# Neap:

Trial Examination 2018

# VCE Mathematical Methods Units 3&4

## Written Examination 1

## Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

### Structure of booklet

| Number of questions | Number of questions to be answered | Number of marks |
|---|---|---|
| 7 | 7 | 40 |

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

**Materials supplied**

Question and answer booklet of 8 pages

Formula sheet

Working space is provided throughout the booklet.

**Instructions**

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

**At the end of the examination**

You may keep the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

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**Question 1** (4 marks)

**a.** Differentiate $x^3 \log_e(x + 2)$ with respect to $x$. 1 mark

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**b.** Let $f(x) = \dfrac{\cos(2x)}{x}$.

Find $f'\left(\dfrac{\pi}{4}\right)$. 3 marks

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**Question 2** (3 marks)

Let $g'(x) = (4 - x)^2$.

Given that $g(1) = 0$, find $g(x)$.

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**Question 3** (4 marks)

**a.** Solve $9^{1-3x} = \dfrac{1}{27}$ for $x$. 2 marks

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**b.** Solve $\log_2(3x) + \log_2(x+4) - \log_2(15) = 0$ for $x$. 2 marks

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**Question 4** (4 marks)

Consider the function $f$ with the rule $f(x) = \dfrac{1}{\sqrt{x}}$, and the function $g$ with the rule $g(x) = 1 - \dfrac{x^2}{4}$.

**a.** Find the rule for $f(g(x))$, expressing your answer in the form $\dfrac{a}{\sqrt{b-x^2}}$ where $a$ and $b$ are integers. 2 marks

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**b.** State the maximal domain for which $f(g(x))$ is defined. 2 marks

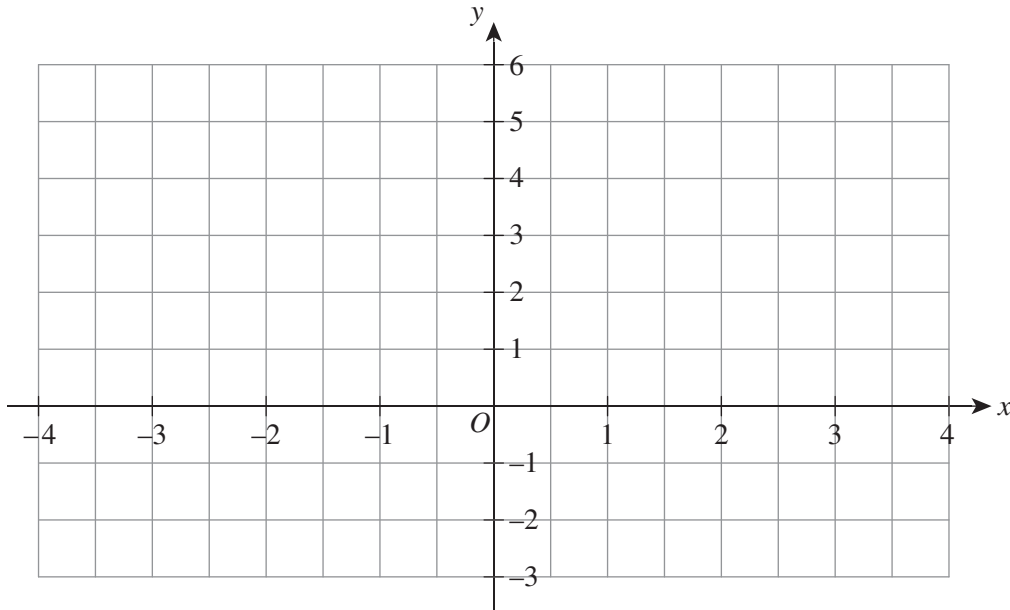_____

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**Question 5** (7 marks)

**a.** Let $f : [-1, 4] \rightarrow R$, where $f(x) = -3\sin\left(\dfrac{\pi x}{2}\right) + 3$.

State the range and the period of the function $f$. 2 marks

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**b.** On the axes below, sketch the graph of $f$, clearly indicating axes intercepts and turning points. Label the end points with their coordinates. 3 marks



**c.** Find the values of $x$ for which $f(x) = \dfrac{3}{2}$. 2 marks

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**Question 6** (9 marks)

The probability of a traffic jam on a particular section of road on any given day is $\frac{1}{4}$. Assume that a traffic jam on any given day is independent of a traffic jam on any other given day.

