MM12 Polynomials and Quadratics Test 2018

Name: ANSWERS.

Skills (Section A and B) ..... / 20 **Analysis (Section C) ..... / 22**

Section A: Short Answer (15 minutes) **No Calculators Allowed**

NOTE: Exact answers are required unless instructed otherwise within the question

1. (a) Without dividing, find the remainder when $x^3 - 2x^2 - 5x + 6$ is divided by $x + 2$

Remainder $= P(-2)$

$= (-2)^3 - 2(-2)^2 - 5(-2) + 6$

$= 0$

(b) Hence, fully factorise $x^3 - 2x^2 - 5x + 6$

$x+2$ is a factor

$$x+2 \overline{)\, x^3 - 2x^2 - 5x + 6}$$

$x^2 - 4x + 3$

$x^3 + 2x^2$

$-4x^2 - 5x$

$-4x^2 - 8x$

$3x + 6$

$3x + 6$

0

$(x+2)(x^2 - 4x + 3)$

$= (x+2)(x-3)(x-1)$. (1+2=3 marks)

2. Expand $(2x - 3)^3$

$(2x)^3 - 3(2x^2)(3) + 3(2x)(3)^2 - (3)^3$

$= 8x^3 - 36x^2 + 54x - 27$

(2 marks)

3. Given $P(x) = x^3 - kx^2 + 4x - 1$ has a remainder of 2 when divided by x – 3 find k.

$P(3) = 27 - 9k + 12 - 1 = 2$

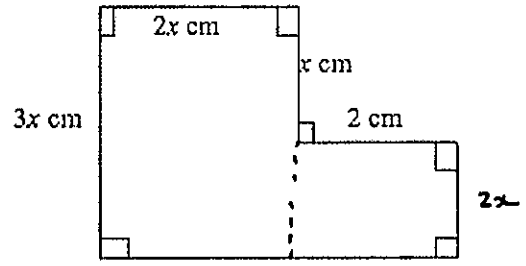
$38 - 9k = 2$

$9k = 36$

$k = 4$.

(2 marks)

4. The area of the shape shown below is 42 cm^2. Find the value of x.



$A = 3x \times 2x + 2x \times 2x$

$= 6x^2 + 4x$

$6x^2 + 4x = 42$

$6x^2 + 4x - 42 = 0$

$2(3x-7)(x+3) = 0$

$x = 7/3$ or -3

since $x > 0$, $x = 7/3$

(3 marks)

5. (a) Express $y = 2x^2 - 5x - 2$ in the form $y = a(x+b)^2 + c$

$y = 2\left(x^2 - \frac{5}{2}x - 1\right)$

$= 2\left[\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) - 1 - \frac{25}{16}\right]$

$= 2\left[\left(x - \frac{5}{4}\right)^2 - \frac{41}{16}\right]$

$= 2\left(x - \frac{5}{4}\right)^2 - \frac{41}{8}$

(b) Hence, or otherwise, write the coordinates of the turning point of $y = 2x^2 - 5x - 2$

$\left(\frac{5}{4}, -\frac{41}{8}\right)$

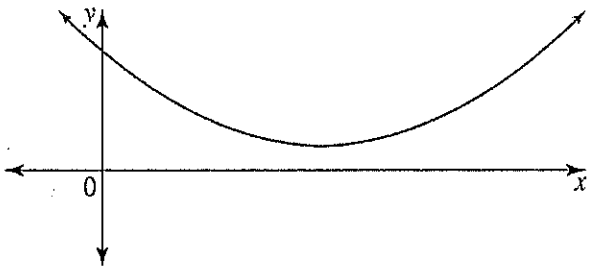
(3+1=4 marks)

End of Section A

Name: ANSWERS

Section B: Multiple Choice Questions **Calculators are allowed**
(30 minutes for Sections B&C)

1. For the graph of the parabola $y = ax^2 + bx + c$ shown, with $\Delta = b^2 - 4ac$, which statement is correct?



A. $a > 0$ and $\Delta > 0$
B. $a > 0$ and $\Delta < 0$
C. $a < 0$ and $\Delta < 0$
D. $a < 0$ and $\Delta > 0$
E. $a > 0$ and $\Delta = 0$

