2019 VCE Mathematical Methods Trial Examination 1

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Victorian Certificate of Education
2019

STUDENT NUMBER

Figures										Letter
Words										

MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 20 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y=\dfrac{\sin(2x)}{x^2}$ if $\dfrac{dy}{dx}=\dfrac{2f(x)}{x^3}$, find the rule for the function $f(x)$.

2 marks

b. Let $g(x)=\tan\left(\dfrac{2}{x}\right)$. Evaluate $g'\left(\dfrac{6}{\pi}\right)$.

2 marks

Question 2 (3 marks)

A footballer has seven shots at goal from the 50 metres line. He estimates that he is most likely to score three goals. If the number of goals scored can be represented as a binomial probability distribution, with seven independent trials, and p the probability of a success on any one trial. Find the value of p where $0 < p < 1$.

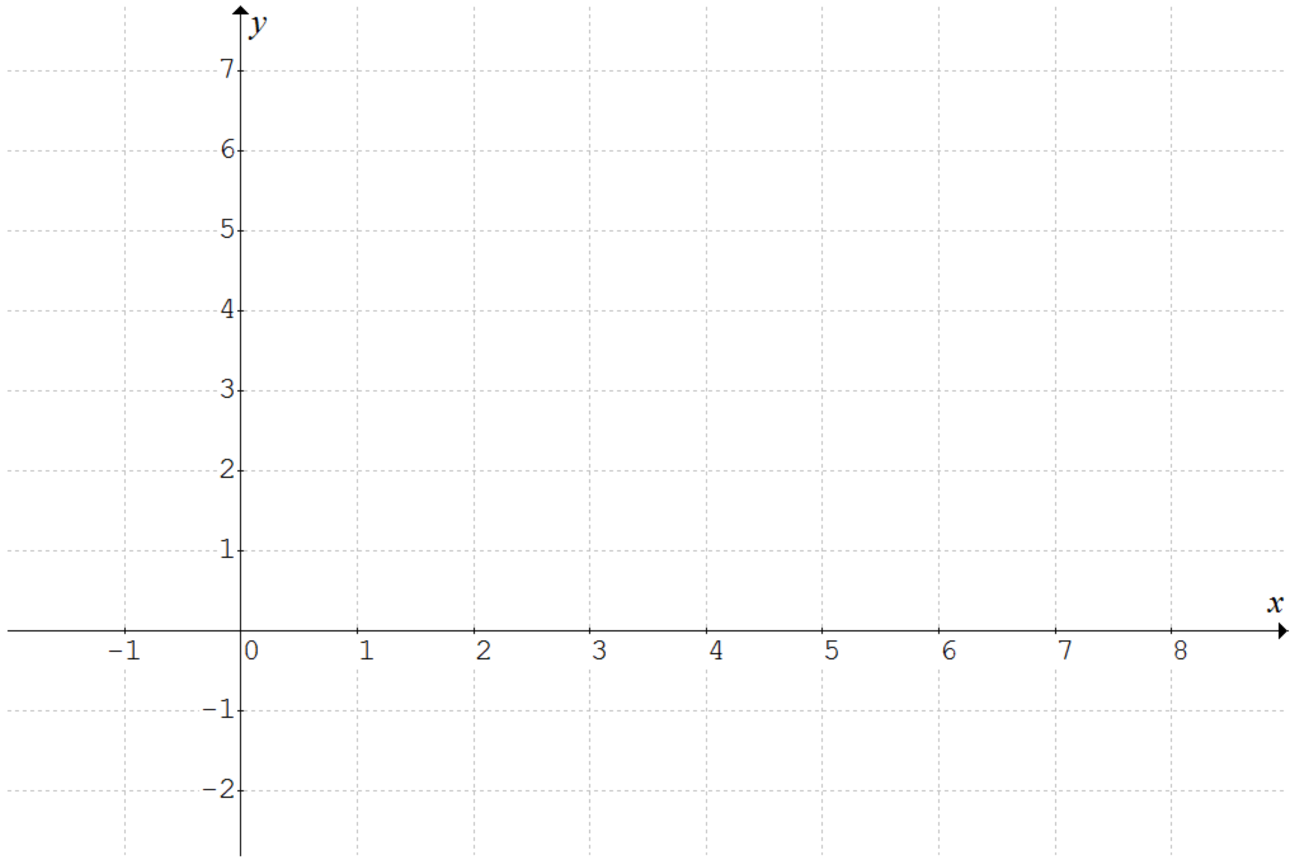
Question 3 (3 marks)

a. Find the general solution of $2-4\cos\left(\frac{\pi x}{3}\right)=0$

1 mark

b. Sketch the graph of the function $f:[0,6]\to R \quad f(x)=2-4\cos\left(\frac{\pi x}{3}\right)$,

stating the coordinates of all axial intercepts, endpoints and turning points. 2 marks



