

The Mathematical Association of Victoria

Trial Examination 2019

**MATHEMATICAL METHODS**

**Trial Written Examination 2 - SOLUTIONS**

**SECTION A: Multiple Choice**

Question	Answer	Question	Answer
1	E	11	D
2	D	12	A
3	C	13	E
4	A	14	C
5	C	15	C
6	E	16	D
7	B	17	A
8	E	18	C
9	B	19	E
10	C	20	E

**Question 1**

**Answer E**

$$y = -3 \cos\left(2\pi x - \frac{\pi}{2}\right)$$

$$\text{period} = \frac{2\pi}{2\pi} = 1$$

$$\text{Amplitude} = 3$$

**Question 2**

**Answer D**

$f(x) = \sin(x)$  for  $x \in [0, 2\pi]$  and  $g(x) = (x+1)^2$  for its maximal domain which is  $\mathbb{R}$ .  
 $g(f(x))$  exists, with the domain of  $g(f(x))$  equalling the domain of  $f$ , which is  $[0, 2\pi]$ .

**Question 3**

**Answer C**

Solve  $2\sin(2x) = 1$  for  $x$ .

$$\sin(2x) = \frac{1}{2}$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \dots$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \dots$$

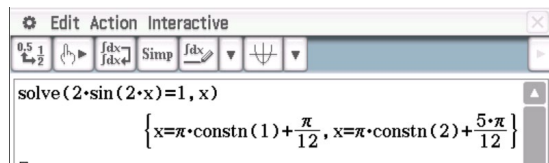
General solution

The period is  $\pi$ .

$$x = \frac{\pi}{12} + n\pi, n \in \mathbb{Z}$$

or

$$x = \frac{5\pi}{12} + n\pi, n \in \mathbb{Z}$$

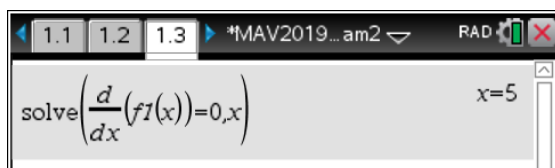
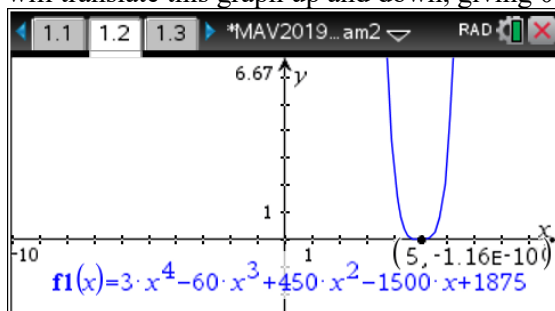


**Question 4**

**Answer A**

$$f(x) = 3x^4 - 60x^3 + 450x^2 - 1500x + k + 1875$$

The graph of  $f_1(x) = 3x^4 - 60x^3 + 450x^2 - 1500x + 1875$  has only one turning point. The value of  $k$  will translate this graph up and down, giving 0, 1 or 2  $x$ -intercepts.



**Question 5**

**Answer C**

$$f: \left[ \frac{3}{2}, \infty \right) \rightarrow \mathbb{R}, f(x) = -\sqrt{2x-3} + 4$$

$$\text{Let } y = -\sqrt{2x-3} + 4.$$

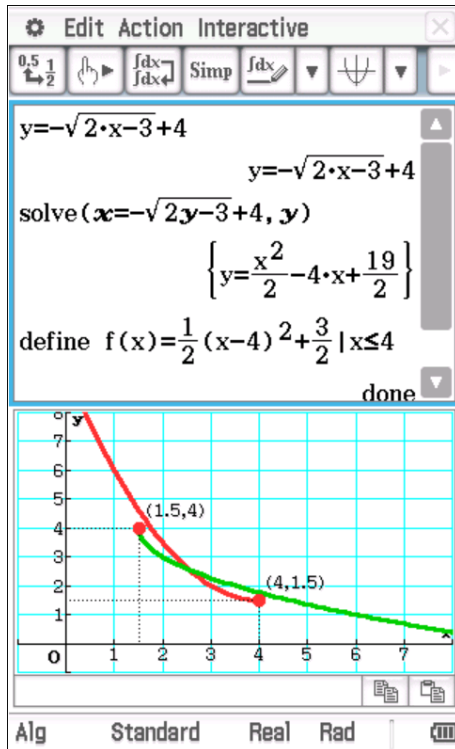
Inverse: swap  $x$  and  $y$  and solve for  $y$ .

$$x = -\sqrt{2y-3} + 4$$

$$y = \frac{1}{2}(x-4)^2 + \frac{3}{2}$$

The range of  $f$  is  $(-\infty, 4]$ . The domain of the inverse is  $(-\infty, 4]$ .

$$f^{-1} : (-\infty, 4] \rightarrow \mathbb{R}, f^{-1}(x) = \frac{1}{2}(x-4)^2 + \frac{3}{2}$$

