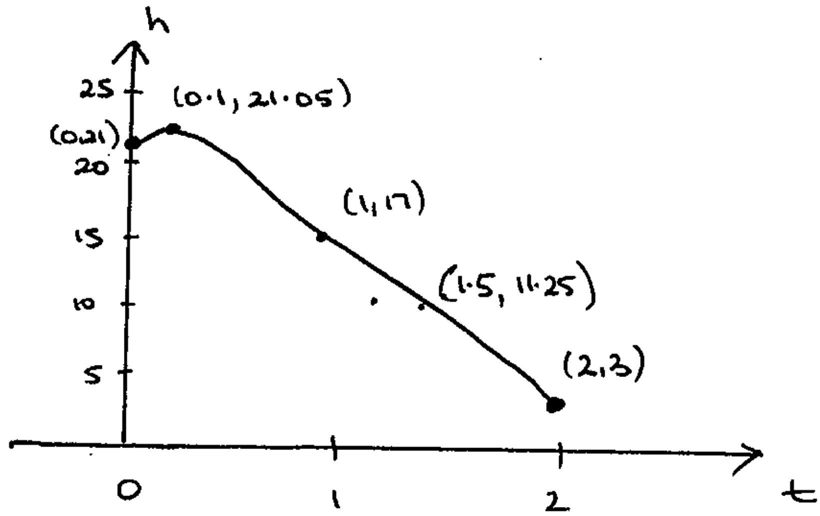
ii) 1.5 seconds

$t = 1.5$

$h = 11.25$

(2 marks)

c) Sketch the graph of h against t for $0 \le t \le 2$. Label coordinates of the end points and a turning point, giving approximate values correct to 1 decimal place.



(3 marks)

d) Find the maximum height above the ground that Delta reached during this stunt. Give your answer in metres correct to 2 decimal places.

21.05 m

(1 mark)