Question 4 (4 marks)

The tangent to the graph of the curve $y=\sqrt{5+cx}$ at the point $x=2$, makes an angle of 135^0 with the positive direction of the x-axis. Determine the value of c.

Question 5 (4 marks)

a. Find $\frac{d}{dx}\left[\log_e\left(x^2+4\right)\right]$. 1 mark

b. **Hence,** find the value of b, if the average value of the function $f(x)=\frac{x}{x^2+4}$ over $0 \le x \le b$ is equal to $\log_e\left(\sqrt[b]{2}\right)$.

3 marks

Question 6 (6 marks)

Sometimes in packets of biscuits, some of the biscuits are broken.

One packet of these biscuits contains 12 biscuits and suppose that $\frac{1}{6}$ of the biscuits are broken.

A random sample of the 12 biscuits in a packet is chosen from the population.
Let $\hat{P}$ represent the sample proportion of broken biscuits in these packets.

a. Find $\Pr\left(\hat{P}=\frac{1}{12}\right)$, giving your answer in the form $c\left(\frac{a}{b}\right)^n$ where a, b, c and n are positive integers. 2 marks

b. For a sample of 144 packets, find the standard deviation of $\hat{P}$. 1 mark

c. Jenny accidently drops a second packet of these biscuits on the floor and several more of the biscuits became broken. The first packet is known to have two broken biscuits. Jenny then randomly selects one of these packets of biscuits, then takes out and eats two biscuits from this packet. If probability that both the biscuits are not broken are is $\frac{23}{66}$, determine the number of broken biscuits in the packet that was dropped onto the floor.

2 marks

d. The weights of these packets of biscuits are normally distributed with a mean of 250 grams, with a standard deviation of 8 grams. Let Z be a random variable with the standard normal distribution. If the probability that a packet of biscuits weighs between 244 and 256 grams, is equal to $2\Pr(w \le Z \le 0)$, find the value of *w*.

1 mark

Question 7 (3 marks)

Consider the function $f : R^{+} \to R$, $f(x) = \frac{1}{x}$.

a. Find the equation of the tangent to the graph of f at the point where $x = a$, and $a > 0$.
Give your answer in the form $y = mx + c$.

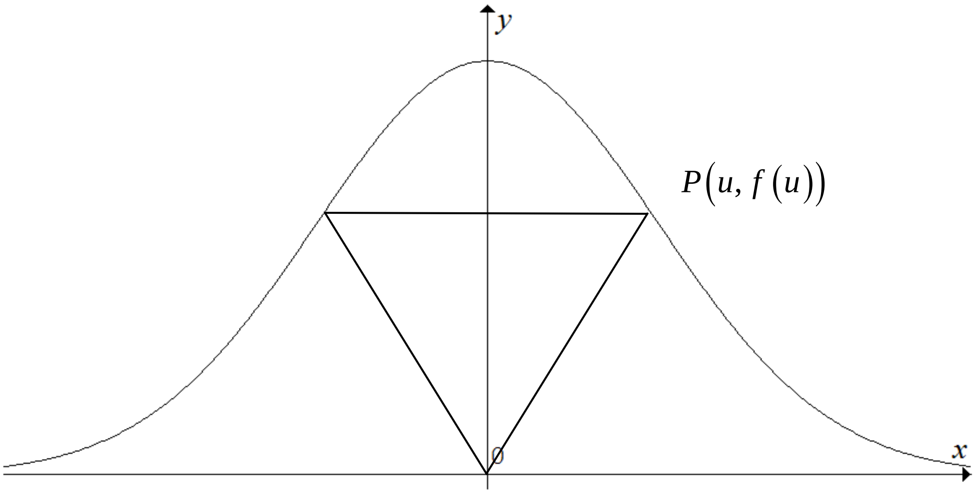
2 marks

b. If O is the origin and this tangent crosses the *x*-axis at the point R and the *y*-axis at the point S, find the coordinates of the points R and S and find the area of the triangle ORS.

