MM12 Assignment 3 Cubics and Functions

Name: ANSWERS.

Skills (Section A and B) .... / 33 **Analysis (Section C) .... / 24**

Section A: Short Answer **No Calculators Allowed**

NOTE: Exact answers are required unless instructed otherwise within the question.

1. **(a)** Fully factorise $P(x) = x^3 + x^2 - 17x + 15$

$P(1) = 1 + 1 - 17 + 15 = 0$

$x - 1$ is a factor

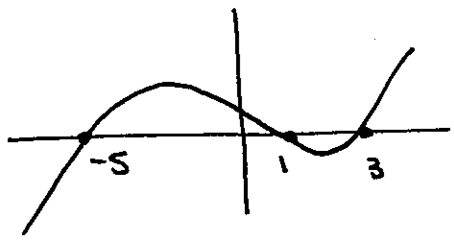
$$\begin{array}{r|l} & x^2 + 2x - 15 \\ \hline x - 1 & x^3 + x^2 - 17x + 15. \\ & \underline{x^3 - x^2} \\ & \quad 2x^2 - 17x \\ & \quad \underline{2x^2 - 2x} \\ & \qquad -15x + 15 \\ & \qquad \underline{-15x + 15} \\ & \qquad\qquad 0 \end{array}$$

$(x-1)(x^2 + 2x - 15)$

$= (x-1)(x+5)(x-3)$

(3 marks)

(b) Hence solve the equation $x^3 + x^2 - 17x + 15 = 0$

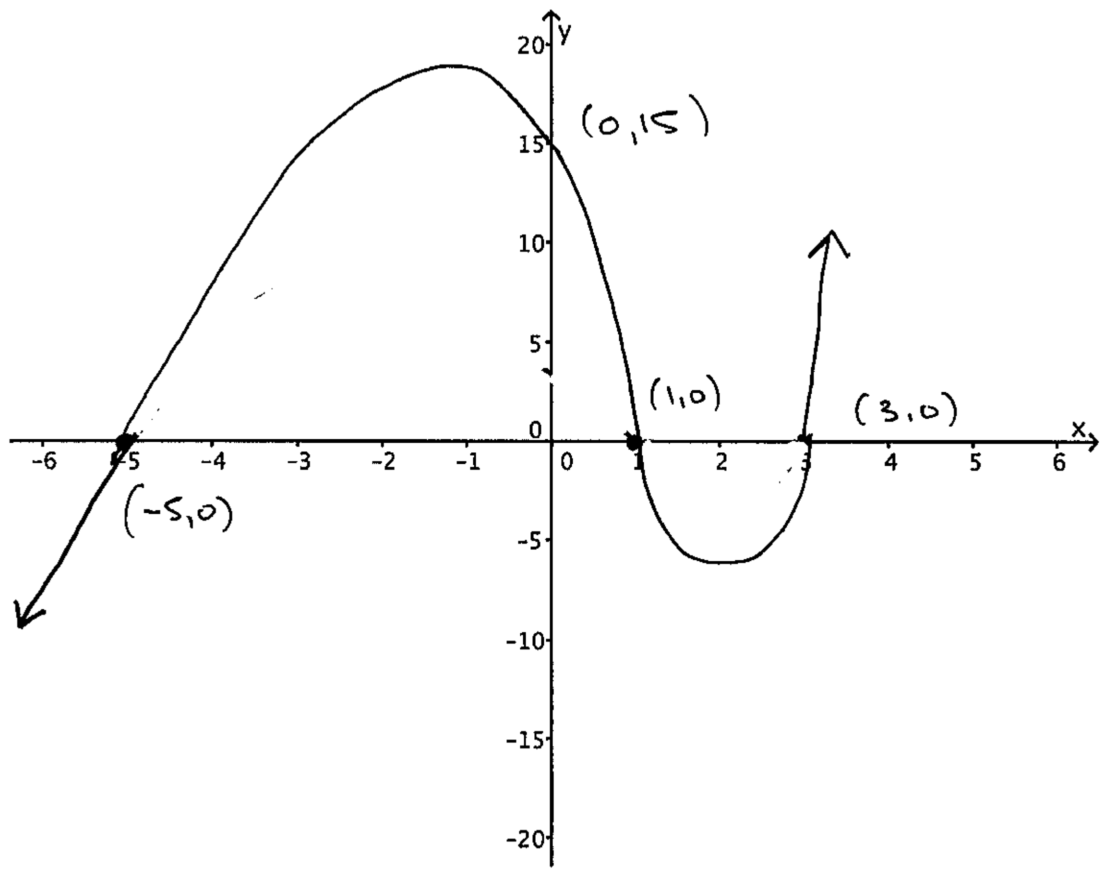


(2 marks)

$(x-1)(x+5)(x-3) = 0$

$x = 1, -5, 3$

(c) Sketch the graph of $y = x^3 + x^2 - 17x + 15$ showing the coordinates of the *x* and *y* intercepts. You <u>do not need</u> to show the coordinates of the turning points.



