Neap:

Trial Examination 2018

VCE Mathematical Methods Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 19 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2018 VCE Mathematical Methods Units 3&4 Written Examination 2.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

The linear function $f : D \to R$, $f(x) = 4 - 3x$ has a range of $(-5, 4]$.

The domain D is

A. $[0, 3)$

B. $(0, 4)$

C. $\left(\frac{4}{3}, 0\right)$

D. $(-8, 19]$

E. R

Question 2

For the function with the rule $f(x) = -x^2 + 3$, the average rate of change with respect to x over the interval $[-1, 2]$ is

A. -2
B. -1
C. 1
D. 2
E. 3

Question 3

The length of the line segment that joins $(-1, 3)$ to $(2, -3)$ is

A. $\sqrt{3}$
B. $5\sqrt{3}$
C. $3\sqrt{5}$
D. $\sqrt{37}$
E. 45

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Question 4

A transformation $T: R^2 \rightarrow R^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ maps the graph of $y = 2^{x+1} + 2$ onto the graph of

A. $y = \frac{1}{2} \times 2^{-x+1} + 2$

B. $y = \frac{1}{2}(2^{-(x+1)} + 2)$

C. $y = 2^{-x} + 2$

D. $y = 2^{-x} + 1$

E. $y = -2^{2(x+1)} - 2$

Question 5

If $2x + a$ is a factor of $2x^3 - 5x^2 + ax$ where $a \in R\backslash\{0\}$, then the value of a is

A. –7
B. –3
C. 3
D. 5
E. 7

Question 6

The number of goals, X, scored by a soccer player over her career is a random variable with the following discrete probability distribution table:

x	0	1	2	3	4
$\Pr(X = x)$	a	b	0.15	0.04	0.01

Given that $E(X) = 0.91$, the values of a and b are

A. $a = 0.35, b = 0.45$
B. $a = 0.4, b = 0.4$
C. $a = 0.45, b = 0.35$
D. $a = 0.455, b = 0.455$
E. $a = 0.55, b = 0.45$

Question 7

If $\int_1^4 f(x)dx = 10$, then $\int_1^4 2(1 - f(x))dx$ is equal to

A. –18
B. –14
C. –6
D. 14
E. 18

Question 8

The graph of $y = x^2 - ax$ has a range of $[-4, \infty)$ where a is a positive constant.

The value of a is

A. 1
B. 2
C. 4
D. 8
E. 16

Question 9

The average value of the function with the rule $f(x) = \log_e(2x + 4)$ over the interval $[0, 2]$ is

A. $\log_e(2)$
B. $\frac{\log_e(2)}{2}$
C. $4(\log_e(2) - 1)$
D. $8\log_e(2) - 2$
E. $\log_e(16) - 1$

Question 10

If $\Pr(B) = 0.7$, $\Pr(A' \cap B) = 0.3$ and $\Pr(A \cap B') = 0.1$, then $\Pr(A)$ is equal to

A. 0.3
B. 0.4
C. 0.5
D. 0.6
E. 0.7

Question 11

The function with the rule $f(x) = 4\tan\left(\frac{\pi x}{3}\right)$ has a maximal domain of

A. R
B. $R\backslash\{3n\}, n \in Z$
C. $R\backslash\{3(n + 1)\}, n \in Z$
D. $R\backslash\left\{\frac{3(n + 1)}{2}\right\}, n \in Z$
E. $R\backslash\left\{\frac{3(2n + 1)}{2}\right\}, n \in Z$

Question 12

The graph of the function f with the rule $f(x) = 2\log_e(x) + 1$ is reflected in the x-axis, then dilated from the y-axis by a factor of 4 before being translated 2 units to the right to become the graph of g.

