The Mathematical Association of Victoria

Trial Examination 2018

MATHEMATICAL METHODS

Trial Written Examination 1 - SOLUTIONS

Question 1

a. $y = 2x\log_e(2x)$.

$\frac{dy}{dx} = \left(2x \times \frac{1}{x}\right) + \left(\log_e(2x) \times 2\right)$ **1M**

$\frac{dy}{dx} = 2 + 2\log_e(2x)$ **1A**

b. $f(x) = \frac{e^{x^2}}{e^x + 1}$.

$f(x) = \frac{\left(e^x + 1\right) \times e^{x^2} \times 2x - e^{x^2} \times e^x}{\left(e^x + 1\right)^2}$ **1M**

$f'(0) = \frac{\left(e^0 + 1\right) \times e^0 \times 0 - e^0 \times e^0}{\left(e^0 + 1\right)^2}$ **1M**

$f'(0) = \frac{0 - 1}{(2)^2} = -\frac{1}{4}$ **1A**

Question 2

a. $f(x) = 18x^2 - ax^4$

$f'(x) = 36x - 4ax^3$

$36x - 4ax^3 = 4x\left(9 - ax^2\right) = 0$ for stationary point **1M**

Gives $x = 0, x = \pm\frac{3}{\sqrt{a}}$

Given stationary point at $x = -\frac{3}{\sqrt{5}}$ where $a \in R^+$

$a = 5$ as required **1M (Show that)**

b. Coordinates

$(0, 0)$ **1A**

$\left(\frac{3}{\sqrt{5}}, \frac{81}{5}\right), \left(-\frac{3}{\sqrt{5}}, \frac{81}{5}\right)$ **1A**

Question 3

a. $g:\left[\frac{1}{2},\infty\right)\to R, g(x)=\sqrt{2x-1}$

$g(x)=(2x-1)^{\frac{1}{2}}$

$g(1)=(2-1)^{\frac{1}{2}}=1$

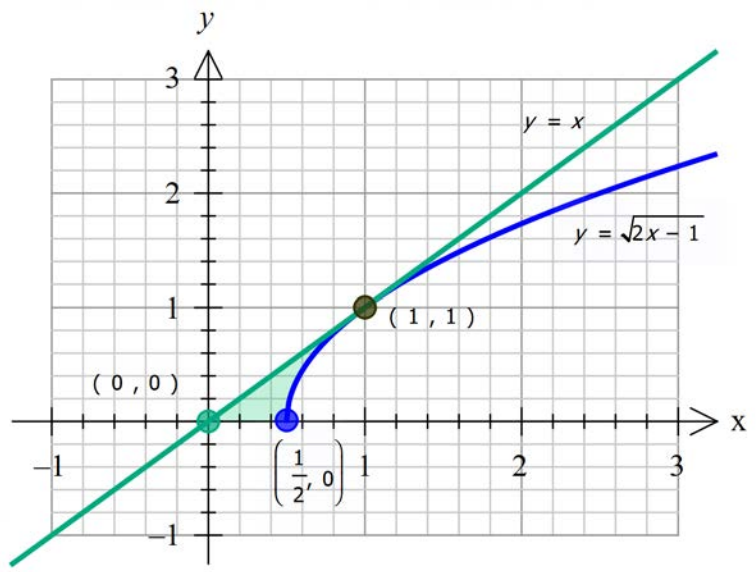
$g'(x)=2\times\frac{1}{2}(2x-1)^{-\frac{1}{2}}=\frac{1}{\sqrt{2x-1}}$ using the chain rule **1A**

$m=g'(1)=\frac{1}{\sqrt{2-1}}=1$

$y-1=1\times(x-1)$

$y=x$ **1M (Show that)**

b.



$A=\int_0^{\frac{1}{2}}(x)dx+\int_{\frac{1}{2}}^{1}\left(x-\sqrt{2x-1}\right)dx$ **1M**

$=\left[\frac{x^2}{2}\right]_0^{\frac{1}{2}}+\left[\frac{x^2}{2}-\frac{(2x-1)^{\frac{3}{2}}}{3}\right]_{\frac{1}{2}}^{1}$

$=\frac{1}{8}+\left(\frac{1}{2}-\frac{1}{3}\right)-\left(\frac{1}{8}-0\right)$

$=\frac{1}{6}$ **1A**

OR

$A = \text{Area of the triangle} - \int_{\frac{1}{2}}^{1} \left(\sqrt{2x-1}\right) dx$ **1M**

