Neap:

Trial Examination 2019

VCE Mathematical Methods Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Number of questions	Number of questions to be answered	Number of marks
8	8	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 9 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = (3\log_e(x))^2$.

Find $\dfrac{dy}{dx}$. 1 mark

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b. Let $f(x) = \dfrac{\cos(3x)}{x^2}$.

Find $f'\left(\dfrac{2\pi}{3}\right)$. 3 marks

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Question 2 (4 marks)

Let $f'(x) = 1 + \dfrac{2}{e^x}$.

a.   Given $f(0) = 4$, find the rule for $f(x)$.   2 marks

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b.   Find the equation of the tangent to $y = f(x)$ at the y-intercept.   2 marks

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Question 3 (2 marks)

Solve $\tan^2(2x) - 1 = 0$ for x, where $x \in [0, \pi]$.

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Question 4 (5 marks)

a. Solve the equation $5 - 2\log_e(x) = \log_e(\sqrt{x})$ for x. 2 marks

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b. Find the value(s) of k for which the equation $5^x = 125^{k - x^2}$ has no solutions for x. 3 marks

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Question 5 (6 marks)

Let $f: (-2, 4] \to R, f(x) = \sqrt{4 - x} - 1$.

a. Sketch the graph of $y = f(x)$ on the axes below. Label the axis intercepts and endpoints with their coordinates. 3 marks
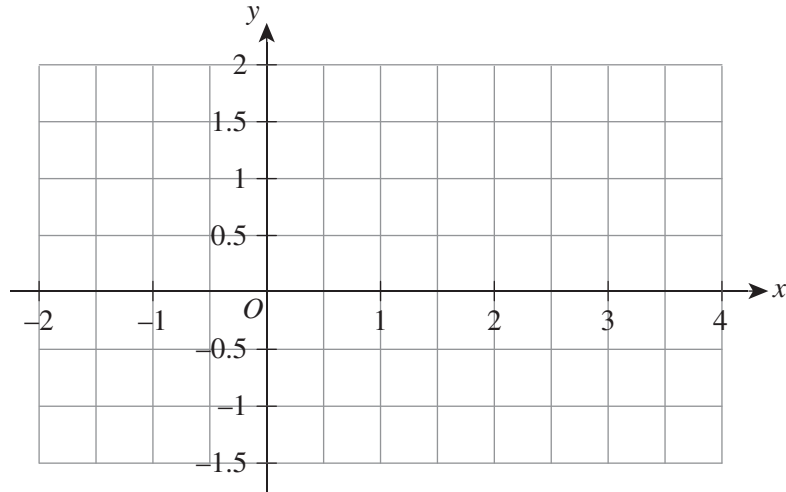
b. Find the area bounded by the graph of $y = f(x)$, the x-axis and the y-axis. 3 marks

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Question 6 (7 marks)

Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} kx^2(1-x) & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

a. Show that $k = 12$. 2 marks

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b. Find the mean of X. 2 marks

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c. Let Y be a continuous random variable with the probability density function

$$g(x) = \begin{cases} qx^2(p-x) & 0 \le x \le p \\ 0 & \text{elsewhere} \end{cases}$$

Given that $p, q > 0$ and the mean of Y is equal to $\dfrac{3}{10}$, find the value of p and q. 3 marks

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Question 7 (5 marks)

For events A and B, from a sample space where A and B are both independent events, it is known that $\Pr(A|B) = \frac{1}{8}$ and $\Pr(B|A) = \frac{1}{3}$.

a. Show that $\Pr(A) = \frac{3}{8}\Pr(B)$.   1 mark

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b. If $\Pr(A \cup B) = \frac{3}{4}$, find $\Pr(B)$.   3 marks

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c. Calculate $\Pr(A' \cap B)$ where A' represents the complement of A.   1 mark

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Question 8 (7 marks)

The graph of the general hyperbola $f(x) = \dfrac{k}{x}$, where $k > 0$, is shown below. The point A is located on the graph with coordinates $\left(a, \dfrac{k}{a} \right)$, where $a > 0$.
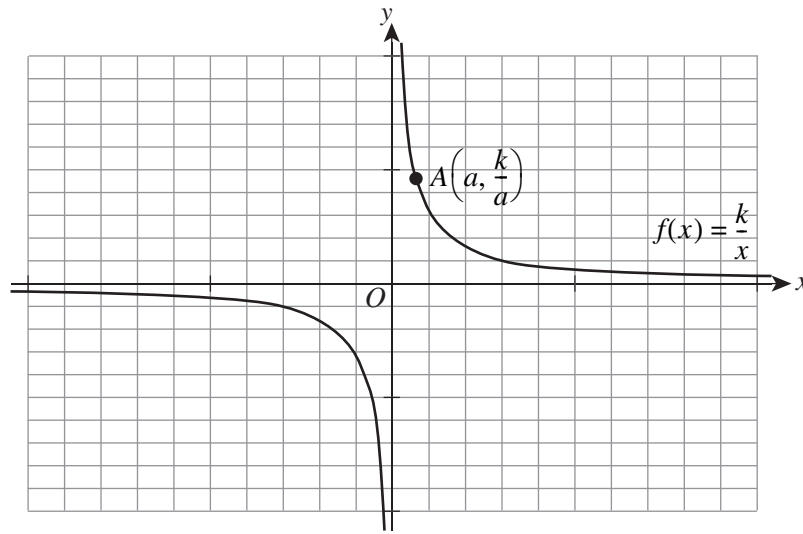
The transformation defined by $T : R^2 \to R^2$, $T\left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ is applied to the point A to create an image point A'.

a. State the coordinates of A' and plot its coordinates relative to A on the graph above. 1 mark

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b. Show that the distance between the points A and A' can be expressed as $\dfrac{2\sqrt{a^4 + k^2}}{a}$. 2 marks

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c. **i.** Find the minimum distance between the two branches of the hyperbola given
by $y = f(x)$. Give your answer in terms of k.   3 marks

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ii. Find the value(s) for k where the minimum distance between the two branches of
the hyperbola is less than 10.   1 mark

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END OF QUESTION AND ANSWER BOOKLET