**Question 6****Answer E**

$$y = e^x \text{ is } y_T = 2e^{-2x} + 3$$

$$y_1 = 2e^x \quad \text{a dilation from the } x\text{-axis by a factor of 2}$$

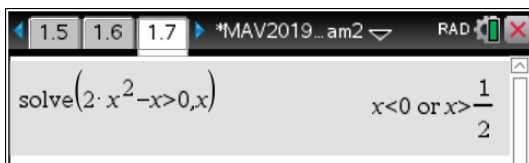
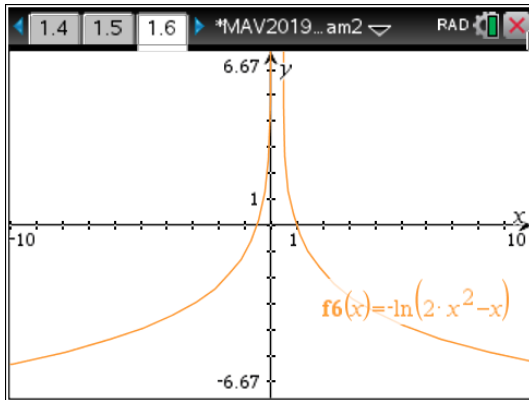
$$y_2 = 2e^{2x} \quad \text{followed by a dilation from the } y\text{-axis by a factor of 0.5}$$

$$y_3 = 2e^{-2x} \quad \text{then a reflection in the } y\text{-axis and}$$

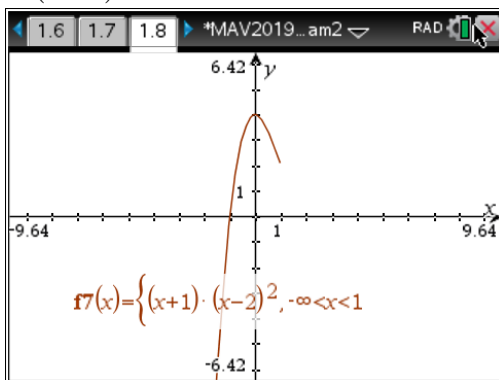
$$y_T = 2e^{-2x} + 3 \quad \text{a translation in the positive direction of the } y\text{-axis by 3 units.}$$

**Question 7****Answer B**

The graph of  $y = -\log_e(2x^2 - x)$  has two vertical asymptotes,  $x = 0$  and  $x = \frac{1}{2}$ .

**Question 8****Answer E**

$l: (-\infty, 1) \rightarrow \mathbb{R}$ ,  $l(x) = (x+1)(x-2)^2$  is a many to 1 function. So the inverse is not a function.

**Question 9****Answer B**

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$x' = -x, \quad x = -x'$$

$$y' = 0.5y + 2, \quad y = 2y' - 4$$

$$y = \frac{3}{x}$$

$$2y' - 4 = -\frac{3}{x'}$$

$$y' = -\frac{3}{2x'} + 2$$

The equation of the image is

$$y = -\frac{3}{2x} + 2$$

The screenshot shows the TI-84 Plus calculator's 'Edit Action Interactive' window. The window title is 'Edit Action Interactive'. The main area contains the following text:

$$\begin{bmatrix} -1 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -x \\ \frac{y}{2} + 2 \end{bmatrix}$$

solve(X=-x, x)

$$\{x=-X\}$$

solve(Y=\frac{y}{2}+2, y)

$$\{y=2 \cdot Y - 4\}$$

solve(2 \cdot Y - 4 = \frac{3}{-X}, Y)

$$\{Y = \frac{-3}{2 \cdot X} + 2\}$$

□

At the bottom, there are tabs for 'Alg', 'Standard', 'Real', and 'Rad', along with a calculator icon.

OR

$$y_1 = -\frac{3}{x} \quad \text{reflection in the } y\text{-axis}$$

$$y_2 = -\frac{3}{2x} \quad \text{dilation by a factor of 0.5 from the } x\text{-axis}$$

$$y_3 = -\frac{3}{2x} + 2 \quad \text{translation of 2 units up}$$

**Question 10**

**Answer C**

$$f(x+2y) = f(x)f(2y)$$

$$f(x) = e^{2x}$$

$$\begin{aligned} LHS &= f(x+2y) = e^{2(x+2y)} \\ &= e^{2x} \times e^{4y} \\ &= f(x)f(2y) = RHS \end{aligned}$$

The screenshot shows the TI-84 Plus calculator's 'Edit Action Interactive' window. The window title is 'Edit Action Interactive'. The main area contains the following text:

$$\text{define } f(x) = e^{2x}$$

done

$$\text{judge}(f(x+2y) = f(x)f(2y))$$

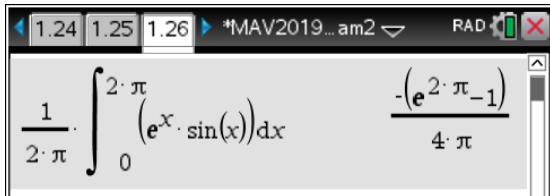
TRUE

At the bottom, there are tabs for 'Alg', 'Standard', 'Real', and 'Rad', along with a calculator icon.

**Question 11**                      **Answer D**

$f(x) = e^x \sin(x)$  over the interval  $[0, 2\pi]$

$$\begin{aligned} \text{Average value} &= \frac{1}{2\pi} \int_0^{2\pi} (e^x \sin(x)) dx \\ &= -\frac{e^{2\pi} - 1}{4\pi} \end{aligned}$$

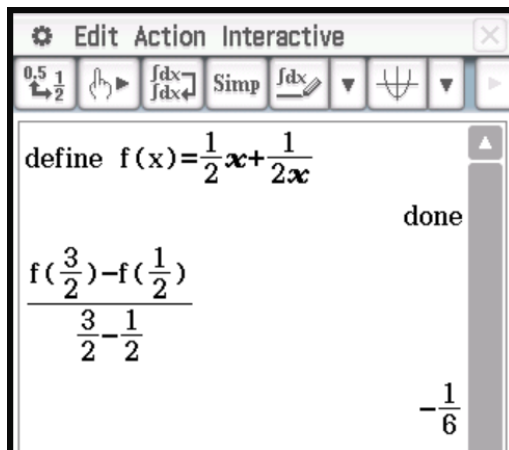


**Question 12**                      **Answer A**

The average rate of change of  $y = \frac{1}{2}x + \frac{1}{2x}$  from  $x = \frac{1}{2}$  to  $x = \frac{3}{2}$ .