The rule for the graph of g is

A. $-2\log_e\left(\frac{x-2}{4}\right) - 1$

B. $-2\log_e\left(\frac{x}{4} - 2\right) - 1$

C. $-2\log_e\left(\frac{x}{4}\right) - 3$

D. $-2\log_e(4x - 2) - 1$

E. $-2\log_e\left(\frac{x-2}{4}\right) + 1$

Question 13

The simultaneous equations $2x - ay = a - 2$ and $ax - 8y = a$ have infinitely many solutions for

A. $a = -4$

B. $a = 4$

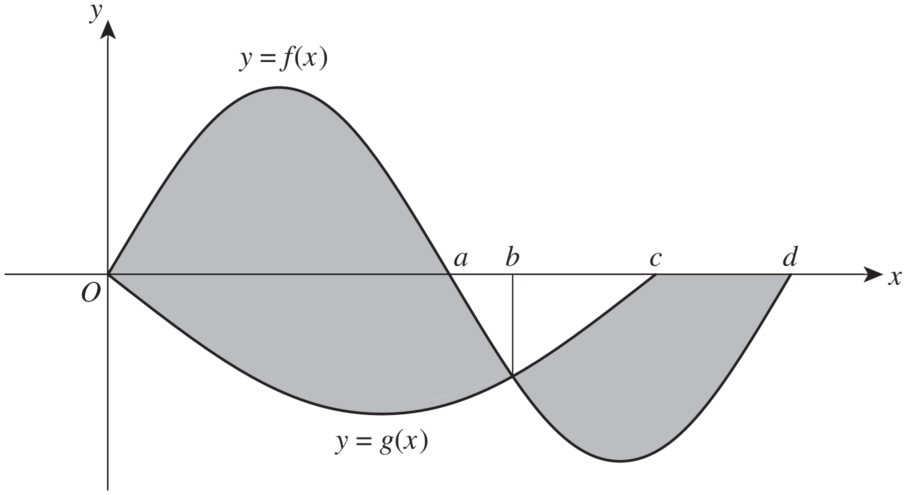
C. $a = -4$ and $a = 4$

D. $a = R\backslash\{-4, 4\}$

E. $a = R\backslash\{4\}$

Question 14

Consider the graphs of the smooth functions f and g shown below, where f is defined for $x \in [0, d]$ and g is defined for $x \in [0, c]$.



The area of the shaded region could be represented by

A. $\int_0^d (f(x) - g(x))dx$

B. $\int_0^b (f(x) - g(x))dx + \int_b^d (g(x) - f(x))dx$

C. $\int_0^a (f(x) + g(x))dx + \int_a^b (f(x) - g(x))dx + \int_b^d (g(x) - f(x))dx$

D. $\int_0^b (f(x) - g(x))dx + \int_c^b (f(x) - g(x))dx + \int_d^c g(x)dx$

E. $\int_0^b (f(x) - g(x))dx + \int_b^c (g(x) - f(x))dx + \int_c^d g(x)dx$

Question 15

The continuous random variable X has a normal distribution with mean of 20 and variance of 4. For a given number a, $\Pr(X > a) = 0.3$.

Correct to two decimal places, a is equal to

A. 17.49
B. 18.95
C. 21.05
D. 22.10
E. 24.30

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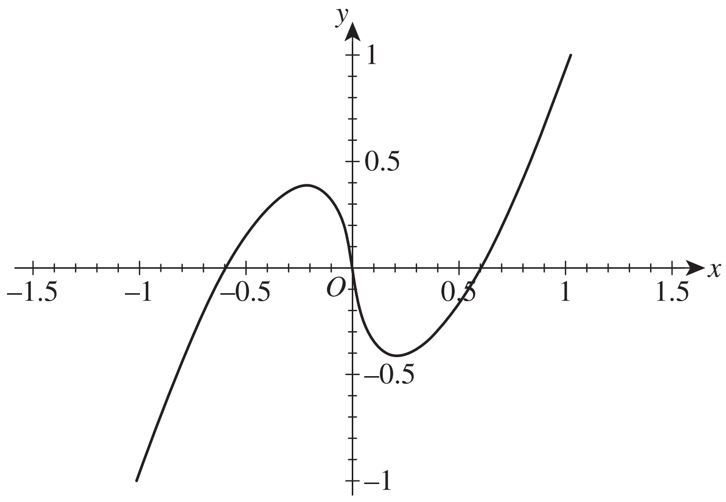
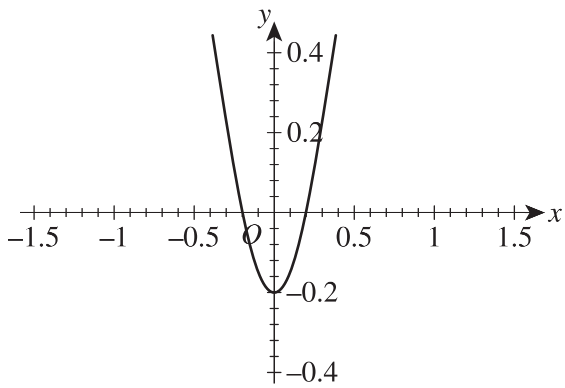
Question 16

The graph of the derivative function $y = f'(x)$ is shown below.