(3 marks)

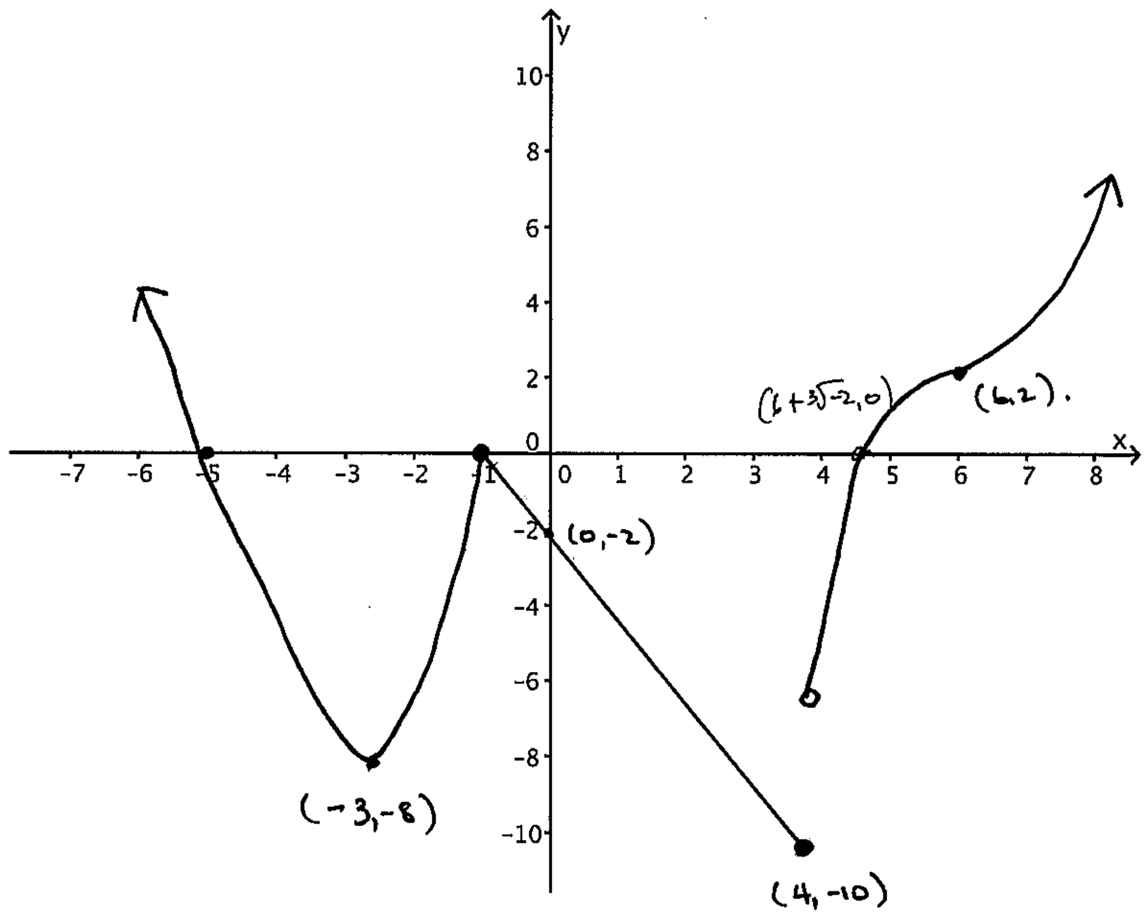
(d) Hence find the interval for which $x^3 + x^2 - 17x + 15 < 0$

$x \in (-\infty, -5) \cup (1, 3)$

(1 mark)

2. For the given hybrid function $g(x)=\begin{cases}2(x+5)(x+1), & x<-1\\ -2(x+1), & -1\le x\le 4\\ (x-6)^3+2, & x>4\end{cases}$

calculate all key points that will enable you to sketch a detailed graph of $g(x)$ on the set of axes provided below.



