2018 Mathematical Methods Trial Exam 1 Solutions

Q1a The function and its inverse intersect at $y = x$.

Let $mx^2 + 1 = x$, $mx^2 - x + 1 = 0$. For one solution, $\Delta = 0$,

$1 - 4m = 0$, $m = \frac{1}{4}$

Q1b $x = -\frac{-1}{2m} = 2$, $(2, 2)$

Q2a Stationary points, $f'(x) = 3(x-1)^2 + m = 0$, $x = 1 \pm \sqrt{\frac{-m}{3}}$.

For two stationary points, $m < 0$.
For one, $m = 0$. For none, $m > 0$.

Q2b Select $m = 1$, $f'(x) = 3(x-1)^2 + 1 = 3x^2 - 6x + 4$

$f(x) = x^3 - 3x^2 + 4x + 1$

Q3a Remainder $= f\left(\frac{1}{2}\right) = 9$

Q3b Translate $f(x)$ in the negative *y*-direction by 9 units.

Q3c Expand and compare coefficients of x^3 and x^2:
$4p - 8 = 8$ and $-2p + 4q = 0$, $p = 4$ and $q = 2$

Q4a $\cos(\sin x) = 1$, $\sin x = 0$ since $-1 \le \sin x \le 1$

.: $x = n\pi$ where n is an integer

Q4b $f'(x) = (-\sin(\sin x))(\cos x)$

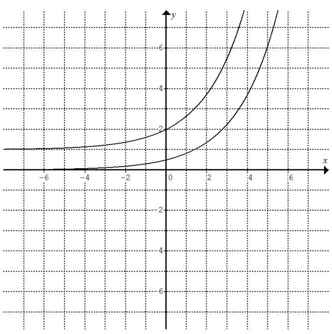
Let $f'(x) = 0$, $(-\sin(\sin x))(\cos x) = 0$

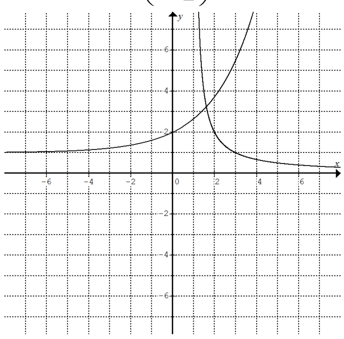
$\sin x = 0$ or $\cos x = 0$, .: $x = \frac{n\pi}{2}$ where n is an integer

Q5a $f(x) = 1 + e^{\frac{x}{2}}$, $f'(x) = \frac{1}{2}e^{\frac{x}{2}}$

The graph of $f'(x)$ is the same as the graph of $f(x) - 1$ dilated parallel to the *y*-axis by a factor of $\frac{1}{2}$.

It has $y = 0$ as an asymptote, and *y*-intercept $\left(0, \frac{1}{2}\right)$.





Q5b $f^{-1}(x) = 2\log_e(x-1)$, $\frac{d}{dx}f^{-1}(x) = \frac{2}{x-1}$ for $x > 1$

The graph of $\frac{d}{dx}f^{-1}(x)$ has asymptotes $y = 0$ and $x = 1$.

Q6a $y = \log_e(a\tan x) = \log_e a + \log_e(\tan x)$

$$\frac{dy}{dx} = \frac{1}{\tan x} \times \sec^2 x = \frac{\cos x}{\sin x} \times \frac{1}{\cos^2 x} = \frac{1}{(\sin x)(\cos x)}$$

Q6b $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{(\sin x)(\cos x)}dx = \left[\log_e(a\tan x)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$$= \log_e\left(a\tan\frac{\pi}{3}\right) - \log_e\left(a\tan\frac{\pi}{6}\right) = \log_e\left(a\sqrt{3}\right) - \log_e\left(\frac{a}{\sqrt{3}}\right) = \log_e 3$$

Q7a $v = \frac{t+6}{(t+1)^2} = \frac{t+1}{(t+1)^2} + \frac{5}{(t+1)^2} = \frac{1}{t+1} + \frac{5}{(t+1)^2}$

Q7b Distance travelled $= \int_0^4 v\,dt = \int_0^4 \left(\frac{1}{t+1} + \frac{5}{(t+1)^2}\right)dt$

$$= \left[\log_e(t+1) - \frac{5}{t+1}\right]_0^4 = \log_e 5 + 4.$$

Average speed $= \frac{\log_e 5 + 4}{4}$ m s^{-1}

Q8 $p \approx \hat{p} = \frac{900}{2500} = 0.36$

95% confidence interval

$$\approx \left(0.36 - 2\sqrt{\frac{0.36 \times 0.64}{2500}}, 0.36 + 2\sqrt{\frac{0.36 \times 0.64}{2500}}\right)$$

$$\approx (0.36 - 0.02, 0.36 + 0.02) = (0.34, 0.38)$$

Q9a

	Male	Not male	
VCE	0.24	0.08	0.32
Not VCE	0.16	0.52	0.68
	0.40	0.60	1

$$\Pr(\text{male} \mid \text{VCE}) = \frac{\Pr(\text{male} \cap \text{VCE})}{\Pr(\text{VCE})} = \frac{0.24}{0.32} = 0.75$$

Q9b Binomial distribution: $n = 25$, $p = \frac{540}{1200} = 0.45$

$\Pr(X = 12 \text{ or } 13) = \Pr(X = 12) + \Pr(X = 13)$

$= {}^{25}C_{12}\left(0.45^{12}\right)\left(0.55^{13}\right) + {}^{25}C_{13}\left(0.45^{13}\right)\left(0.55^{12}\right)$

Q10a $1.5a \times 2 + a \times 2 = 1$, $a = 0.2$

Q10b Let M be the median.

$$\int_1^M 1.5 \times 0.2\,dx = [0.3x]_1^M = 0.3(M-1) = 0.5,\ M = \frac{8}{3}$$

Q10c $\overline{X} = \int_{-\infty}^{\infty} xf(x)dx = \int_1^3 0.3x\,dx + \int_4^6 0.2x\,dx$

$$= \left[\frac{0.3x^2}{2}\right]_1^3 + \left[\frac{0.2x^2}{2}\right]_4^6 = 1.2 + 2 = 3.2$$

Please inform mathline@itute.com re conceptual and/or mathematical errors