↑↑ upright shape so $a > 0$

no x intercepts, so $\Delta < 0$

2. The parabola with equation $y = x^2$ is translated so that its image has its vertex at $(-4, 3)$. The equation of the image is:

A. $y = (x-4)^2 + 3$
B. $y = (x-3)^2 + 4$
C. $y = (x+4)^2 + 3$
D. $y = (x+3)^2 - 4$
E. $y = -4x^2 + 3$

3. The expression $(4x + 1)^2 + 3x - 2$ is equal to

A. $4x^2+5x-1$
B. $4x^2+11x-1$
C. $16x^2+5x-1$
D. $16x^2+5x+1$
E. $16x^2+11x-1$

$(4x+1)^2 + 3x - 2$

$= 16x^2 + 8x + 1 + 3x - 2$

$= 16x^2 + 11x - 1$

4. If $ax^3 + 2x^2 + 5$ is exactly divisible by $x+1$, the value of a is

A. 1

B. 7

C. –1

D. 3

E. –7

$P(-1) = 0$

$-a + 2 + 5 = 0$

$a = 7$

5. If the graph of $y = 2x^2 - kx + 6$ touches the x-axis, the possible value(s) of k is/are

A. $k = 2$ or $k = -3$

B. $k = 1$

C. $k = -2$ or $k = -3$

D. $k = 4\sqrt{3}$ or $k = -4\sqrt{3}$

E. $k = 12\sqrt{2}$ or $k = -12\sqrt{2}$

$\Delta = 0$

$(-k)^2 - 4 \times 2 \times 6 = 0$

$k^2 - 48 = 0$

$k = \pm\sqrt{48}$

$k = \pm 4\sqrt{3}$

6. The expression $27x^3 - 8$ expressed as a product of a linear and a quadratic factor is equal to

A. $(3x + 2)(9x^2 + 6x + 4)$

B. $(3x + 2)(9x^2 + 24x + 4)$

C. $(3x + 2)(9x^2 + 12x + 4)$

D. $(3x - 2)(9x^2 + 6x + 4)$

E. $(3x - 2)(9x^2 - 12x + 4)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

End of Section B

Section C: Extended Response

1. The polynomial $P(x) = 6x^3 + ax^2 + bx - 2$ where a and b are constants, has a factor of $x + 2$ and gives a remainder of 4 when divided by $x + 1$. Find the values of a and b.

$P(-2) = 0$

$-48 + 4a - 26 - 2 = 0$

$4a - 26 = 50$ (1)

$P(-1) = 4$

$-6 + a - b - 2 = 4$

$a - b = 12$ (2)

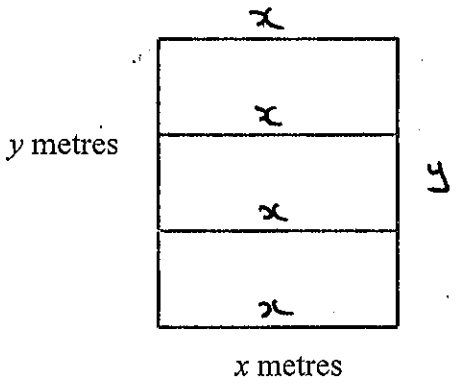
Solve (1) + (2) using CAS

$a = 13$

$b = 1$

(4 marks)

2. A rectangular gate is made from 10 metres of metal tubing. The frame has a length of y metres and a width of x metres. The design is shown in the diagram below.



(a) Find an expression for the total length of tubing in terms of x and y.

Total length is $2y + 4x$

(b) Find the equation expressing y in terms of x.

$2y + 4x = 10$

$2y = 10 - 4x$

$y = 5 - 2x$

The gate is to be covered with a sheet of metal.