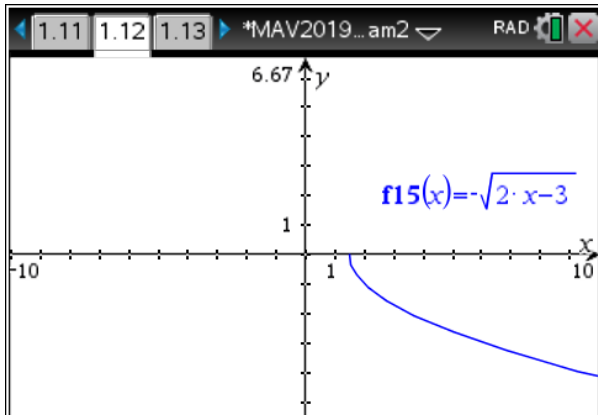
Let  $f(x) = \frac{1}{2}x + \frac{1}{2x}$ .

$$\text{Average rate of change} = \frac{f\left(\frac{3}{2}\right) - f\left(\frac{1}{2}\right)}{\frac{3}{2} - \frac{1}{2}} = -\frac{1}{6}$$



**Question 13**                      **Answer E**

$f: \left(\frac{3}{2}, \infty\right) \rightarrow \mathbb{R}$ ,  $f(x) = -\sqrt{2x-3}$  is a strictly decreasing function over the given domain.

**Question 14****Answer C**

$$a = 5t + 3$$

$$v = \int (5t + 3) dt$$

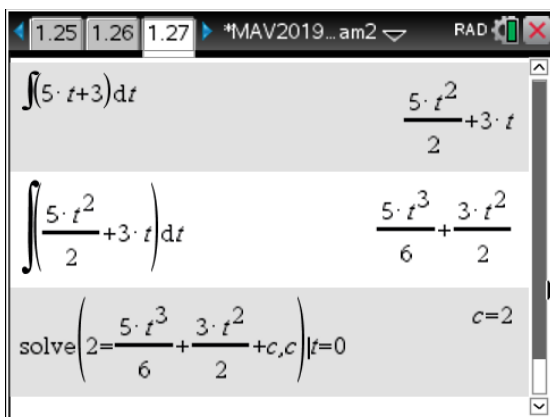
$$= \frac{5t^2}{2} + 3t + c, c = 0$$

$$= \frac{5t^2}{2} + 3t$$

$$x = \int \left( \frac{5t^2}{2} + 3t \right) dt$$

$$= \frac{5t^3}{6} + \frac{3t^2}{2} + d, \text{ when } t = 0, x = 2$$

$$x = \frac{5t^3}{6} + \frac{3t^2}{2} + 2$$



**Question 15****Answer C**Let  $f(x) = x^3$ Area of the rectangles =  $0.2(f(1) + f(1.2) + f(1.4) + f(1.6) + f(1.8)) = 3.08$ 

$$\text{Actual area} = \int_1^2 f(x) dx = \frac{15}{4}$$

$$\frac{3.08}{3.75} \times 100\% \approx 82.13\%$$

1.1 \*MAVExam2MC RAD

Define  $f(x)=x^3$  Done

$$\frac{0.2 \cdot (f(1)+f(1.2)+f(1.4)+f(1.6)+f(1.8))}{\int_1^2 x^3 dx} \cdot 100$$

82.1333333333

**Question 16****Answer D**

$x$	0	1	2	3
$\Pr(X = x)$	0.2	$a$	$b$	$c$

$$0.2 + a + b + c = 1 \dots(1)$$

$$a + 2b + 3c = 1.9 \dots(2)$$

$$a + 4b + 9c - 1.9^2 = 1.29 \dots(3)$$

$$a = 0.1, b = 0.3, c = 0.4$$

Edit Action Interactive

0.5 1/2 (h) fdx/fdx Simp fdx

$$\begin{cases} 0.2 + a + b + c = 1 \\ a + 2b + 3c = 1.9 \\ a + 4b + 9c - 1.9^2 = 1.29 \end{cases} \quad a, b, c$$

$$\left\{ a = \frac{1}{10}, b = \frac{3}{10}, c = \frac{2}{5} \right\}$$

□



**Question 17****Answer A**

$$np = 20, np(1-p) = 10$$

$$n = 40, p = \frac{1}{2}$$

$$\Pr(X = 15 | X < 18) = \frac{\Pr(X = 15)}{\Pr(0 \leq X \leq 17)} = 0.170 \text{ correct to three decimal places}$$

**Question 18****Answer C**

$$\Pr(A) = 0.7 \text{ and } \Pr(A' \cap B') = 0.2$$

For independent events  $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A) \times \Pr(B)$$

Solve  $0.8 = 0.7 + \Pr(B) - 0.7 \times \Pr(B)$  for  $\Pr(B)$

$$\Pr(A \cap B) = \frac{7}{10} \times \frac{1}{3} = \frac{7}{30}$$

**Question 19****Answer E**

$$f(x) = \frac{1}{4\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-3}{4}\right)^2}, \quad x \in R$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad x \in R$$

mean is 3, variance is 16

**Question 20****Answer E**

Solve  $\int_0^m (e^{x+2}) dx = \frac{1}{2}$  for  $m$

The screenshot shows a CAS interface with the following elements:

- Window title: Edit Action Interactive
- Toolbar: Contains icons for fraction conversion (0.5 to 1/2), undo, redo, differentiation (dx/dx), integration (dx/dx), simplify (Simp), solve (fdx), and other mathematical symbols.
- Input area:  $\text{solve}\left(\int_0^m e^{x+2} dx = \frac{1}{2}, m\right)$
- Output area:  $\left\{m = \ln\left(e^2 + \frac{1}{2}\right) - 2\right\}$

**SECTION B****Question 1**

a. Total Surface Area =  $4 \times \frac{1}{2}xs + x^2$  with TSA given as  $1100 \text{ cm}^2$

$$4 \times \frac{1}{2}xs + x^2 = 1100$$

$$2xs + x^2 = 1100$$

Giving  $s = \frac{1100 - x^2}{2x}$  as required **1M Show that**

b. Right angled triangle formed by vertical height,  $h$ , half the side length,  $x$  and slant height,  $s$ .