$= \frac{1}{2} - \left[\frac{(2x-1)^{\frac{3}{2}}}{3}\right]_{\frac{1}{2}}^{1}$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1}{6}$ **1A**

Question 4

a. $\hat{p} = \frac{0.132 + 0.024}{2} = \frac{0.156}{2} = 0.078$ **1A**

b. $0.132 - 0.078 = 0.054$

$2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 2\sigma_{\bar{X}} = 0.054$ **1M**

$\sigma_{\bar{X}} = 0.027$ **1A**

Question 5

a. $\sqrt{3}\sin(2x) = \cos(2x)$

$\frac{\sqrt{3}\sin(2x)}{\cos(2x)} = 1$

$\tan(2x) = \frac{1}{\sqrt{3}}$ **1A**

$2x = \frac{\pi}{6} + n\pi, n \in Z$

$x = \frac{\pi}{12} + \frac{n\pi}{2}, n \in Z$ **1A**

b. $\frac{d}{dx}\left(\tan(2x)\right) = \frac{2}{\cos^2(2x)}$

$y = \sqrt{3}\tan(2x)$

$\frac{dy}{dx} = \frac{2\sqrt{3}}{\cos^2(2x)}$

For stationary point

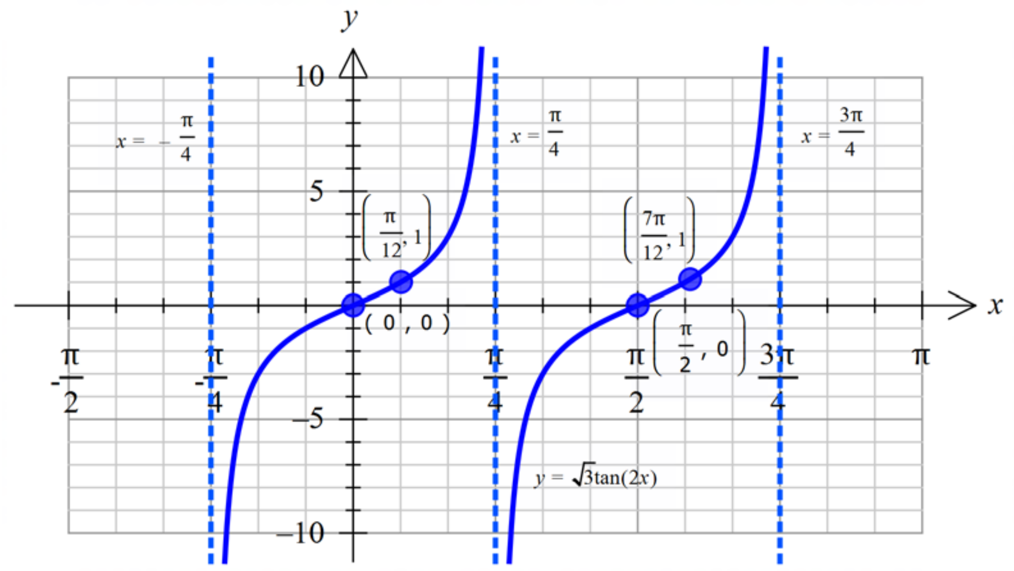
$\frac{dy}{dx} = \frac{2\sqrt{3}}{\cos^2(2x)} = 0$ gives no solution for x.

So no stationary point. **1M (Show that)**

c. Shape **1A**

Vertical asymptotes at $x=-\frac{\pi}{4},\frac{\pi}{4},\frac{3\pi}{4}$ **1A**

Points $\left(\frac{\pi}{12},1\right),\left(\frac{7\pi}{12},1\right)$, (0, 0), $\left(\frac{\pi}{2},0\right)$ **1A**



Question 6

$2\log_2(x-2)+\log_2(x)=0$

$\log_2(x-2)^2+\log_2(x)=0$

$\log_2\left(x(x-2)^2\right)=0$ **1A**

$x(x-2)^2=2^0=1$

$x(x^2-4x+4)=1$

$x^3-4x^2+4x-1=0$ **1A**

$(x-1)(x^2-3x+1)=0$ **1A**

$x\neq 1,\ x\neq\frac{3-\sqrt{5}}{2}$ as $x>2$

$x=\frac{3+\sqrt{5}}{2}$ only **1A**

Question 7

Average value $= \frac{1}{12-0}\int_0^{12}\left(-20\sin\left(\frac{\pi t}{12}+10\right)+20\right)dt$ **1M**