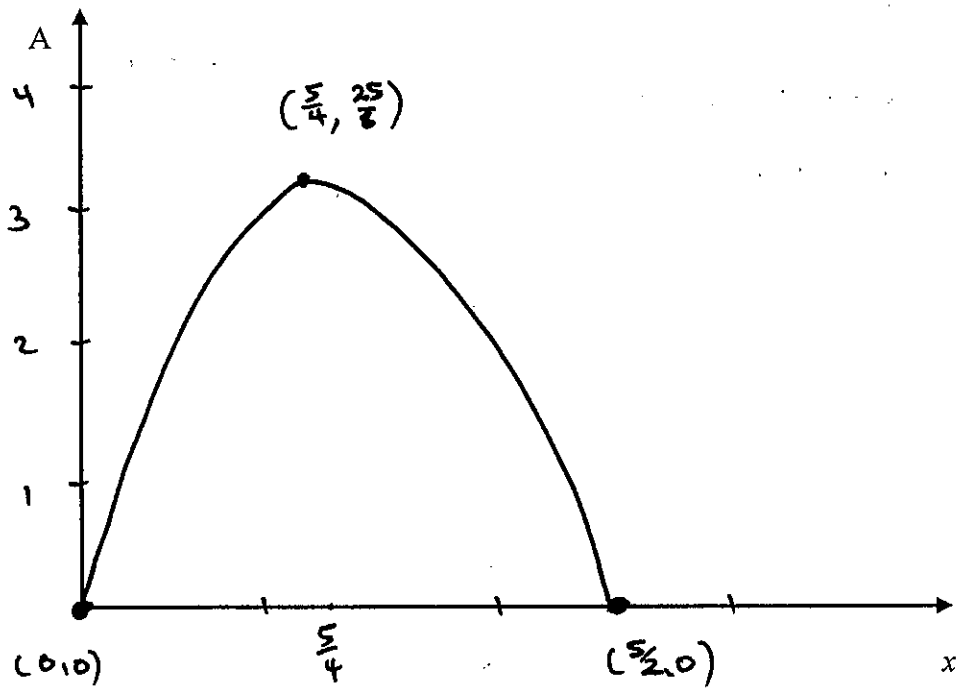
(c) Find an equation for the area of the gate, A, in terms of x.

$A = L \times w$

$= x(5-2x)$

$A = 5x - 2x^2$

(d) Sketch the graph of the equation found in part (c). Label the coordinates of the axial intercepts and the turning point.



(e) What is the maximum possible area of the gate?

$\frac{25}{8}$ m^2

(f) If the area of the gate is 1.6 m^2, find the approximate dimensions of the gate, correct to 2 decimal places.

$5x - 2x^2 = 1.6$

$x = 0.38$ or 2.12

$y = 4.24$ 0.76

(1+1+2+3+1+2 = 10 marks)

width 0.38 m

length 4.24 m

3. Consider the following two functions, where k is an unknown constant.

$y = kx^2 - 3x + 1$ and $y = kx + 2$

(a) Show that at the point(s) where the graphs of these functions intersect must satisfy the equation $kx^2 - (3 + k)x - 1 = 0$

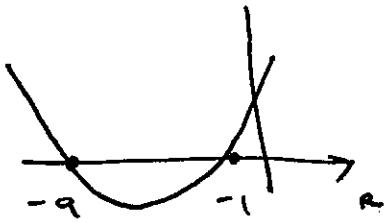
$$kx^2 - 3x + 1 = kx + 2$$
$$kx^2 - 3x - kx + 1 - 2 = 0$$
$$kx^2 - (3+k)x - 1 = 0$$

(b) Show that the discriminant of the quadratic $kx^2 - (3 + k)x - 1 = 0$ is equal to $k^2 + 10k + 9$.

$$\Delta = [-(3+k)]^2 - 4 \times k \times -1$$
$$= 9 + 6k + k^2 + 4k$$
$$= k^2 + 10k + 9$$

(c) Hence, find the values of k for which the graphs of the two functions given at the start of this problem do not intersect.

$$\Delta < 0$$
$$k^2 + 10k + 9 < 0$$
$$(k+9)(k+1) < 0$$



$$-9 < k < -1$$

(2+2+4 = 8 marks)

End of Section C