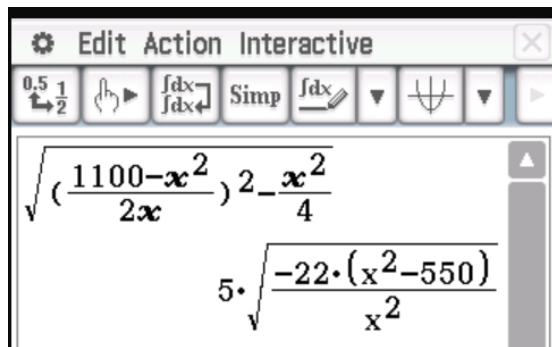
$$\left(\frac{x}{2}\right)^2 + h^2 = s^2$$

$$\text{Giving } h^2 = s^2 - \frac{x^2}{4}$$

$$\text{From } s = \frac{1100 - x^2}{2x} \text{ we have } h^2 = \left(\frac{1100 - x^2}{2x}\right)^2 - \frac{x^2}{4}$$

$$\text{Positive vertical height, } h = \sqrt{\left(\frac{1100 - x^2}{2x}\right)^2 - \frac{x^2}{4}}$$

$$\text{Simplified } h = 5\sqrt{\frac{-22(x^2 - 550)}{x^2}} = \frac{5\sqrt{-22(x^2 - 550)}}{x} \quad (\text{accept different forms}) \quad \mathbf{1A}$$



c. Volume of square based pyramid =  $\frac{1}{3}x^2h$

$$\text{giving } V = \frac{1}{3}x^2 \times 5\sqrt{\frac{-22(x^2 - 550)}{x^2}}$$

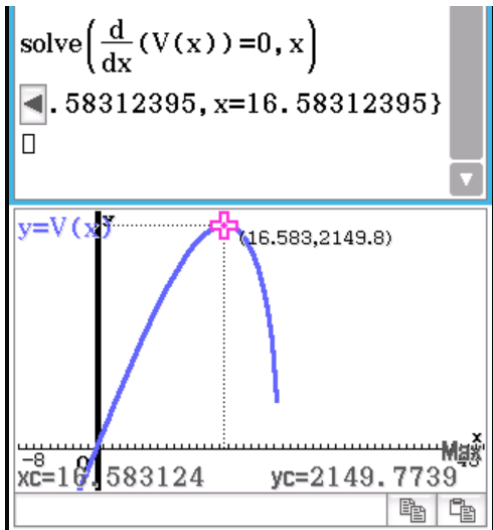
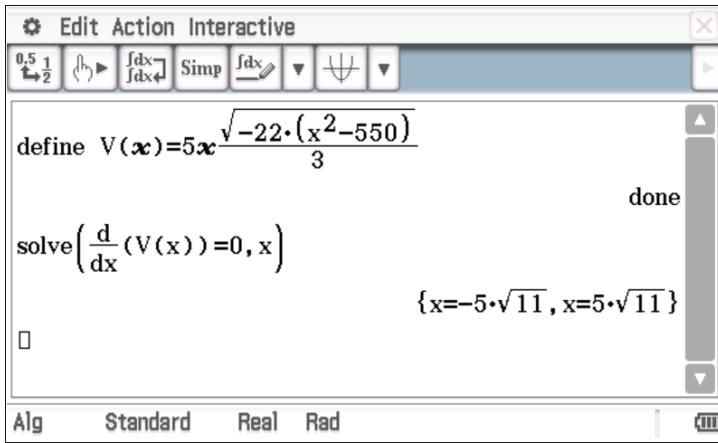
$$\text{for } x > 0, V = \frac{5x\sqrt{-22(x^2 - 550)}}{3} \quad \mathbf{1M Show that}$$

d. For maximum volume,  $\frac{dV}{dx} = 0$

gives  $x = \pm 5\sqrt{11}$

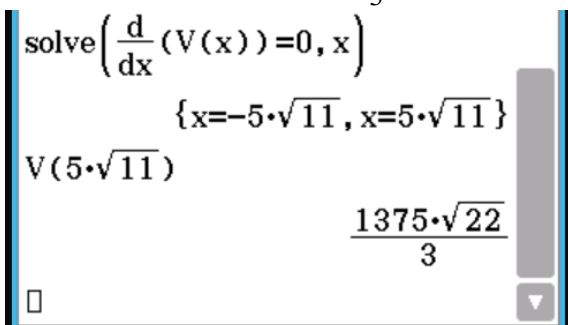
from graph of  $V$  against  $x$ , and  $x > 0$ , maximum volume occurs at  $x = 5\sqrt{11}$

1A



e. Maximum volume,  $V = \frac{1375\sqrt{22}}{3}$

1A



f. Given  $\frac{dV}{dt} = 0.2t$

$$V = \int (0.2t) dt = 0.1t^2 + c$$

$$t = 0, V = 0, c = 0$$

$$V = 0.1t^2 \quad \mathbf{1M}$$

Equating  $0.1t^2 = \frac{1375\sqrt{22}}{3}$  for full volume

Gives  $t = 146.62$  minutes correct to two decimal places  $\mathbf{1A}$

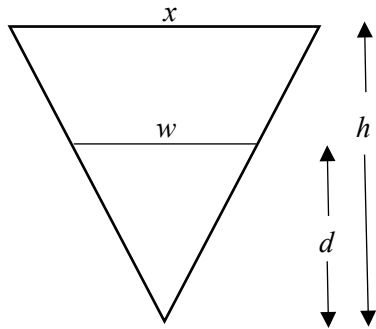
A calculator screenshot showing the equation  $\text{solve}\left(0.1 \cdot t^2 = \frac{1375 \cdot \sqrt{22}}{3}, t\right)$  and the result  $t = 146.6210725$ .

