Mathematical Methods

2019

Trial Examination 1
(1 hour)

Instructions

Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 The following diagram shows a triangle with vertices $(-4, 0)$, $(0, 4)$ and $(4, 0)$, and a parabola with vertex $(0, 3)$.
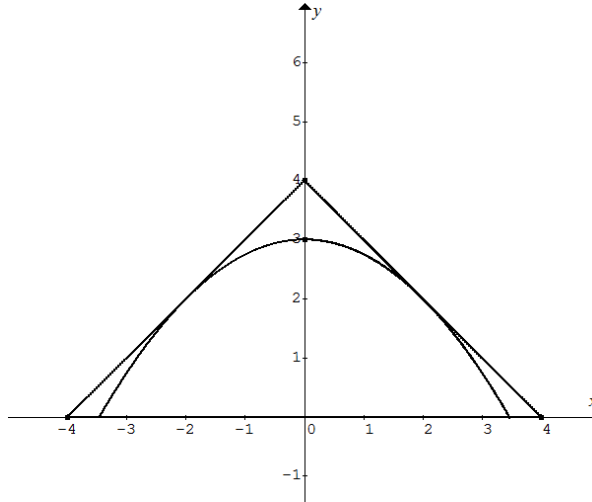
a.   Write down the equation of the parabola in turning point form in terms of dilation factor k in the y-direction.

1 mark

b.   The parabola can just fit inside the triangle. Determine the value of k in part a.

2 marks

c.   Determine the coordinates of the x-intercepts of the parabola.
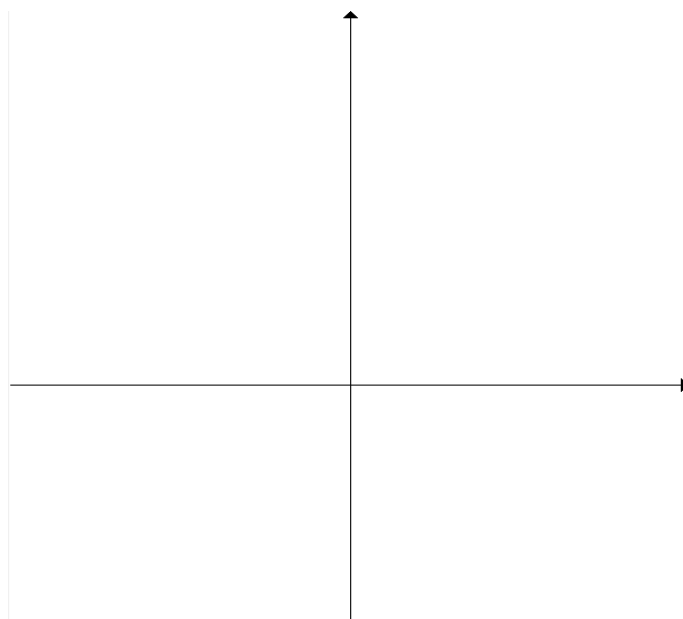
1 mark

Question 2 Polynomial $P(x) = x^6 - 1$ can be expressed in the form of 6 factors for $x > 0$.

a. Firstly, expand $\left(a + \sqrt{a} + 1\right)\left(a - \sqrt{a} + 1\right)$. 1 mark

b. Given $P(x) = \left(x^2 - 1\right)\left(x^4 + mx^2 + 1\right)$, find the value of m. 1 mark

c. Hence or otherwise express $P(x)$ in the form of 6 factors. 2 marks

d. Sketch the graph of $P(x)$. 1 mark

Question 3 Solve $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = 7$ for e^{-x}.

2 marks

Question 4 Consider $f(x) = \log_e (ax)^2$ where $a \in R \setminus \{-1, 1\}$.

a. Show that $f(-x) = f(x)$.

1 mark

b. Show that $f(xy) \neq f(x) + f(y)$.

2 marks

c. Find $f'(x)$ and show that $f'(-x) = f'(x)$.

2 marks

Question 5 Given $f(x) = x^{\frac{2}{3}}$ and $g(x) = x^{\frac{3}{2}}$, solve $f(g(x)) - g(f(x)) = 0$ for x.

2 marks

Question 6 $y = \frac{1}{2}\cos\left(n\left(x + \frac{\pi}{10}\right)\right) - 2$ is the transformation of $y = 2\cos(x) + 1$.
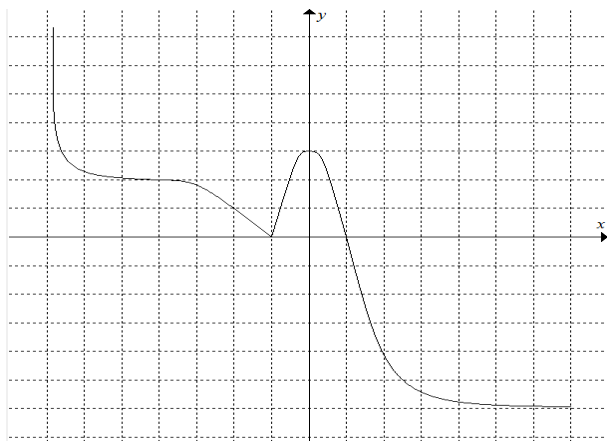
A sequence of transformations can change $y = \frac{1}{2}\cos\left(n\left(x + \frac{\pi}{10}\right)\right) - 2$ back to $y = 2\cos(x) + 1$.

Write down the required transformations in correct order.

5 marks

Question 7 Sketch on the same axes the graph of the derivative function of the function shown below.

4 marks

Question 8 The graph of function $f(x)$ intersects the x-axis at $x = a$ and $x = b$, where $b > a > 0$ and $f(x) \geq 0$ for $x \in [a, b]$.

Let A be the area of the region bounded by $f(x)$ and the x-axis.

a. Write a definite integral for A. 1 mark

The graph of $f(x)$ is dilated in both x and y directions by the same factor of $\sqrt{2}$ and then translated in the positive y-direction by $\sqrt{2}$ units.

b. Write a definite integral for the area of the region after the stated transformations.

2 marks

c. Find the area of the region after the stated transformations in terms of A, a and b.

1 mark

Question 9 Country town A has a population of $100\,000$, and $10\,000$ are overweight.
A sample of 400 people is selected randomly.
Let $\hat{P}$ be the proportion of overweight people in the sample.

a. Calculate the standard deviation of $\hat{P}$. 1 mark

b. Find $\Pr(\hat{P} > 0.10)$. 1 mark

c. Let N be the number of overweight people in another sample of 400 people from country town A.
Find an approximate value of $\Pr(10 \le N \le 70)$ in the sample. 1 mark

d. A random sample of 400 people is taken from country town B. If there are 40 overweight people in the sample, calculate an approximate 95% confidence interval for the population proportion p of country town B.

1 mark

Question 10 $f(x) = 2\sin(2x)$ over $x \in \left[\dfrac{\pi}{4} - \alpha, \dfrac{\pi}{4} + \alpha\right]$ is a probability density function.

a. Calculate the value of α. 3 marks

b. Given $\cos(\theta) = \sqrt{\dfrac{1 + \cos(2\theta)}{2}}$, find the exact value of $\Pr\left(\dfrac{5\pi}{24} < X < \dfrac{7\pi}{24}\right)$. 2 marks

End of Exam