The Mathematical Association of Victoria

Trial Exam 2018

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale .
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

© The Mathematical Association of Victoria, 2018

THIS PAGE IS BLANK

© The Mathematical Association of Victoria, 2018

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (5 marks)

a. Find $\frac{dy}{dx}$ when $y = 2x\log_e(2x)$. 2 marks

b. Find $f'(0)$ when $f(x) = \frac{e^{x^2}}{e^x + 1}$. 3 marks

TURN OVER

© The Mathematical Association of Victoria, 2018

Question 2 (4 marks)

The graph of the function $f(x) = 18x^2 - ax^4$ where $a \in R^+$ has a stationary point at $x = -\frac{3}{\sqrt{5}}$.

a. Show that the value of *a* is 5. 2 marks

b. Hence state the coordinates of the stationary points for the graph of f. 2 marks

© The Mathematical Association of Victoria, 2018

Question 3 (4 marks)

Let $g : \left[\frac{1}{2}, \infty\right) \to R$, where $g(x) = \sqrt{(2x-1)}$.

a. Show that the equation of the tangent to the curve at $x = 1$ is $y = x$. 2 marks

b. Find the area bounded by the graph of the function g, the line $y = x$ and the x-axis. 2 marks

TURN OVER

© The Mathematical Association of Victoria, 2018

Question 4 (3 marks)
A company is investigating the efficiency of one of its machines which packages muesli. A random sample of 400 packets was taken and the 95% confidence interval for the proportion of underweight muesli packets was found to be (0.024, 0.132).

a. Find the sample proportion from which this interval was obtained. 1 mark

_____

_____

_____

b. Using the nearest integer value for z, determine the standard deviation of the sample. 2 marks

_____

_____

_____

_____

_____

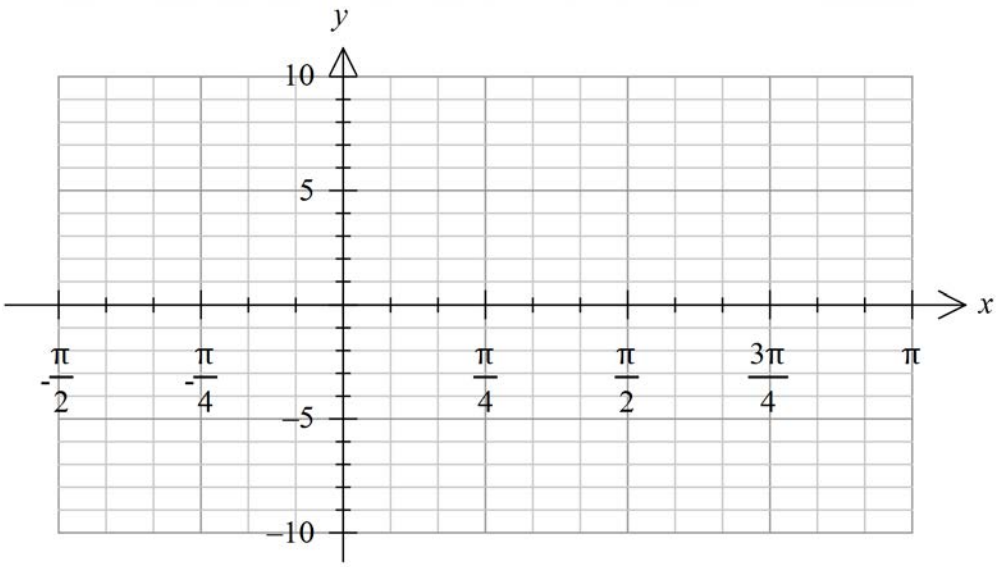
© The Mathematical Association of Victoria, 2018

Question 5 (6 marks)

a. Find the general solution to $\sqrt{3}\sin(2x) = \cos(2x)$. 2 marks

b. Show that there are no stationary points in the graph of $y = \sqrt{3}\tan(2x)$. 1 mark

c. **Hence** sketch the graph of $y = \sqrt{3}\tan(2x)$ for $x \in \left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$. On your graph label the points where $\sqrt{3}\tan(2x) = 1$ and the *x*-intercepts with their coordinates. Label any asymptotes with their equations. 3 marks



TURN OVER

© The Mathematical Association of Victoria, 2018

Question 6 (4 marks)
Solve $2\log_2(x-2)+\log_2(x)=0$ for x.