g. Average volume =  $\frac{1}{146.621 - 0} \int_0^{146.621} (0.1t^2) dt$   $\mathbf{1M}$

Average volume =  $716.6 \text{ cm}^3$  correct to one decimal place  $\mathbf{1A}$

A calculator screenshot showing the integral calculation  $\frac{1}{146.621} \int_0^{146.621} 0.1 \cdot t^2 dt$  and the result  $716.590588$ .

h. Using similar triangles  $\frac{w}{d} = \frac{x}{h}$  **1M**



With side length  $x = 5\sqrt{11}$ , we have  $h = 5\sqrt{\frac{-22\left((5\sqrt{11})^2 - 550\right)}{(5\sqrt{11})^2}} = 5\sqrt{22}$  **1A**

$\frac{w}{d} = \frac{5\sqrt{11}}{5\sqrt{22}}$ ,  $w = \frac{d}{\sqrt{2}}$  **1M**

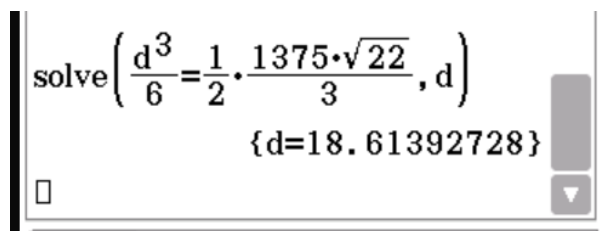
Volume of water  $= \frac{1}{3}w^2d$

Giving  $V_w = \frac{1}{3}w^2d = \frac{1}{3}\left(\frac{d}{\sqrt{2}}\right)^2 d$

Giving  $V_w = \frac{d^3}{6}$  with domain  $0 \leq d \leq 5\sqrt{22}$  **1A**

i. Let  $\frac{d^3}{6} = \frac{1}{2} \times \frac{1375\sqrt{22}}{3}$

giving  $d = 18.6$  cm, correct to one decimal place **1A**



**Question 2**

$$h: [0, 60] \rightarrow \mathbb{R}, h(t) = \frac{1}{2} \cos\left(-\frac{4\pi}{3}\left(t - \frac{1}{2}\right)\right) + 2$$

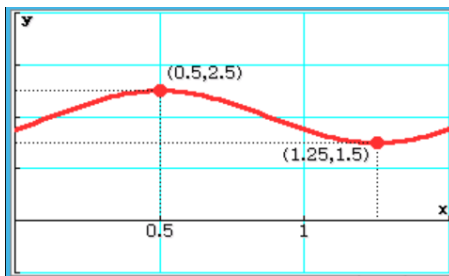
a.  $h(0) = \frac{7}{4}$

Initial height 1.75 metres **1A**

The screenshot shows a TI-84 Plus calculator interface. At the top, it says "Edit Action Interactive". Below that, there are several icons for calculator functions. The main display area shows the definition of a function:  $h(t) = \frac{1}{2} \cos\left(-\frac{4\pi}{3}\left(t - \frac{1}{2}\right)\right) + 2$ . Below this, the user has entered  $h(0)$  and the calculator has returned the result  $\frac{7}{4}$ . The word "done" is visible on the right side of the screen.

b. period =  $\frac{2\pi}{\frac{4\pi}{3}} = \frac{3}{2}$  **1A**

range =  $\left[-\frac{1}{2} + 2, \frac{1}{2} + 2\right] = [1.5, 2.5]$  **1A**



c. **1A 3 correct, 2A all correct**

- dilate from  $t$ -axis by factor of  $\frac{1}{2}$
- dilate from  $h$ -axis by factor of  $\frac{3}{4\pi}$
- reflect in the  $h$ -axis
- translate in positive direction of  $t$ -axis by  $\frac{1}{2}$  units
- translate in positive direction of  $h$ -axis by 2 units

$$\text{d. } T \begin{pmatrix} t \\ h \end{pmatrix} = \begin{bmatrix} b & 0 \\ 0 & c \end{bmatrix} \begin{pmatrix} t \\ h \end{pmatrix}.$$

Dilated by a factor of  $\frac{2}{3}$  from the  $t$ -axis gives  $b=1$ ,  $c=\frac{2}{3}$ . **1A**

$$\text{e. } h_1(t) = \frac{2}{3} \left( \frac{1}{2} \cos \left( -\frac{4\pi}{3} \left( t - \frac{1}{2} \right) \right) + 2 \right)$$

$$h_1(t) = \frac{1}{3} \cos \left( -\frac{4\pi}{3} \left( t - \frac{1}{2} \right) \right) + \frac{4}{3} \quad \text{1A}$$

$$\text{f. } T \begin{pmatrix} t \\ h \end{pmatrix} = \begin{bmatrix} -\frac{3}{4\pi} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \begin{pmatrix} t \\ h \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ \frac{4}{3} \end{pmatrix} \quad \text{1A}$$

g. Solve  $\frac{d}{dt}(h_2'(t)) = 0$  or observe from the graph of the derivative

Solve  $\frac{d}{dt}(h_2'(t)) = 0$  and substitute a suitable constant **1M**

The screenshot shows a CAS window titled "Edit Action Interactive". The input field contains the definition of a function  $h(t) = \frac{5}{12} \cos\left(-\frac{4\pi}{3}(t-2)\right) + 2$  and the command  $\text{solve}\left(\frac{d^2}{dt^2}(h(t))=0, t\right)$ . The output field shows the solution  $\left\{t = \frac{3 \cdot \text{constn}(1)}{4} + \frac{13}{8}\right\}$ . The interface includes a toolbar with various mathematical symbols and a bottom status bar with "Alg", "Standard", "Real", and "Rad" modes.

First at a maximum at  $t = \frac{1}{8}$  minutes **1A**

**OR**

Sketch derivative graph **1M**



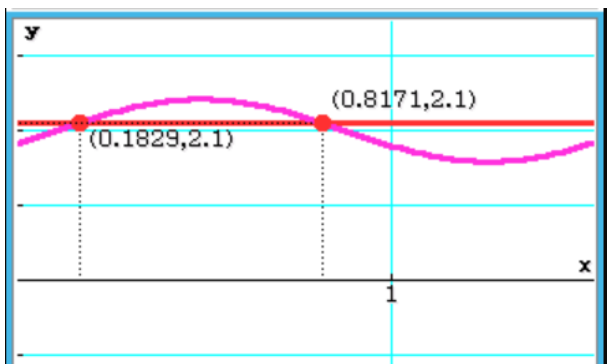


First at a maximum at  $t = \frac{1}{8}$  minutes **1A**

**h.** one wave cycle is 1.5 minutes

Solve  $h_2(t) > 2.1$ ,  $t \in [0, 1.5]$  **1M**

A graph helps to visualise the situation



**Edit Action Interactive** ✕

$\frac{0.5}{2}$   $\frac{1}{2}$   $\int dx$   $\int dx$   $\int dx$   $\int dx$   $\int dx$   $\int dx$   $\int dx$   $\int dx$   $\int dx$   $\int dx$   $\int dx$

define  $h(t) = \frac{5}{12} \cos\left(-\frac{4\pi}{3}(t-2)\right) + 2$  done

solve( $h(t) = 2.1 \mid 0 \leq t \leq 1.5, t$ )

$\{t = 0.1828605848, t = 0.8171394152\}$

northern path under water for  $\frac{0.8171 - 0.1829}{1.5} = 0.4228$

proportion of one wave cycle = 0.42 correct to two decimal places **1A**

**Question 3**

a. No, only the first cage was selected. **1A**

b. Give every mouse a number from 1 to 90 and generate 12 numbers using a suitable random number generator. **1A**

**OR**

Randomly select a tub and then randomly select 4 mice from each tub (stratified sampling). **1A**

$$\text{c. } \frac{\binom{10}{2} \binom{20}{2}}{\binom{30}{4}} \quad \mathbf{1M}$$

$$= \frac{190}{609} \quad \mathbf{1A}$$

TI-84 Plus calculator screenshot showing the calculation of the probability in part c. The expression  $\frac{nCr(10,2) \cdot nCr(20,2)}{nCr(30,4)}$  is entered, resulting in  $\frac{190}{609}$ .

**OR**

$$6 \times \frac{10}{30} \times \frac{9}{29} \times \frac{20}{28} \times \frac{19}{27} \quad \mathbf{1M}$$

$$= \frac{190}{609} \quad \mathbf{1A}$$

d. Solve  $\frac{18-\mu}{\sigma} = -0.841\dots$  and  $\frac{24-\mu}{\sigma} = 0.524\dots$  **1M**

$$\mu = 21.7 \text{ correct to one decimal place} \quad \mathbf{1A}$$

$$\sigma = 4.4 \text{ correct to one decimal place} \quad \mathbf{1A}$$

TI-84 Plus calculator screenshot showing the solution for part d. It uses the invNorm function to find z-scores and then solves a system of equations for  $\mu$  and  $\sigma$ .

invNorm(0.2,0,1)      -0.841621233465

invNorm(0.7,0,1)      0.524400510099

solve( $\frac{18-a}{b} = -0.84162123346456$  and  $\frac{24-a}{b} = 0.524400510099$ )

$a = 21.6966669268$  and  $b = 4.39231661448$

**OR**

TI-84 Plus calculator screenshot showing an alternative solution for part d using the solve function with invNorm.

solve( $\frac{18-a}{b} = \text{invNorm}(0.2,0,1)$  and  $\frac{24-a}{b} = \text{invNorm}(0.7,0,1)$ )

$a = 21.6966669268$  and  $b = 4.39231661448$

e.  $E(\hat{p}) = p = 0.8$  **1A**

Standard deviation of  $\hat{p} = \sqrt{\frac{p(1-p)}{n}}$  where  $p = 0.8$ ,  $n = 100$

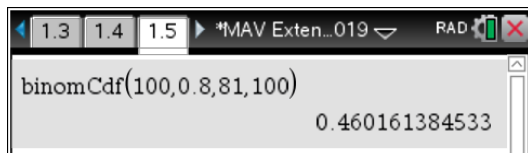
$= \sqrt{\frac{0.8 \times 0.2}{100}} = 0.04$  **1A**

f. Let  $X \sim \text{Bi}(100, 0.8)$

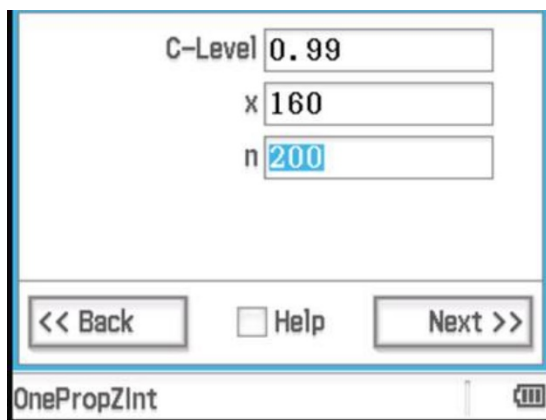
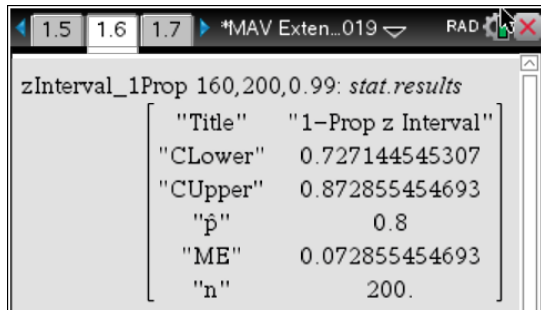
$\Pr(\hat{p} > p) = \Pr(\hat{p} > 0.8)$

