2018 VCE Mathematical Methods 2 (NHT) examination report

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Section A – Multiple-choice questions

Question	Answer	Comments
1	B	
2	C	
3	D	
4	E	
5	D	
6	D	
7	C	
8	E	
9	B	
10	E	
11	B	
12	D	
13	D	
14	A	$(x,\ y)\rightarrow\left(x,\ \frac{y}{2}\right)\rightarrow\left(-x,\ \frac{y}{2}\right)\rightarrow\left(-x-\frac{\pi}{6},\ \frac{y}{2}-4\right)$ $x'=-x-\frac{\pi}{6},\ x=-\left(x'+\frac{\pi}{6}\right)$ $y'=\frac{y}{2}-4,\ y=2\left(y'+4\right)$ $g(x)=3\cos\left(x-\frac{\pi}{6}\right),\ 2\left(y'+4\right)=3\cos\left(-x'-\frac{\pi}{6}-\frac{\pi}{6}\right)$ $y'=\frac{3}{2}\cos\left(-x'-\frac{\pi}{3}\right)-4,\ f(x)=\frac{3}{2}\cos\left(-x-\frac{\pi}{3}\right)-4$
15	A	
16	E	
17	E	
18	D	Dilate the graph of *f* by a factor of 2 from the *y*-axis and reflect the image in the *x* and *y* axes.
19	D	
20	A	$f(3)=7,\ f'(3)=2,\ g(x)=f^{-1}(x),\ g'(7)=\frac{1}{2}$ since $f'(x)\times f'(y)=1, g(x)=f'(y)=\frac{1}{f'(x)}$

Section B

Question 1a.

$a = 2,\ b = 4,\ c = 4$

Question 1b.

$x = -1.29...,\ m = -2,\ n = -1$

Question 1ci.

$y = 4x - 8$

Question 1cii.

$y = -11$

Question 1ciii.

$P\left(-\frac{3}{4}, -11\right),\ Q(1, -11)$

Question 1di.

$d = -\frac{7}{4}$

Question 1dii.

$m' = -\frac{15}{4}, n' = -\frac{11}{4}$

Question 2ai.

$S \sim \text{Bi}(5, 0.08)$, $\Pr(S = 2) = 0.0498$, correct to four decimal places

Question 2aii.

$\Pr(S = 2 \mid S \leq 2) = \frac{\Pr(S = 2 \cap S \leq 2)}{\Pr(S \leq 2)} = \frac{0.0498...}{0.9954...} = 0.0501$, correct to four decimal places

Question 2b.

$(0.0169, 0.1431)$, correct to four decimal places. This could have been obtained directly from using technology; there was no need to use the formula.

Question 2c.

$\Pr(M > 20.6) = 0.0228$, correct to four decimal places

Question 2d.

$1 - \Pr(19.5 < M < 20.5) = 0.0956$, correct to four decimal places

Question 2e.

2 sensors and 0 motors, or 0 sensors and 2 motors, or 1 sensor and 1 motor

$\Pr(S=0)=0.6590..., \Pr(S=1)=0.2865..., \Pr(S=2)=0.0498...$

$M \sim \text{Bi}(5, 0.0955...)$

$\Pr(M=0)=0.6051..., \Pr(M=1)=0.3197..., \Pr(M=2)=0.0675...$

$\Pr(2 \text{ defective}) = 0.0498.. \times 0.6050.. + 0.6590.. \times 0.0675.. + 0.2865.. \times 0.3197.. = 0.166$, correct to three decimal places

Question 2f.

$\Pr(K < 30.4) = 0.99$, $\dfrac{30.4-30}{\sigma} = 2.326...$, $\sigma = 0.17$, correct to two decimal places

Question 2g.

$$\text{Mean} = \int_{290}^{330} \left(\frac{3w}{640000}(330-w)^2(w-290) \right) dw = 306$$

Question 2h.

$$\int_{290}^{306} \left(\frac{3}{640000}(330-w)^2(w-290) \right) dx = 0.5248$$, correct to four decimal places

Question 3a.

$$\text{Area} = \int_0^{80} \left(y_1(x) - y_2(x) \right) dx = 357.04 \text{ m}^2$$, correct to two decimal places

Question 3b.