© The Mathematical Association of Victoria, 2018

Question 7 (4 marks)

The height of water in a dam is modelled by the function *h* with rule $h(t) = -20\sin\left(\frac{\pi t}{12} + 10\right) + 20$, where *h* is the height in metres at time *t* hours after 9 am.

Find the average value of the height of water in a dam, from 9 am to 9 pm on a certain day, giving your answer in the form $a\cos(b) + c$, where *a*, *b* and *c* are real constants.

© The Mathematical Association of Victoria, 2018

Question 8 (5 marks)
A random variable X has the following probability distribution, where a and b are real constants.

x	0	1	2	3
$\Pr(X = x)$	0.1	a	b	0.1

a. **i.** Find b in terms of a. 1 mark

ii. Hence show that $\mathrm{E}(X) = -a + 1.9$. 1 mark

b. If $\mathrm{Var}(X) = 0.56$ find all possible sets of values for a and b. 3 marks

© The Mathematical Association of Victoria, 2018

Question 9 (5 marks)
Let $f(x) = 2e^{1-x}$.

a. Find the rule and state the range for the inverse function, f^{-1}. 3 marks

_____

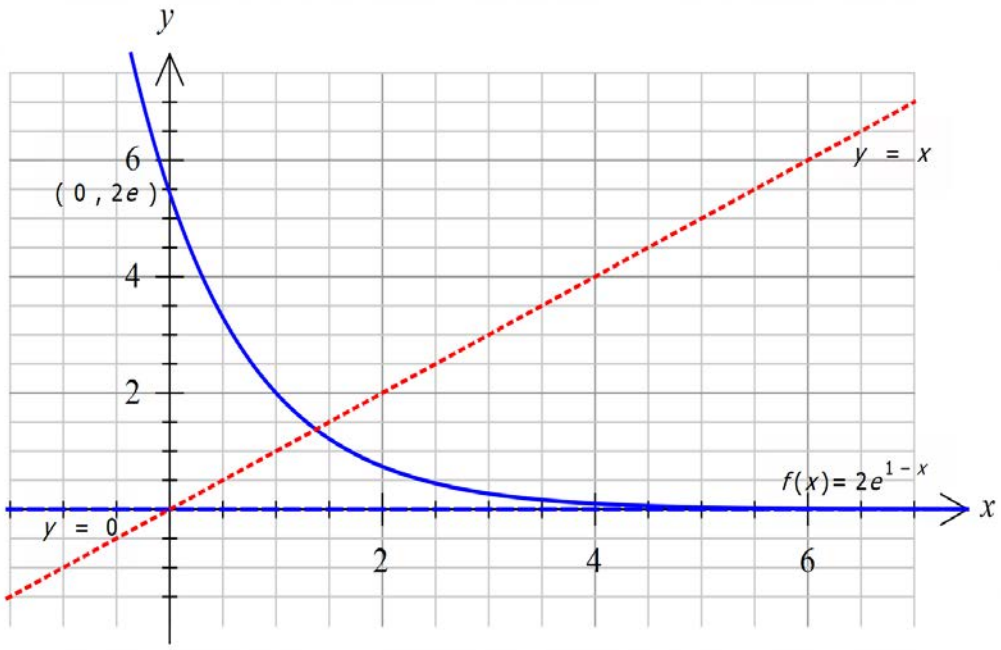
_____

_____

_____

_____

b. Part of the graph of f is shown below. Sketch the graph of f^{-1} on the same set of axes. Label the asymptote with its equation and axial intercept with its coordinates. 2 marks



END OF QUESTION AND ANSWER BOOK

© The Mathematical Association of Victoria, 2018

© The Mathematical Association of Victoria, 2018