$= \Pr(X > 80)$  **1M**

$= 0.460$  correct to three decimal places **1A**



g. (0.727, 0.873) **1A**



Lower 0.7271445  
 Upper 0.8728555  
 $\hat{p}$  0.8  
 n 200

<< Back    Help

OnePropZInt

**h.**  $0.99 \times 300 = 297$       **1A**

**i.** Solve  $2.575 \dots \sqrt{\frac{0.8 \times 0.2}{n}} < 0.02$       **1M**

$n = 2654$       **1A**

1.5 1.6 1.7 \*MAV Exten...019 RAD

invNorm(0.995,0,1) 2.575829303

solve(2.5758293030016 \*  $\sqrt{\frac{0.8 \cdot 0.2}{n}} < 0.02, n$ )  
 $n > 2653.95863928$

Note:  $Z \sim N(0,1)$ ,  $\Pr(Z < z_{0.99}) = 0.995$ ,  $z_{0.99} = 2.575 \dots$

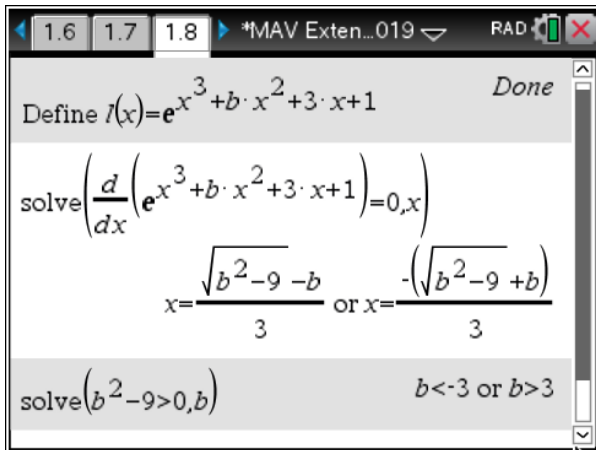
**Question 4**

a. Solve  $\frac{d}{dx}(l_b(x)) = 0$  for  $x$ .

$$x = \frac{-b \pm \sqrt{b^2 - 9}}{3} \quad \mathbf{1M}$$

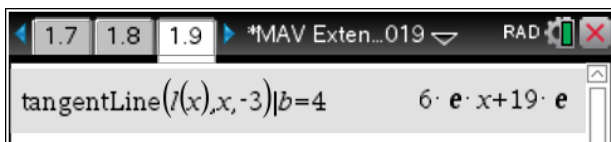
For two solutions  $b^2 - 9 > 0$ .

$$b < -3 \text{ or } b > 3 \quad \mathbf{1A}$$

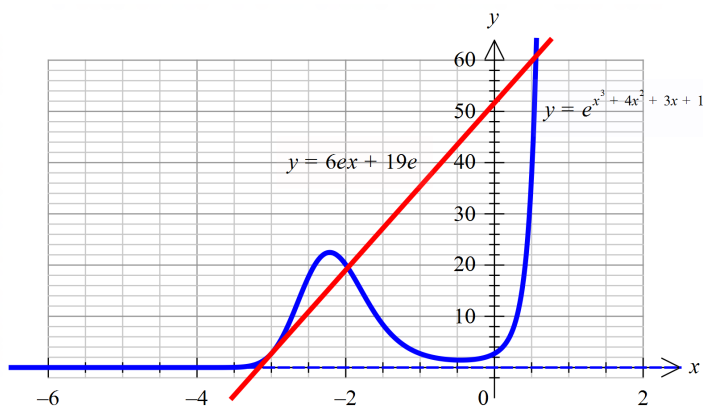


b.  $-3 \leq b \leq 3$       **1A**

c.  $y = 6ex + 19e$       **1A**

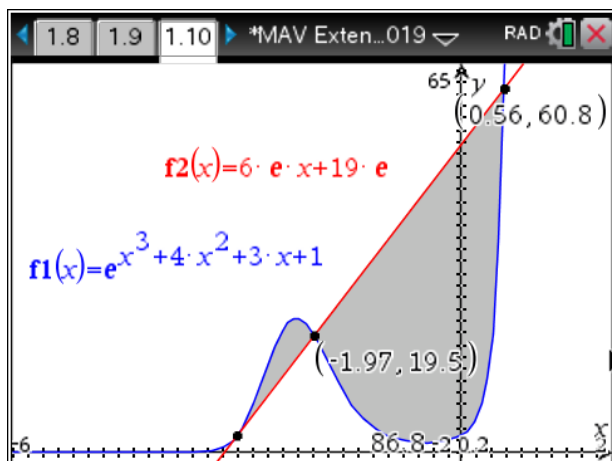


The tangent line drawn correctly on the graph.      **1A**



d.  $\int_{-3}^{-1.97} (l_4(x) - y_t) dt + \int_{-1.97}^{0.56} (y_t - l_4(x)) dt$       **1M**

= 86.8      **1A**



e.  $g: R \rightarrow R$ ,  $g(x) = e^x$  and  $h: [-1, \infty) \rightarrow R$ ,  $h(x) = x^2 + 2x + 1$

