TSSM
Creating VCE Success

2018 Trial Examination

Letter

STUDENT NUMBER

MATHEMATICAL METHODS
Units 3 & 4 – Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION & ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 11 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer **all** questions in the space provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $f:(-3, \infty) \to R, \ f(x) = \dfrac{2x}{3+x}$

Differentiate f with respect to x. 2 marks

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b. Let $g(x) = (1 - x^4)^2$

Evaluate $g'(1)$ 2 marks

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Question 2 (4 marks)

Let $y = x\cos(x)$

a. Find $\frac{dy}{dx}$ 2 marks

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b. Hence, calculate $\int_0^\pi (x\sin(x) + 1)dx$ 2 marks

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Question 3 (4 marks)
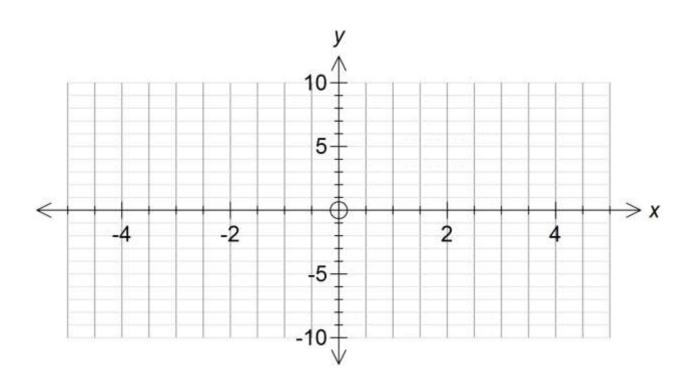
Let $f:[-2,2] \to R$, $f(x) = 2(1-x)^2(x+2)$

a. Show that $f(x) = 2x^3 - 6x + 4$ 1 mark

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b. Sketch the graph of *f* on the axes below. Label the axis intercepts and any stationary points with their coordinates.   3 marks

Question 4 (2 marks)

In a large population of mice, the proportion of white mice is $\frac{1}{3}$.

Let $\hat{P}$ be the random variable that represents the sample proportion of white mice for samples of size n drawn from the population.

Find the smallest integer value of n such that the standard deviation of $\hat{P}$ is less than or equal to $\frac{1}{72}$

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TURN OVER

Question 5 (5 marks)

For Jom to qualify for the next stage of the Quiz show he must answer one question correctly. Each question has four choices to choose from. If Jom answers the first question wrongly, he is asked another question. A maximum of three questions must be asked and Jom guesses each question.

a. What is the probability that Jom will answer a question correctly?   1 mark

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b. What is the probability that Jom will not qualify for the next stage?   2 marks

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c. Calculate the probability that Jom qualifies for the next stage of the show.   2 marks

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Question 6 (4 marks)

Let $(\sin(\theta) - 1)(2\sin(\theta) + 1)\sin(\theta) = 0$

a. State all possible values of $\sin(\theta)$. 1 mark

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b. Find all possible solutions for $(\sin(\theta) - 1)(2\sin(\theta) + 1)\sin(\theta) = 0$, where $0 \leq \theta \leq \pi$. 3 marks

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TURN OVER

Question 7 (4 marks)

Let $f \, (0, 1] \to R, \ f(x) = \log_e(2x)$

a. State the range of f. 1 mark

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b. Let $g: [b, 3) \to R, \ g(x) = \sqrt{3 - x}, \ \text{where } b > 0$

   i. Find the smallest value of b so that $f\big(g(x)\big)$ exists. 2 marks

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   ii. Find the rule for $f\big(g(x)\big)$. 1 mark

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Question 8 (4 marks)

For events A and B from a sample space, $\Pr(A \cap B) = \frac{1}{6}$ and $\Pr(B) = \frac{1}{3}$.
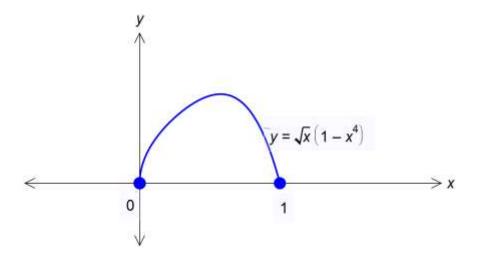
Let $\Pr(A) = p$.

a. Find $\Pr(B|A)$ in terms of p.   1 mark

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b. Find $\Pr(B'|A')$ in terms of p.   2 marks

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c. Find the value of p when $\Pr(B'|A') = \frac{1}{2}$.   1 mark

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TURN OVER

Question 9 (9 marks)

The graph of $f:[0\ 1] \to R, \ f(x) = \sqrt{x}(1-x^4)$ is shown below.

a. Calculate the area between the graph of f and the x-axis. 2 marks

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b. Show that the gradient of the tangent to the graph of f is $\frac{(1-9x^4)}{2\sqrt{x}}$. 1 mark

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c. A tangent to the graph of f makes an angle of $\tan^{-1}(-4)$ with the positive direction of the x-axis.

    i.   Show that this is possible when $x = 1$.   2 marks

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    ii.   Hence find the equation of this tangent.   2 marks

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d. The graph of f is dilated by a factor of 2 from the x-axis and then translated 1 unit vertically up.

Write down the equation of the tangent to the transformed graph at $x = 1$.   2 marks

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END OF QUESTION AND ANSWER BOOK