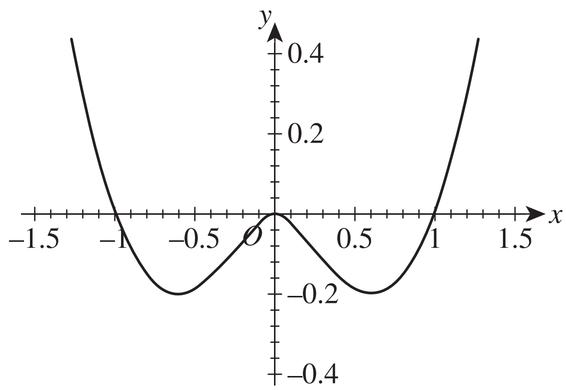


Which of the following could be the graph of the original function $y = f(x)$?

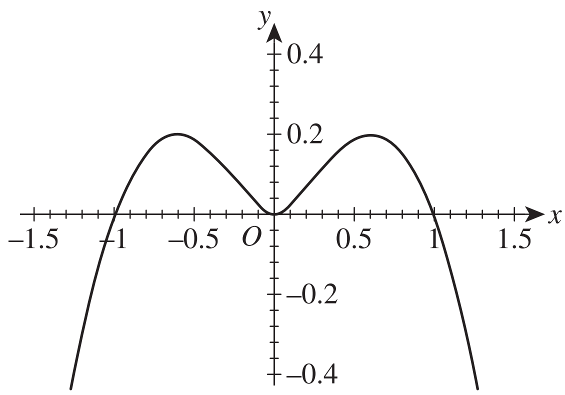
A.



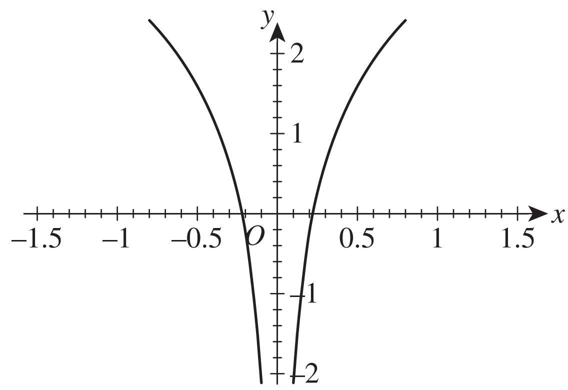
B.



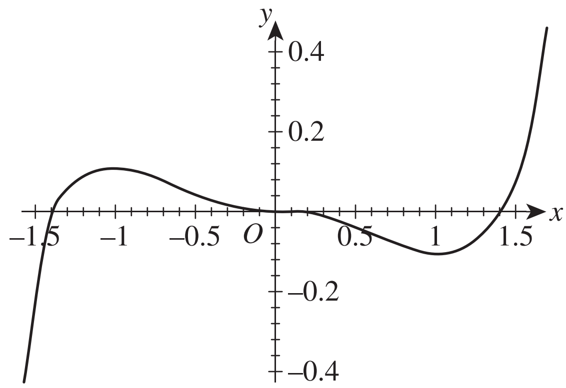
C.



D.



E.



Question 17

It is known that 96% of train travellers on a certain route had a valid ticket.

A sample of 500 travellers is taken. For samples of 500 travellers, $\hat{P}$ is the random variable of the distribution of sample proportions of travellers **without** a valid ticket.

$\Pr\left(\hat{P} \le \frac{3}{100}\right)$, when approximated by a normal distribution, is closest to

A. 0.1269
B. 0.1333
C. 0.1513
D. 0.4991
E. 0.8731

Question 18

Let $f(x) = x^2$ and $g(x) = \log_e(4 - 2x)$.