$D(x) = y_1(x) - y_2(x)$, $D'(x) = 0$, $x = 5\pi,\ 15\pi,\ 25\pi$, minimum when $x = 5\pi$ and 25π,

$D(5\pi) = \dfrac{5}{4}$ or 1.25 m

Question 3c.

$$\frac{1}{80}\int_0^{80} \left(y_1(x) - y_2(x) \right) dx = 4.46 \text{ m}$$, correct to two decimal places

Question 3d.

The minimum distance between $c_1(x)$ and $c_2(x)$ occurs when $x = 5\pi$,

$c_1(5\pi) = 15 + p,\ c_2(5\pi) = p^2 + 10$, hence there is one point of intersection when

$15 + p - \left(p^2 + 10 \right) = 0$, $p = \dfrac{\sqrt{21}+1}{2}$. So the interval for no intersection is $\left(0,\ \dfrac{\sqrt{21}+1}{2} \right)$.

Question 3e.

$$A(a) = \int_0^{80} (c_1(x) - c_2(x))dx,\ A'(a) = 0,\ a = \frac{1}{2},\ A\left(\frac{1}{2}\right) = 402.86 \text{ m}^2$$, correct to two decimal places

Question 4ai.

$f'(x) = -\log_e(x),\ \sqrt{e} < x < e$

Question 4aii.

$\frac{1}{e} < x < \frac{1}{\sqrt{e}}$

Question 4bi.

$y = -\log_e(a)x + a$

Question 4bii.

Tangent at $x = \frac{1}{a},\ y = -\log_e\left(\frac{1}{a}\right)x + \frac{1}{a},\ -\log_e(a)x + a = -\log_e\left(\frac{1}{a}\right)x + \frac{1}{a},\ \left(\frac{a^2-1}{2a\log_e(a)}, \frac{a^2+1}{2a}\right)$

Question 4biii.

$\left(\frac{e^2-1}{2e},\ \frac{e^2+1}{2e}\right)$

Question 4ci.

$\frac{2}{\log_e(2)} = \frac{b}{\log_e(b)},\ b = 2 \text{ or } 4, \text{ since } b > e,\ b = 4$

Question 4cii.

$\frac{p}{\log_e(p)} = \frac{q}{\log_e(q)},\ p\log_e(q) = q\log_e(p),\ \log_e\left(q^p\right) = \log_e\left(p^q\right),\ q^p = p^q$

Question 4d.

$y = -\frac{x}{2} + e^{\frac{1}{2}}$

Question 4ei.

$E\left(2\sqrt{e},\ 0\right),\ F\left(2\sqrt{e}-2,\ 1\right)$

Question 4eii.

$\text{Area} = \frac{1}{2} \times \left(2\sqrt{e} - 3 + 2\sqrt{e} - 1\right) = 2\sqrt{e} - 2$

Question 4eiii.

$\text{Area} = \frac{e-1}{2}$

Question 4eiv.

$$\text{average} = \frac{1}{2}\left(2\sqrt{e} - 2 + \frac{e-1}{2}\right) = \frac{1}{4}\left(e + 4\sqrt{e} - 5\right)$$

Question 4ev.

$$\text{error} = \int_1^e \left(x - x\log_e(x)\right) dx - \frac{1}{4}\left(e + 4\sqrt{e} - 5\right),$$

$$\sqrt{}$$

$$\%\text{error} = \frac{\int_1^e \left(x - x\log_e(x)\right) dx - \frac{1}{4}\left(e + 4\sqrt{e} - 5\right)}{\int_1^e \left(x - x\log_e(x)\right) dx} \times 100\% = 1.73\%$$