$= \frac{20}{12}\int_0^{12}\left(-\sin\left(\frac{\pi t}{12}+10\right)+1\right)dt$

$= \frac{5}{3}\int_0^{12}\left(-\sin\left(\frac{\pi t}{12}+10\right)+1\right)dt$

$= \frac{5}{3}\left[\frac{12}{\pi}\cos\left(\frac{\pi t}{12}+10\right)+t\right]_0^{12}$ **1M**

$= \frac{5}{3}\left[\left(\frac{12}{\pi}\cos(\pi+10)+12\right)-\left(\frac{12}{\pi}\cos(0+10)+0\right)\right]$

Average height

$= \frac{5}{3}\left[\left(\frac{12}{\pi}\cos(\pi+10)+12\right)-\left(\frac{12}{\pi}\cos(10)\right)\right]$

$= \frac{20}{\pi}\cos(\pi+10)-\frac{20}{\pi}\cos(10)+20$ **1A**

$\cos(\pi+x)=-\cos(x)$

So the expression simplifies to

$= -\frac{20}{\pi}\cos(10)-\frac{20}{\pi}\cos(10)+20$

$= -\frac{40}{\pi}\cos(10)+20$ **1A**

Question 8

a.i. $0.1+a+b+0.1=1$

$a+b=0.8,\ b=0.8-a$ **1A**

ii. $\mathrm{E}(X)=a+2b+0.3$

$= a+1.6-2a+0.3$

$= -a+1.9$ **1M (Show that)**

b. $\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$

$a+4b+0.9-(-a+1.9)^2=0.56$ **1M**

$a+4b+0.9-(a^2-3.8a+3.61)=0.56$

$a^2-0.8a+0.07=0$ **1A**

$(a-0.1)(a-0.7)=0$

$a=0.1,\ b=0.7$ OR $a=0.7,\ b=0.1$ **1A**

Question 9

a. Let $y = f(x) = 2e^{1-x}$

Inverse swap x and y.

$x = 2e^{1-y}$ **1M**

$\frac{x}{2} = e^{1-y}$

$1 - y = \log_e\left(\frac{x}{2}\right)$

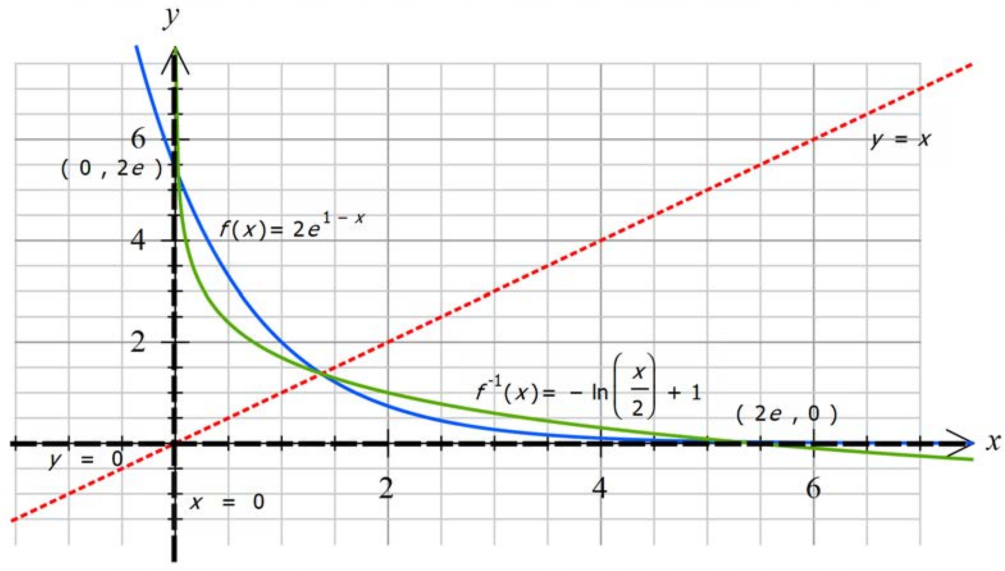
$y = -\log_e\left(\frac{x}{2}\right) + 1$

$f^{-1}(x) = -\log_e\left(\frac{x}{2}\right) + 1$ **1A**

Range is R **1A**

b. Shape and three points of intersection **1A**

Asymptote $x = 0$ and intercept $(2e, 0)$ **1A**



END OF SOLUTIONS