The maximal domain of f for the composite function $g(f(x))$ to exist is

A. $x \in \left(-\infty, -\frac{1}{2}\right)$

B. $x \in \left(-\infty, -\frac{\sqrt{2}}{2}\right] \cup \left[\frac{\sqrt{2}}{2}, \infty\right)$

C. $x \in \left(-\infty, -\frac{\sqrt{2}}{2}\right) \cup \left(\frac{\sqrt{2}}{2}, \infty\right)$

D. $x \in \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

E. $x \in \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$

Question 19

The equation $\sin(kx) = 1$ where $x \in [0, 2\pi]$ has **no solutions** when

A. $k \in R$

B. $k \in \pi$

C. $k \in \left[\frac{1}{4}, \infty\right)$

D. $k \in \left[0, \frac{1}{4}\right]$

E. $k \in \left(-\frac{3}{4}, 0\right]$

Question 20

Let $f : R \to R,\ f(x) = e^{2x}$.

For all $u \in R$, $f(u) - f(-u)$ is equal to

A. 0

B. $2f(u)$

C. $\dfrac{f(u) - 1}{f(u)}$

D. $\dfrac{(f(u))^2 - 1}{f(u)}$

E. $\dfrac{f(u^2) - 1}{f(u)}$

END OF SECTION A

SECTION B

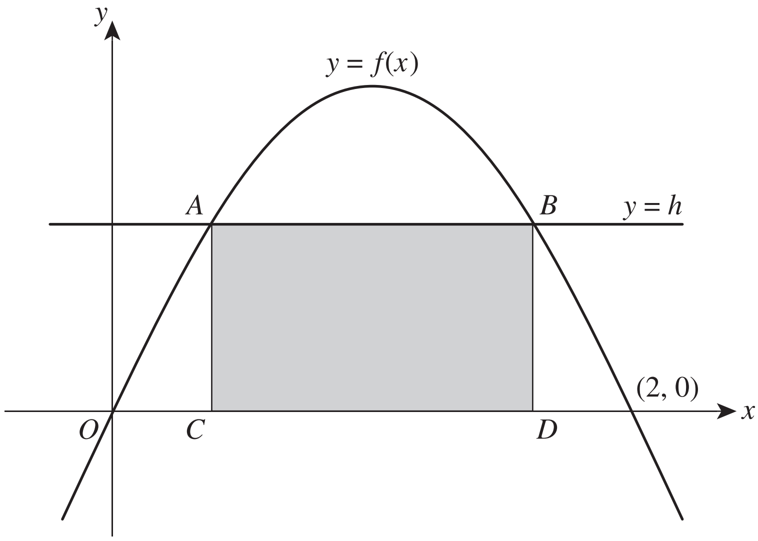
Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (14 marks)

Let $f: R \rightarrow R,\ f(x) = x(2 - x)$.

The line $y = h$ intersects f at the points A and B as indicated on the graph below.



a. **i.** Show that the coordinates of the point A can be given as $(1 - \sqrt{1 - h}, h)$. 2 marks

ii. Hence or otherwise, show that the length of the line AB can be given by $2\sqrt{1 - h}$. 1 mark

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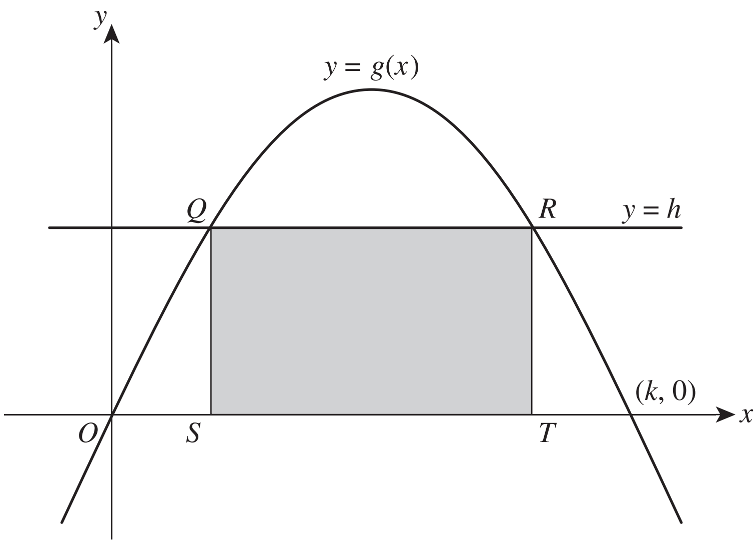
b. Find the area of the rectangle *ABCD*, expressing your answer in terms of h. 1 mark

c. Find the maximum area of the rectangle *ABCD*. 3 marks

Let $g : R \to R$, $g(x) = x(k - x)$ where $k > 0$.