$f(x) = g(h(x))$  exists because the range of  $h$ ,  $[0, \infty)$  is a subset of the domain of  $g$ ,  $R$ . **1A**

f.  $f(x) = g(h(x)) = e^{x^2 + 2x + 1}$

Let  $y = e^{x^2 + 2x + 1}$

Inverse swap  $x$  and  $y$

$$x = e^{y^2 + 2y + 1}$$

$$y^2 + 2y + 1 = \log_e(x) \quad \mathbf{1M}$$

$$(y + 1)^2 = \log_e(x)$$

$$y = \sqrt{\log_e(x)} - 1 \text{ as the range is } [-1, \infty)$$

$$f^{-1}(x) = \sqrt{\log_e(x)} - 1 \quad \mathbf{1M \text{ Show that}}$$

g. Let  $y = f_a(x) = e^{x^2 + 2x + 1} + a$

Inverse swap  $x$  and  $y$

$$(y + 1)^2 = \log_e(x - a)$$

$$f_a^{-1}(x) = \sqrt{\log_e(x - a)} - 1 \quad \mathbf{1A}$$

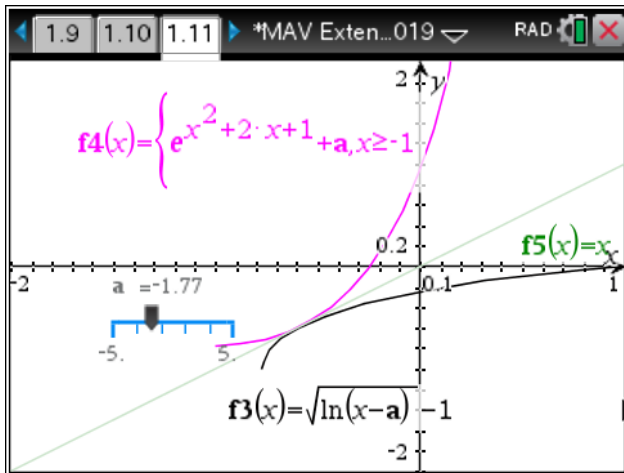
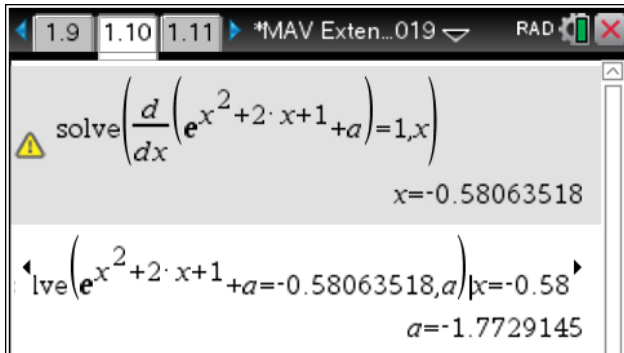
h. The graphs will be tangential when they touch the line  $y = x$ . The gradient will be equal to 1.

Solve  $\frac{d}{dx}(e^{x^2 + 2x + 1} + a) = 1$  for  $x$ .

$$x = -0.58 \text{ correct to two decimal places} \quad \mathbf{1A}$$

Solve  $e^{(-0.58\dots)^2 + 2 \times -0.58\dots + 1} + a = -0.58\dots$  for  $a$ . **1M**

$$a = -1.77 \text{ correct to two decimal places} \quad \mathbf{1A}$$

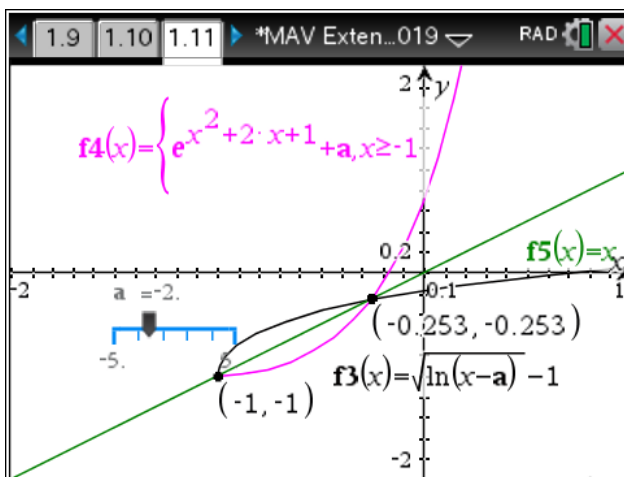


i. Translate the graph of  $f$ , 2 units down to get the maximum bounded area.

$a = -2$  **1A**

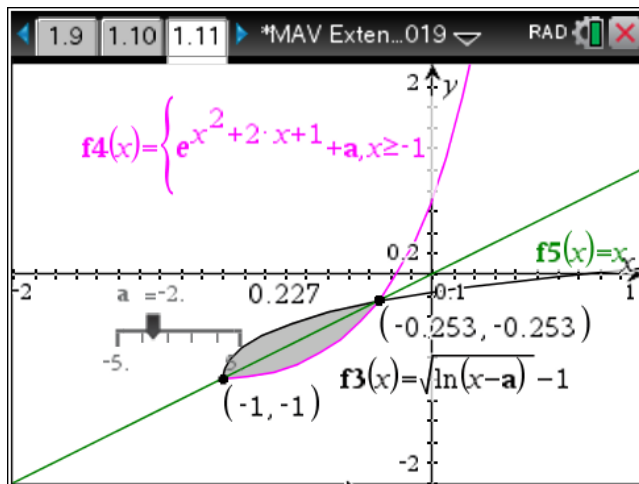
$x = -1$  **1A**

$x = -0.253$  correct to three decimal places **1A**



$$\mathbf{j. \text{ area} = 2 \int_{-1}^{-0.253\dots} (x - f_{-2}(x)) dx \quad \mathbf{1M}}$$

$$= 0.23 \text{ correct to two decimal places} \quad \mathbf{1A}$$



**END OF SOLUTIONS**