(7 marks)

Work Space:

parabola T.P. $x=-3$

$y=-8$

line $x=-1$ $y=0$

$x=4$ $y=-10$

$x=0$ $y=-2$

cubic $x=4$ $y=-6$

SPOI $(6,2)$

x int $(6+\sqrt[3]{-2},0)$

3. A circle has rule $x^2 + 2x + y^2 - 6y + 1 = 0$.

(a) Show that this rule can be written in the form $(x + 1)^2 + (y - 3)^2 = 9$.

$$(x^2+2x+1)+(y^2-6y+9)-1-9+1=0$$

$$(x+1)^2+(y-3)^2-9=0$$

$$(x+1)^2+(y-3)^2=9$$

(3 marks)

(b) State the coordinates of:

(i) the centre of the circle and

$(-1,3)$

(ii) its radius

3

(2 marks)

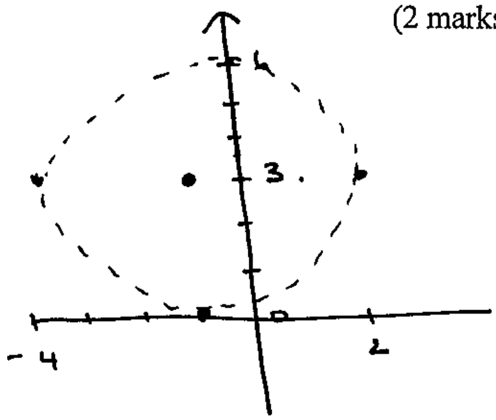
(c) How many x-intercepts does the circle have? Why?
Draw a diagram to assist.

x int, $y=0$

$$(x+1)^2+(-3)^2=9$$

$$(x+1)^2=0$$

$x=-1$ is the only solution



(3 marks)

(d) State:

(i) the domain and

$x \in [-4,2]$

(ii) the range of this relation.

$y \in [0,6]$

(2 marks)

End of Section A

Total: /26

Full Name: ____________________

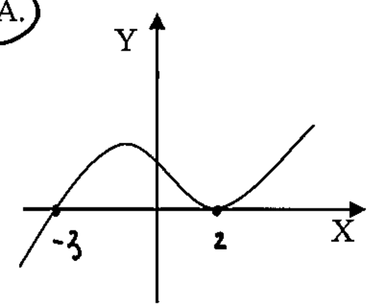
Section B: Multiple Choice Questions **Calculators Are Allowed**

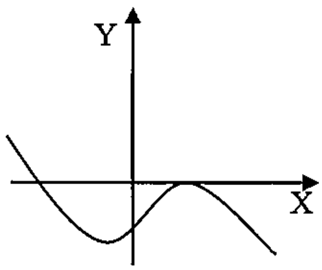
Circle Correct Response

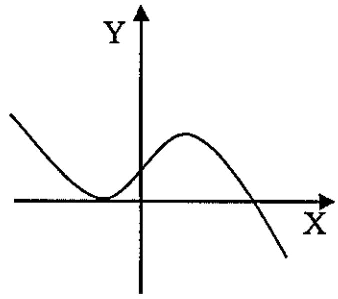
1. The graph with equation $y = -3(x+1)^2(2x-3)$

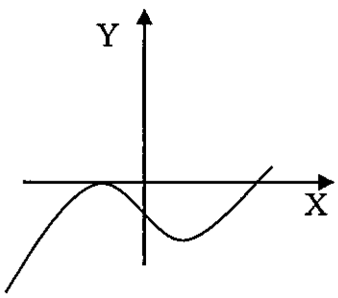
A. has 3 distinct x-intercepts ×
B. has a turning point at $x = -1$ ✓
C. has a turning point at $x = 1.5$ ×
D. is the graph of $y = x^3$ translated 1 unit to the left and 1.5 units down.
E. all of the above

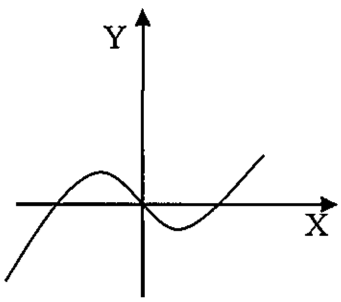
2. The graph of $y = (x-2)^2(x+3)$ could be:

A.


B.


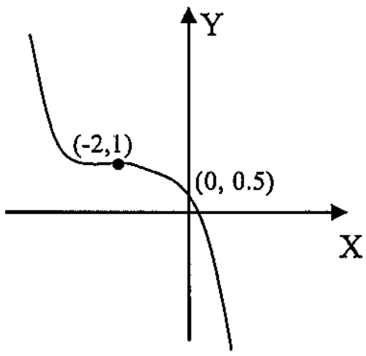
C.


D.


E.


3. The graph shown has rule

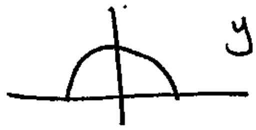
A. $y = (x-2)^3 + 1$ ×
B. $y = (x+2)^3 + 0.5$ ×
C. $y = -\frac{1}{16}(x+2)^3 + 1$
D. $y = (x-1)^3 + 0.5$ ×
E. $y = (x+2)^3 + 1$ ×



4. If $f(x) = 5(x+2)^2 + 7$, then $f(-1)$ equals

A. 52
B. 17
C. 27
D. 12
E. 54

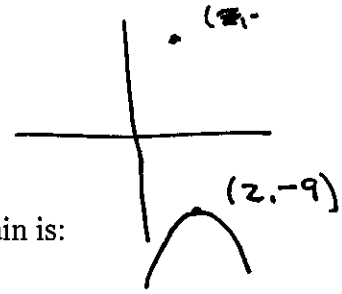
5. Which of the following functions is **not** a one-to-one function?
   A. $y = 3x^2 - 3x, x > 3$
   B. $y = \sqrt{3 - x^2}$
   C. $y = 3x^2, x > 0$
   D. $y = 3\sqrt{x}$
   E. $y = 3x$



$y = \sqrt{3-x^2}$

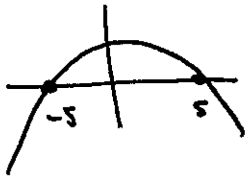
6. The function $f: R \to R$, where $f(x) = -[(x-2)^2 + 9]$, has range
   A. $(-\infty, 9)$
   B. $(-\infty, 2)$
   C. $(-\infty, -9]$
   D. $[9, \infty)$
   E. $[-9, \infty)$

$f(x) = -(x-2)^2 - 9$



7. For the function with rule $f(x) = \sqrt{25 - x^2}$, the maximal domain is:
   A. $[0, 25]$
   B. $[-5, 5]$
   C. $(-\infty, 5]$
   D. $(-5, 5)$
   E. $[-5, \infty)$

$25 - x^2 \geq 0$



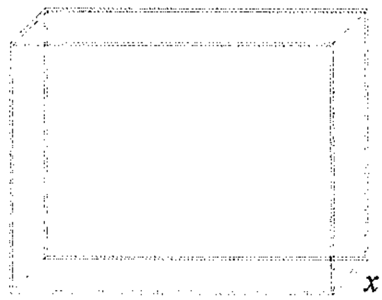
End of Section B

Total: /7

Section C: Analysis: **Calculator Allowed**

Exact answers are required unless instructed otherwise within the question

1. A cuboid (rectangular prism) has dimensions x metres, h metres and $4x$ metres, as shown on the diagram. The edges of the cuboid are made from 640 m of wire.



(a) Find h in terms of x.

$$20x + 4h = 640$$

$$h = 160 - 5x$$

(2 marks)

(b) Show that the volume, V m^3, of the cuboid is equal to $20x^2(32-x)$.

$$\begin{aligned} V &= x \times 4x \times (160 - 5x) \\ &= 4x^2 (160 - 5x) \\ &= 4x^2 \times 5(32 - x) \\ &= 20x^2(32 - x) \end{aligned}$$

(2 marks)

(c) Find V when $x = 11$.

$$V = 50820 \text{ m}^3.$$

(2 marks)

(d) Find the possible values of x for the cuboid to exist.

$$0 < x < 32$$

Since $x > 0$ and $160 - 5x > 0$

(2 marks)

(e) Find the possible value(s) of x, correct to 2 decimal places, when V = 60 000.

$60{,}000 = 20x^2(32-x)$

$x = 12.35$ or 28.24 or -8.60

$x = 12.35$ or 28.24 since $0 < x < 32$

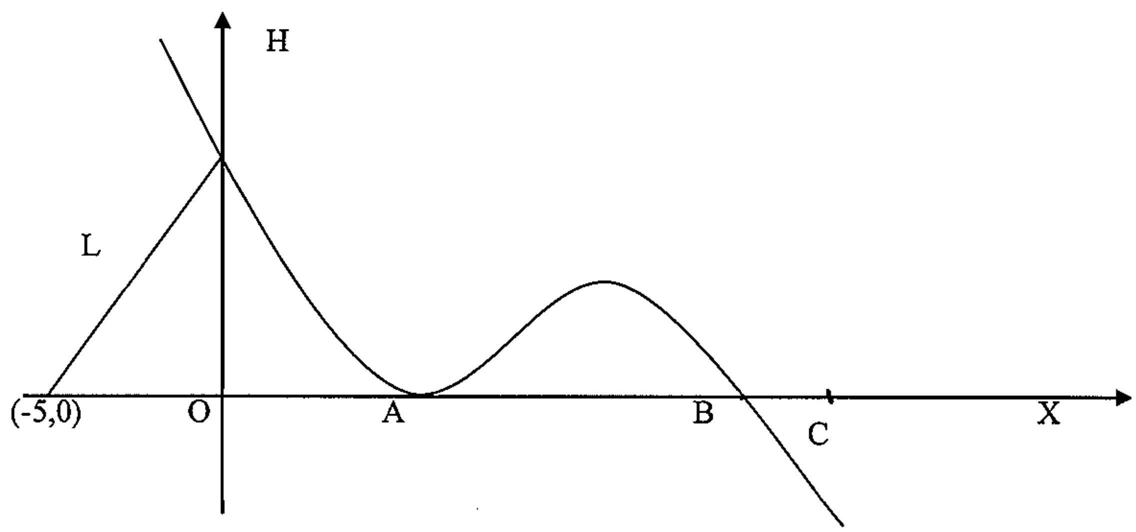
(2 marks)

(f) Find the maximum volume of the cuboid and the value of x for which it occurs. Give your answer correct to 1 decimal place.

$x = 21.3$ m. $V = 970897$ m^3

(2 marks)

2. It is proposed to have a slippery dip at the new aquatic complex. The diagram below shows that the slippery dip is to be based on a cubic function with rule $\boldsymbol{H = -0.2(x^3 - 9x^2 + 24x - 20)}$, where $\boldsymbol{H}$ metres is the height of the slippery dip above the water level at a distance of $\boldsymbol{x}$ metres from the edge of the pool at the origin. **L** represents a ladder that goes to the top of the slippery dip.



(a) What is the height of the slippery dip at the edge of the pool?

4 m

(2 marks)

(b) (i) Show algebraically that $\boldsymbol{H = -0.2(x-2)^2(x-5)}$.

Let $P(x) = x^3 - 9x^2 + 24x - 20$

$P(2) = 8 - 36 + 48 - 20 = 0$

$x - 2$ is a factor

$$\begin{array}{r|l} & x^2 - 7x + 10 \\ \hline x-2 & x^3 - 9x^2 + 24x - 20 \\ & x^3 - 2x^2 \\ \hline & -7x^2 + 24x \\ & -7x^2 + 14x \\ \hline & 10x - 20 \\ & 10x - 20 \\ \hline & 0 \end{array}$$

$(x-2)(x^2 - 7x + 10)$

$(x-2)(x-2)(x-5)$

$(x-2)^2(x-5)$

$H = -0.2\, P(x)$

$H = -0.2\,(x-2)^2(x-5)$

(3 marks)

(ii) Find the coordinates of the points marked $\boldsymbol{A}$ and $\boldsymbol{B}$ where the slippery dip is level with the water.

$A\ (2, 0)$ $\quad B\ (5, 0)$

(2 marks)

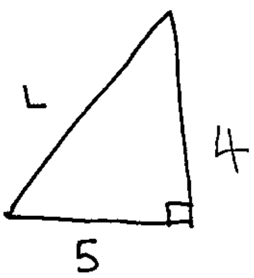
(c) After the slippery dip enters the water at $\boldsymbol{B}$, it continues for 1 metre horizontally. What is the depth of the water at the end of the slippery dip at $\boldsymbol{C}$?

$C\ (6, -3.2)$.

Depth is 3.2 m.

(1 marks)

(d) Find the length, **L** metres, of the ladder used to climb **onto** the slippery dip at the edge of the pool, where $x = 0$. Give your answer correct to 1 decimal point.



$L = \sqrt{16 + 25}$

$= 6.4$ m

(3 marks)

End of Section C

Total: /24