2019 Mathematical Methods Trial Exam 1 Solutions

Q1a $y=-kx^2+3$

Q1b $-kx^2+3=x+4$, $kx^2+x+1=0$, $\Delta=0$, $k=\frac{1}{4}$

Q1c $-\frac{1}{4}x^2+3=0$, $x^2=\pm\sqrt{12}=\pm2\sqrt{3}$, $\left(-2\sqrt{3},0\right)$, $\left(2\sqrt{3},0\right)$

Q2a $\left(a+\sqrt{a}+1\right)\left(a-\sqrt{a}+1\right)=a^2+a+1$

$\left(a+\sqrt{3a}+1\right)\left(a-\sqrt{3a}+1\right)=a^2-a+1$

Q2b

$x^6-1=\left(x^2-1\right)\left(x^4+mx^2+1\right)=x^6+(m-1)x^4+(1-m)x^2-1$,

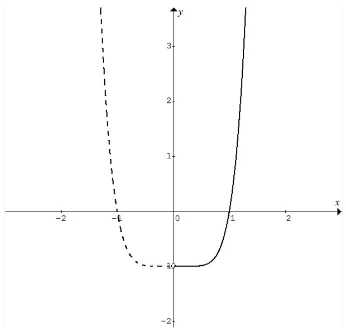
$m=1$

Q2c

$P(x)=\left(x^2-1\right)\left(x^4+x^2+1\right)=(x+1)(x-1)\left(x^2+x+1\right)\left(x^2-x+1\right)$

$=(x+1)(x-1)\left(x+\sqrt{x}+1\right)\left(x-\sqrt{x}+1\right)\left(x+\sqrt{3x}+1\right)\left(x-\sqrt{3x}+1\right)$

Q2d



Q3 $e^x+e^{-x}=7e^x-7e^{-x}$, $6e^x-8e^{-x}=0$, $6-8\left(e^{-x}\right)^2=0$,

$\left(e^{-x}\right)^2=\frac{3}{4}$, $e^{-x}=\frac{\sqrt{3}}{2}$

Q4a

$f(-x)=\log_e(a(-x))^2=\log_e\left(a^2(-x)^2\right)=\log_e\left(a^2x^2\right)=\log_e(ax)^2=f(x)$

Q4b $f(x)=\log_e(ax)^2$, $f(y)=\log_e(ay)^2$

$f(x)+f(y)=\log_e(ax)^2+\log_e(ay)^2=\log_e(ax)^2(ay)^2=\log_e\left(a^2xy\right)^2$

$f(xy)=\log_e(axy)^2$, .: $f(xy)\neq f(x)+f(y)$

Q4c $f(x)=\log_e(ax)^2$, $f'(x)=\frac{1}{(ax)^2}\times2a^2x$

.: $f'(x)=\frac{2}{x}$ for $x\in R\setminus\{0\}$

.: $f'(-x)=\frac{2}{-x}$ for all $x\in R\setminus\{0\}$, this statement is the same as

$f'(x)=\frac{2}{x}$ for all $x\in R\setminus\{0\}$

Also, $f(-x)=f(x)=\log_e(ax)^2$, .: $f'(-x)=f'(x)$

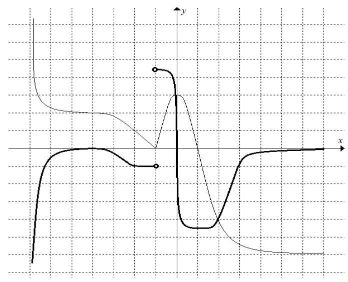
Q5 $f(g(x))=f\left(x^{\frac{3}{2}}\right)=\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}}=x$ for $x\geq0$

$g(f(x))=g\left(x^{\frac{2}{3}}\right)=\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}}=x$ for all $x\in R$.

.: $f(g(x))-g(f(x))=0$ for $x\geq0$.

Q6 Translate $\frac{\pi}{10}$ units to the right, dilate in the *y*-direction by factor 4, dilate in the *x*-direction by factor n, translate 9 units up.

Q7



Q8a $A=\int_a^b f(x)\,dx$

Q8b $\int_{\sqrt{2}a}^{\sqrt{2}b}\left(\sqrt{2}f\left(\frac{x}{\sqrt{2}}\right)+\sqrt{2}\right)dx$

Q8c $\sqrt{2}\times\sqrt{2}\times A+\sqrt{2}\times\left(\sqrt{2}\,b-\sqrt{2}\,a\right)=2(A+b-a)$

Q9a $p=\frac{10000}{100000}=0.1$, $\text{sd}\left(\hat{P}\right)=\sqrt{\frac{0.1\times0.9}{400}}=0.015$

Q9b $\Pr\left(\hat{P}>0.10\right)=0.5$

Q9c $\Pr(10\leq N\leq70)=\Pr\left(0.025\leq\hat{P}\leq0.175\right)=\Pr(-5\leq Z\leq5)\approx1$

Q9d $(0.07,0.13)$

Q10a $\int_{\frac{\pi}{4}-\alpha}^{\frac{\pi}{4}+\alpha}2\sin(2x)\,dx=1$, $\left[-\cos(2x)\right]_{\frac{\pi}{4}-\alpha}^{\frac{\pi}{4}+\alpha}=1$

$-\cos\left(\frac{\pi}{2}+2\alpha\right)+\cos\left(\frac{\pi}{2}-2\alpha\right)=1$, $\sin(2\alpha)+\sin(2\alpha)=1$

$\sin(2\alpha)=\frac{1}{2}$, $2\alpha=\frac{\pi}{6}$, $\alpha=\frac{\pi}{12}$

Q10b $\Pr\left(\frac{5\pi}{24}<X<\frac{7\pi}{24}\right)=\int_{\frac{5\pi}{24}}^{\frac{7\pi}{24}}2\sin(2x)\,dx=\left[-\cos(2x)\right]_{\frac{5\pi}{24}}^{\frac{7\pi}{24}}$

$=-\cos\left(\frac{7\pi}{12}\right)+\cos\left(\frac{5\pi}{12}\right)=\cos\left(\frac{5\pi}{12}\right)+\cos\left(\frac{5\pi}{12}\right)=2\cos\left(\frac{5\pi}{12}\right)$

$=2\sqrt{\frac{1+\cos\frac{5\pi}{6}}{2}}=\sqrt{2-\sqrt{3}}$

Please inform mathline@itute.com re conceptual and/or mathematical errors