**a.** Find the probability that there will be a traffic jam on 3 consecutive days. 1 mark

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**b.** Find the probability that there will be a traffic jam on 3 consecutive days given that there was a traffic jam on at least 1 of these 3 days. 2 marks

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A new roundabout is installed that reduces the probability of a traffic jam on this section of road. The new probability of a traffic jam is given by *p*.

**c.** If the probability of exactly 1 traffic jam during any 2 consecutive days is now $\frac{8}{25}$, find the value of *p*. 3 marks

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The expected waiting time for a motorist queuing at the new roundabout is a normally distributed variable with a mean of 20 seconds and a standard deviation of 6 seconds.

The random variable *Z* is the standard normal distribution and $\Pr(Z > 1) = 0.16$.

**d.** Find the probability that a motorist spends less than 14 seconds queuing. 1 mark

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**e.** Find the probability that a motorist spends between 14 and 20 seconds queuing, given that they spent more than 14 seconds queuing. 1 mark

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**f.** The probability that a motorist spend more than 30 seconds queuing is equal to *k*.

Find the probability that a motorist will spend between 10 and 26 seconds queuing in terms of *k*. 1 mark

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**Question 7** (9 marks)

Let $f : (0, \infty) \to R$, $f(x) = \dfrac{k}{x^2}$ and $g : (0, \infty) \to R$, $g(x) = \dfrac{-k}{x^2}$ where $k > 0$.

The triangle $ABC$ is formed by the intersection of the tangents to $f$ and $g$ at $x = k$ and the $y$-axis.
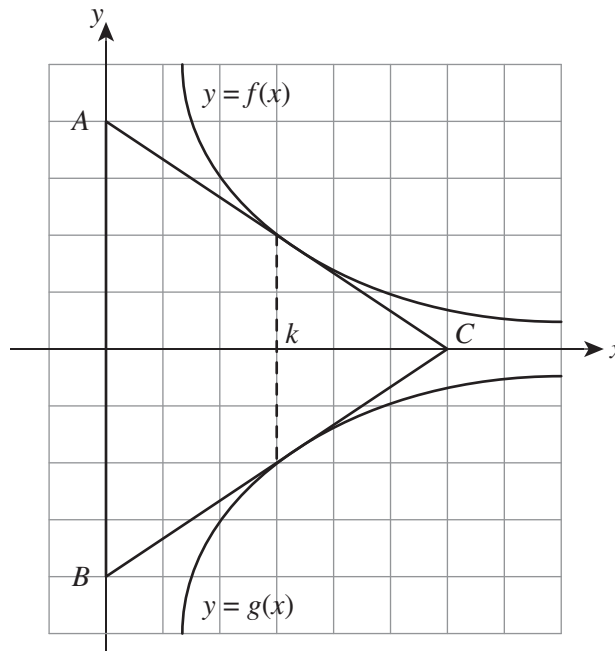


**a.** Find the equation of the tangent to $f$ at the point where $x = k$.     2 marks

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**b.** Find the coordinates of the points $A$, $B$ and $C$.     3 marks

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**c.** Show that the area of the triangle $ABC$ is independent of $k$.     2 marks

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**d.** If triangle *ABC* is equilateral, find the value of *k*, expressing your answer in the form $p^{\frac{1}{q}}$, where $p, q \in Z^+$. 2 marks

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**END OF QUESTION AND ANSWER BOOKLET**