The line $y = h$ intersects g at the points Q and R as indicated on the graph below, where $h > 0$.



d. Find the length of QR in terms of h and k. 2 marks

e. **i.** Find the maximum area of the rectangle *QRST* in terms of k. 4 marks

ii. Hence or otherwise, find the value of k for which the maximum area of the rectangle *QRST* is produced when *QRST* is a square. 1 mark

Question 2 (13 marks)

A bakery makes three types of bread rolls: white, brown and wholegrain. It is known that 60% of the rolls baked are white. The bread rolls are sold in large packets of 12 or small packets of 4. The bread rolls within each packet are randomly selected and the type of any one bread roll is independent from any other type of bread roll.

a. Find the probability that at least half of the bread rolls in a large packet are white. Give your answer correct to four decimal places. 2 marks

b. Find the probability that there are exactly 8 white bread rolls in a large packet given at least half the rolls are white. Give your answer correct to four decimal places. 2 marks

c. The probability that there are exactly 2 brown bread rolls in a small packet is 0.0486.

Find the expected number of brown rolls in a large packet. 3 marks

The total amount of flour that the bakery uses in any given day is a normally distributed random variable with a mean of 36 kg and a standard deviation of 5.2 kg.

d. Find the probability that the bakery uses at least 40 kg on any given day. Give your answer correct to four decimal places. 2 marks

e. The bakery records the number of days each week (7 days) where it uses at least 40 kg of flour.

Find the median number of days each week that the bakery uses at least 40 kg of flour. 2 marks

The manager of the bakery wants to estimate the proportion of customers who are satisfied with the quality of the bread rolls. The manager conducts a random survey of 80 customers and finds that 76 of these customers are satisfied with the quality of the rolls.

f. Determine a 90% confidence interval for the manager's estimate of the customers' proportion of satisfaction. Give your answer to 4 decimal places. 1 mark

g. After the original survey was completed, it was subsequently discovered that one of the customer's responses was invalid and needs to be removed from the results, and a new confidence interval calculated.

If the new 90% confidence interval is [0.9088, 0.9899] to four decimal places, find the new value of the sample proportion $\hat{p}$. 1 mark

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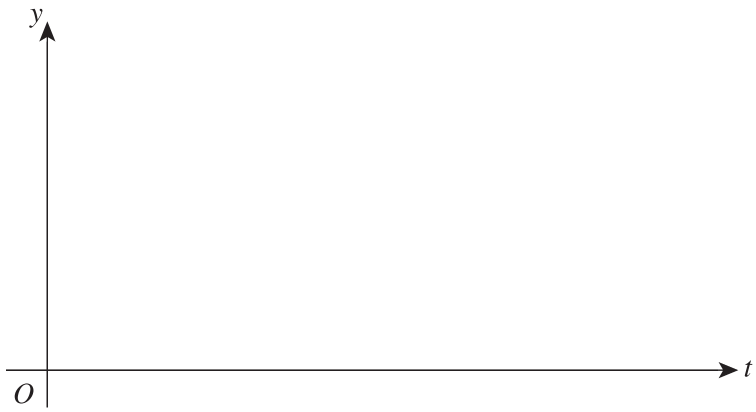
Question 3 (8 marks)

The temperature (in °C) of an oven used to bake bread is a continuous random variable *T*, and has a probability density function *f*, where

$$f(t)=\begin{cases} m(t-190) & 190 \le t \le 200 \\ -m(t-210) & 200 < t \le 210 \\ 0 & \text{otherwise} \end{cases}$$

a. Show that $m = 0.01$. 2 marks

b. Sketch the graph of $y = f(t)$ on the axes provided below. 2 marks



For the bread to be perfectly cooked the oven needs to be between 192°C and 208°C.

c. Find the probability that the bread is perfectly cooked. Give your answer correct to four decimal places. 2 marks

d. Find the probability that the bread rolls are perfectly cooked given it is known the temperature of the oven was greater than 205°C. 2 marks

Question 4 (14 marks)

Let $f : [0, \infty) \to R$, $f(x) = \sqrt{x}$ and $g(x) = f(x) \times (f(x) - 4)$.

a. Show that $g(x) = x - 4\sqrt{x}$. 1 mark

b. Find $g'(x)$ and hence find the coordinates of the stationary points of g. 3 marks

c. State the domain and range of g. 1 mark

Let $h : [s, \infty) \to R$, $h(x) = g(x)$.

d. Find the minimum value of s for the inverse function h^{-1} to exist. 1 mark

e. Show that the rule for the inverse function h^{-1} can be written as $h^{-1}(x) = (\sqrt{x + 4} + 2)^2$. 2 marks

f. State the domain and range of the inverse function h^{-1}. 1 mark

g. Let $d(x) = h^{-1}(x) - h(x)$.

i. Find the maximal domain of d. 1 mark

ii. Find the rule of the function $d(x)$. 1 mark

iii. Hence, show that graphs of $y = h(x)$ and $y = h^{-1}(x)$ do **not** intersect. 2 marks

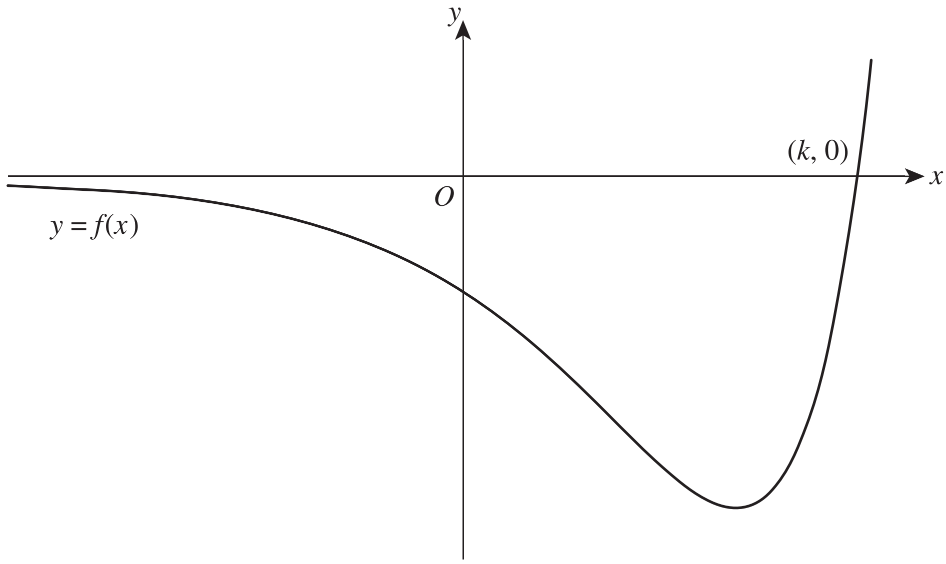
h. Let $q(x) = h(x) - c$ where $c \in R$.

For what value(s) of c will the equation $q(x) = h^{-1}(x)$ have exactly one solution? 1 mark

Question 5 (11 marks)

Let $f : R \rightarrow R, f(x) = (x - k)e^{x}$ where $k \geq 1$.

The graph of f is shown below.



a. **i.** Find $f'(x)$. 1 mark

ii. Hence, find the coordinates of the stationary point of f. 2 marks

b. Find the value(s) of n, where n is a real number, for which $f(x) = n$ has two solutions. 1 mark

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c. **i.** Show by differentiation that $\frac{d}{dx}[xe^x] = (x+1)e^x$. 1 mark

ii. Hence, find $\int f(x)dx$. 2 marks

d. Find the area enclosed by the graph of $y = f(x)$, the x-axis and the y-axis. Give your answer in terms of k. 2 marks

e. Let $g : R \to R$, $g(x) = 2f(4x)$.

Find the value of k for which the area enclosed by the graph $y = g(x)$, the x-axis and the y-axis is equal to $4 - \log_e(3)$. Give your answer correct to four decimal places. 2 marks

END OF QUESTION AND ANSWER BOOKLET