1 mark

Question 8 (3 marks)

An isosceles triangle has two vertices on the graph of the function $f(x) = e^{-x^2}$ one at the point $P(u, f(u))$ the other at $Q(-u, f(u))$ where $u > 0$ and one at the origin as shown in the diagram below.



Find the value of u, for which the area of the triangle is a maximum.

Question 9 (4 marks)

a. The transformation $T: R^2 \to R^2$ with the rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$

maps the graph of $y = x^2$ onto the graph of $y = 11 + 2x - x^2$.

Find the values of a, b, h and k.

1 mark

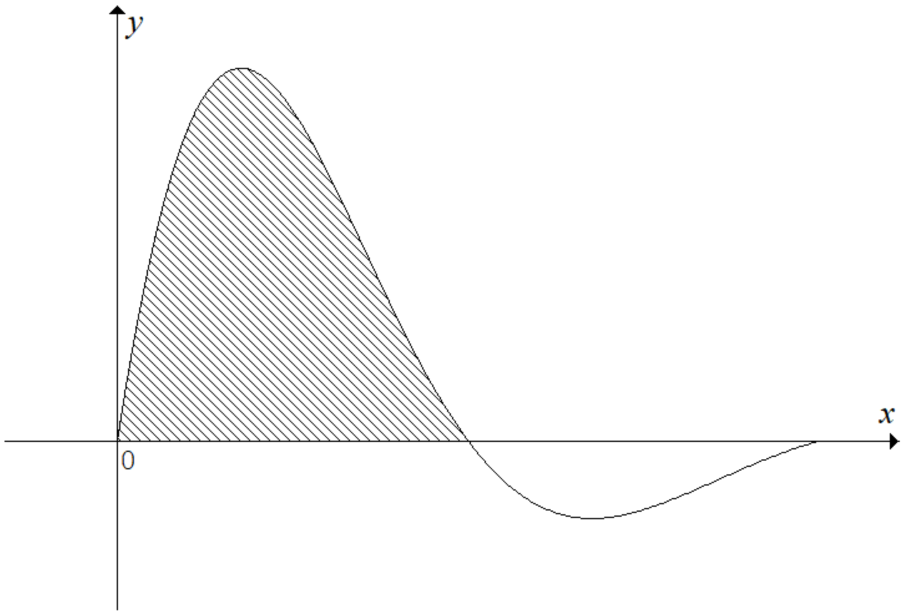
Consider the functions with the rules $f(x) = \dfrac{1}{\sqrt{x+4}}$ and $g(x) = 11 + 2x - x^2$, defined on their maximal domains.

b. Show that $f(g(x))$ does not exist. 1 mark

c. If $g: D \to R$, $g(x) = 11 + 2x - x^2$, find the largest subset D of R, such that $f(g(x))$ is defined. 2 marks

Question 10 (6 marks)

The graph of the function $f:[0,\pi]\to R$, $f(x)=e^{-2x}\sin(2x)$ is shown below.



a. Find the *x*-coordinates of the stationary points on the graph of *f*.

2 marks

b. Differentiate $e^{-2x}\left(\cos(2x)+\sin(2x)\right)$ with respect to *x*.

1 mark

c. **Hence,** find area of the shaded region. 3 marks

End of question and answer book for the
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EXTRA WORKING PAGE

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=na(ax+b)^{n-1}$		$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A)=1-\Pr(A')$		$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$	
$\Pr(A\mid B)=\dfrac{\Pr(A\cap B)}{\Pr(B)}$			
mean	$\mu=E(X)$	variance	$\text{var}(X)=\sigma^2=E\left((X-\mu)^2\right)=E(X^2)-\mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)\,dx$	$\mu=\int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2\,f(x)\,dx$

Sample proportions

$\hat{P}=\dfrac{X}{n}$		mean	$E(\hat{P})=p$
standard deviation	$\text{sd}(\hat{P})=\sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p}-z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\,,\ \hat{p}+z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET