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Year 12 *Trial Exam Paper*

2018

MATHEMATICAL METHODS

Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	4	4	60
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring blank sheets of paper and/or correction fluid/tape into the examination.

Materials provided

- Question and answer book of 27 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions

- Write your **name** in the box provided above on this page, and on your answer sheet for multiple-choice questions.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The gradient of a line perpendicular to the line that passes through (–3, 2) and (1, 5) is

A. $\frac{3}{4}$

B. $\frac{4}{3}$

C. $-\frac{3}{4}$

D. $-\frac{4}{3}$

E. $\frac{2}{3}$

Question 2

The range of the function $f:(-1,\ 7]\rightarrow R,\ f(x)=-x^2+4x-b,$ is

A. $[-21-b,\ \ -5-b)$

B. $(-21-b,\ \ -5-b]$

C. $[-21-b,\ \ -12-b]$

D. $(-5-b,\ \ 4-b]$

E. $[-21-b,\ \ 4-b]$

Question 3

The function with rule $f(x) = -2\tan\left(-\frac{x}{3} + \frac{2\pi}{5}\right) + 4$ has period

A. 6π

B. 3π

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

E. $\frac{\pi}{5}$

Question 4

The graph of the function $f: (0, \infty) \to R,\ f(x) = \frac{1}{\sqrt{x}}$ is reflected in the y-axis and then translated 4 units left and 3 units down. Which one of the following is the rule of the transformed graph?

A. $y = \frac{1}{\sqrt{-x-4}} - 3$

B. $y = \frac{1}{\sqrt{x-4}} - 3$

C. $y = \frac{1}{\sqrt{-x+4}} - 3$

D. $y = \frac{-1}{\sqrt{x-4}} + 3$

E. $y = \frac{-1}{\sqrt{x+4}} - 3$

Question 5

Which one of the following is the inverse function of the function $f: [-1, \infty) \to R,\ f(x) = 1 - 2\sqrt{x+1}$?

A. $f^{-1}: (-\infty, 1] \to R,\ f^{-1}(x) = \frac{1}{4}(x-1)^2 + 1$

B. $f^{-1}: [1, \infty) \to R,\ f^{-1}(x) = \frac{1}{4}(x-1)^2 - 1$

C. $f^{-1}: (-\infty, 1] \to R,\ f^{-1}(x) = -\frac{1}{4}(x-1)^2 + 1$

D. $f^{-1}: [1, \infty) \to R,\ f^{-1}(x) = \frac{1}{4}(x-1)^2 + 1$

E. $f^{-1}: (-\infty, 1] \to R,\ f^{-1}(x) = \frac{1}{4}(x-1)^2 - 1$

SECTION A – continued
TURN OVER

Question 6

The largest value of a for which the inverse function of $g: (-a, a) \to R$, $g(x) = x\cos(x)$ will exist is approximately

A. $a = 0.83$

B. $a = 0.84$

C. $a = 0.85$

D. $a = 0.86$

E. $a = 0.87$

Question 7

Two functions, f and g, are defined as follows:

$$f: \left(-\infty, -\frac{2}{3}\right) \to R, \; f(x) = \log_e(2-3x)$$

$$g: B \to R, \; g(x) = 3x^2 - 4\sqrt{2}x + 1$$

The maximal domain for which the function with rule $f(g(x))$ will be defined is

A. $B = \left\{x : \frac{-2\sqrt{2}-\sqrt{17}}{3} < x < \frac{-2\sqrt{2}+\sqrt{17}}{3}\right\}$

B. $B = \left\{x : \frac{2\sqrt{2}-\sqrt{7}}{3} < x < \frac{2\sqrt{2}+\sqrt{7}}{3}\right\}$

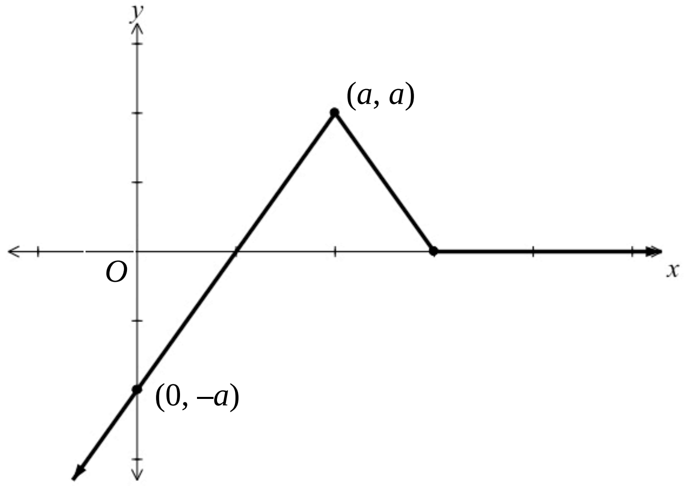
C. $B = \left\{x : \frac{2\sqrt{2}-\sqrt{3}}{3} < x < \frac{2\sqrt{2}+\sqrt{3}}{3}\right\}$

D. $B = \left\{x : x < \frac{2\sqrt{2}-\sqrt{7}}{3}\right\}$

E. $B = \left\{x : x < \frac{\sqrt{3}-2\sqrt{2}}{3}\right\}$

Question 8

Part of the graph of a function *f* is shown below.



Which one of the following is the average value of f over the interval $[0,\ 2a]$?

A. $\frac{3a}{8}$

B. $\frac{a}{6}$

C. $\frac{a}{8}$

D. $\frac{a}{2}$

E. $\frac{1}{2}$

Question 9

The normal to the graph of $y=\frac{1}{2}\sin(3x)+1$ at the point where $x=-\frac{\pi}{18}$ is

A. $y=\frac{4\sqrt{3}}{9}x+\frac{2\sqrt{3}\pi-162}{81}$

B. $y=\frac{4\sqrt{3}}{9}x+\frac{2\sqrt{3}\pi+162}{81}$

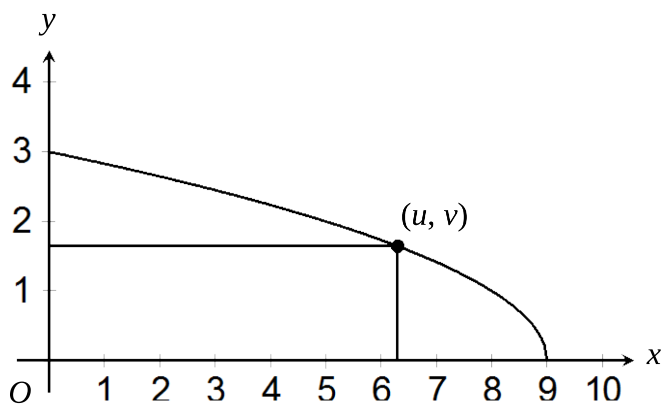
C. $y=-\frac{4\sqrt{3}}{9}x-\frac{162+2\sqrt{3}\pi}{81}$

D. $y=-\frac{4\sqrt{3}}{9}x+\frac{2\sqrt{3}\pi-162}{81}$

E. $y=-\frac{4\sqrt{3}}{9}x-\frac{8\pi\sqrt{3}-243}{324}$

Question 10

A rectangle is formed by using part of the coordinate axes and a point (u, v), where $u > 0$, on the graph of $y = \sqrt{9 - x}$.



What is the maximum area of the rectangle?

A. $\dfrac{13\sqrt{5}}{2\sqrt{2}}$

B. $6\sqrt{3}$

C. 6

D. 10

E. $4\sqrt{5}$

Use the following information to answer Questions 11 and 12.

Let f and g be two functions defined such that $f(0) = 3,\ f'(0) = -6,\ g(0) = 4$ and $g'(0) = 2$.

Question 11

If $h(x) = \dfrac{g(x)}{f(x)}$, then the value of $h'(0)$ is equal to

A. $-\dfrac{1}{3}$

B. $\dfrac{10}{3}$

C. $\dfrac{8}{3}$

D. $\dfrac{10}{9}$

E. $\dfrac{5}{8}$

Question 12

If $k(x) = f(x)\sqrt{g(x)}$, then the value of $k'(0)$ is equal to

A. $-6\sqrt{2}$

B. $3\sqrt{2} - 12$

C. $\dfrac{3\sqrt{2} - 24}{2}$

D. $-\dfrac{45}{4}$

E. $-\dfrac{21}{2}$

Question 13

Let the functions $f: R \to R,\ f(x) = x(x+2)$ and $g: \left[-6,\ -\frac{3}{2}\right] \to R,\ g(x) = 3x - 2.$

If the function h has the rule $h = f - g,$ then the domain of the inverse function of h is

A. $\left[-\frac{4}{3},\ \frac{1}{6}\right]$

B. $\left[-\frac{3}{4},\ 24\right]$

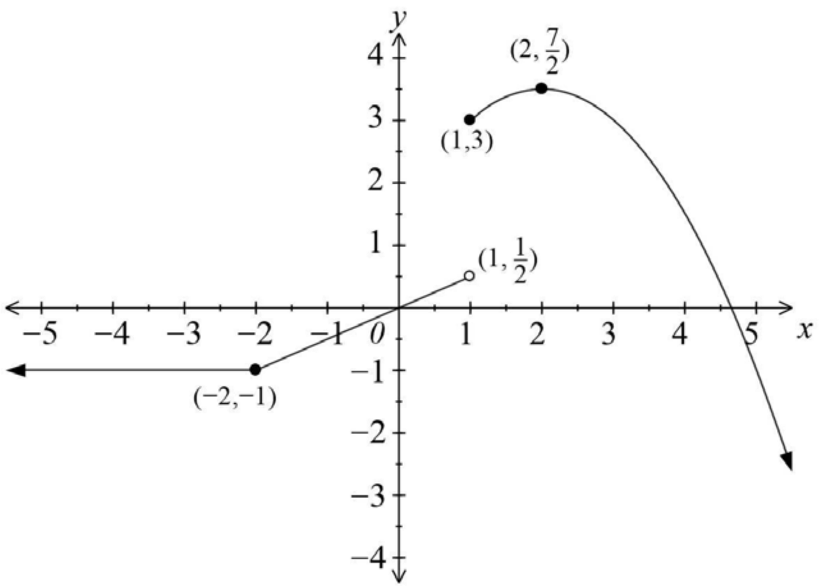
C. $\left[\frac{23}{4},\ 44\right]$

D. $\left[\frac{7}{4},\ 40\right]$

E. $\left[-6,\ -\frac{3}{2}\right]$

Question 14

The graph of the function f is shown below.



The maximal set of values of x for which f is a strictly increasing function is

A. $[-2,\ 2]$

B. $(-2,\ 2)$

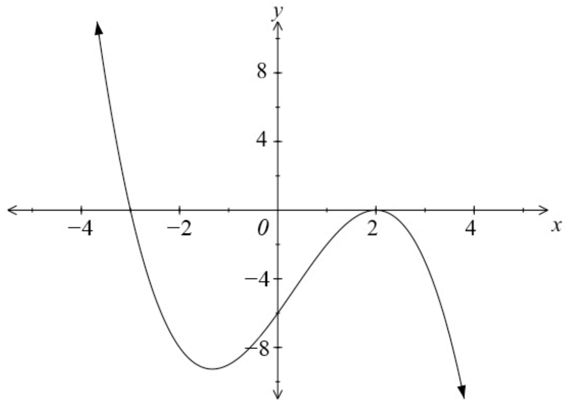
C. $(-2,\ 1) \cup (1,\ 2)$

D. $[-2,\ 1) \cup (1,\ 2)$

E. $[-2,\ 1) \cup (1,\ 2]$

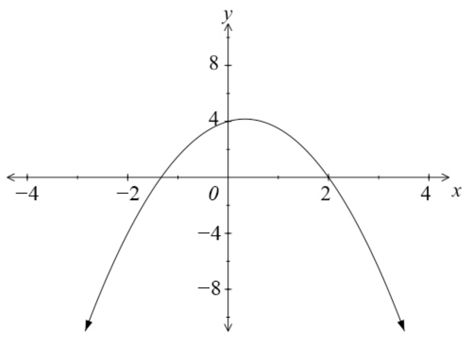
Question 15

Part of the graph of $y = f(x)$ is shown below.

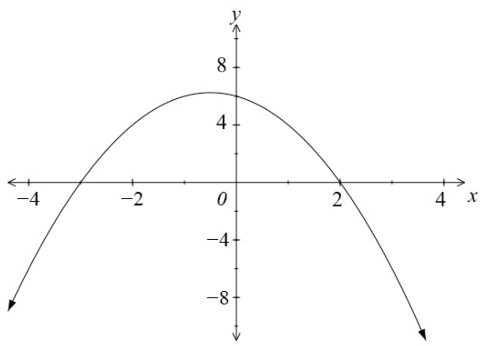


Which one of the following could be the graph of $y = f'(x)$?

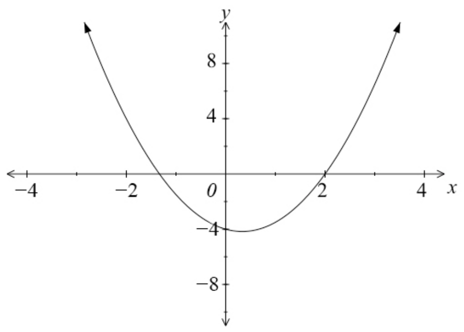
A.



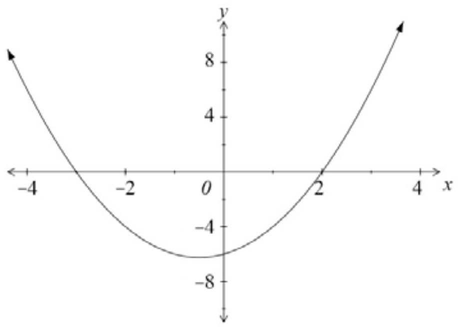
B.



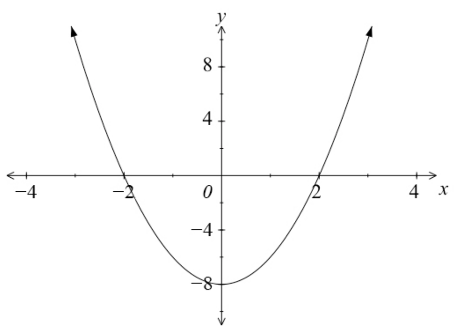
C.



D.



E.



Question 16

The binomial random variable X has $\text{E}(X) = \dfrac{40}{7}$ and $\Pr(X = 1) = \dfrac{5120}{5\,764\,801}$.

The value of $\Pr(X = 4)$ is closest to

A. 0.2371

B. 0.2549

C. 0.1937

D. 0.1536

E. 0.1214

Question 17

A 95% confidence interval for a population proportion is calculated to be (0.3023, 0.4377).
The 90% confidence interval for the same population proportion is closest to

A. (0.3257, 0.4143)

B. (0.3203, 0.4197)

C. (0.3132, 0.4268)

D. (0.2897, 0.4503)

E. (0.2810, 0.4590)

Question 18

The random variable, X, has a probability density function

$$f(x) = \begin{cases} 4kx^3 + 3x^2 - k^2 & 0 < x \le 1 \\ 0 & \text{otherwise} \end{cases}, \text{ where } k \in R.$$

The possible values of k are

A. 1

B. 0

C. 0 or 1

D. $0 \le k \le 1$

E. $k \le 0$ or $k \ge 1$

Question 19

The probability distribution for the discrete random variable X is defined by the following:

x	–2	–1	0	1
$\Pr(X = x)$	a	$2a$	$3a$	$2a$

The variance of X equals

A. $\frac{15}{16}$

B. $\frac{13}{16}$

C. $\frac{11}{16}$

D. $\frac{9}{16}$

E. $\frac{7}{16}$

Question 20

The random variable, X, has a normal distribution with mean 1.5 and standard deviation 0.4.

If $\Pr(X < a) = \Pr\left(Z > \frac{a}{3}\right)$, where the random variable, Z, has the standard normal distribution, then the value of a is

A. $\frac{89}{67}$

B. $\frac{33}{25}$

C. $\frac{46}{35}$

D. $\frac{45}{34}$

E. $\frac{4}{3}$

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (15 marks)

When a drug is given to the body, it is absorbed and mixes with the blood. As time passes, the amount of drug in the bloodstream rises and then falls. An equation that models this process for the drug *Spuriosa-A* is $x = 14t^{0.3}e^{-0.2\,t}$, where x is the concentration of the drug in the bloodstream, in milligrams per litre, t hours after the drug is first given.

John is given a dose of *Spuriosa-A* at 9 a.m. on a particular day.

a. Find the amount of time it takes for the concentration of *Spuriosa-A* in John's bloodstream to reach its peak value. Express the answer in hours.

1 mark

b. Find the amount of time it takes for the concentration of *Spuriosa-A* in John's bloodstream to fall to half of its peak value, correct to the nearest minute.

2 marks

c. What is the average value of the concentration of *Spuriosa-A* in John's bloodstream between 9 a.m. and 1 p.m. on this day? Express the answer in milligrams per litre, correct to four decimal places.

2 marks

d. The average rate of change of the concentration of *Spuriosa-A* in John's bloodstream over the first t_1 hours from when it was given is 2 milligrams per litre per hour.

Find the value of t_1, in hours, correct to two decimal places.

2 marks

e. John is given a second dose of *Spuriosa-A* at 4.30 p.m. on the same day as the first dose.

Find the concentration of *Spuriosa-A* in John's bloodstream at 6 p.m. on the same day. Express the answer in milligrams per litre, correct to four decimal places.

2 marks

A family of functions that can be used to model the amount of drug in the bloodstream has the form $x = at^b e^{-ct}$, where x is the concentration of the drug in the bloodstream, in milligrams per litre, t hours after the drug is first given. a, b and c are positive constants.

f. Find, in terms of a, b and c, the maximum value of x.

2 marks

g. The drug *Spuriosa-B*, which can be modelled by the family of functions $x = at^b e^{-ct}$, is known to reach a maximum concentration of 12 milligrams per litre after 90 minutes and its concentration after 7.5 hours is half of the maximum value.

i. Find the value of b. Express the answer in the form $\dfrac{\log_e(p)}{p^2 - \log_e(q)}$, where p and q are positive integers.

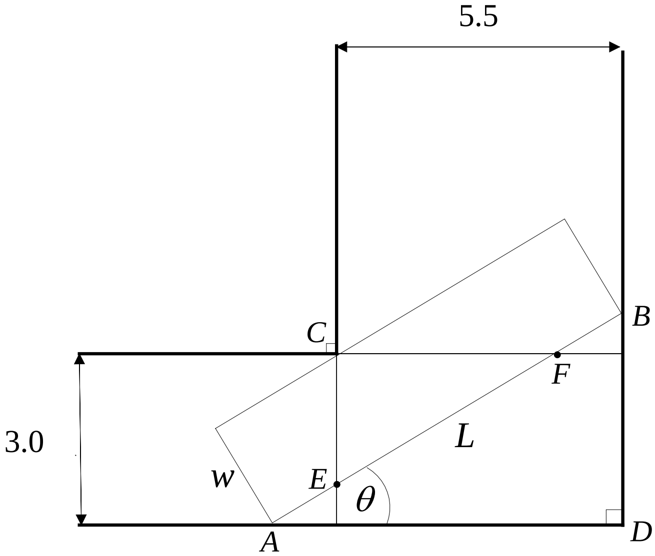
3 marks

ii. State the values of a and c, correct to four decimal places.

1 mark

Question 2 (14 marks)

Consider the problem of positioning a rectangle of width *w* units and length *L* units inside the figure with dimensions shown below. The remaining dimensions of the figure are unspecified.



The rectangle makes contact with the inside corner at *C* and at the sides of the figure at points *A* and *B*. The angle θ is measured in radians. The points *E* and *F* show where the continuation of the sides of the figure would intersect with the rectangle.

a. **i.** State, in terms of *w* and θ, an expression for length *EC* and length *CF*.

2 marks

ii. Hence, show that length $EF = \dfrac{w}{\sin(\theta)\cos(\theta)}$.

1 mark

b. **i.** Show that length $AE = \dfrac{3\cos(\theta) - w}{\sin(\theta)\cos(\theta)}$.

2 marks

ii. Find, in terms of w and θ an expression for length FB.

3 marks

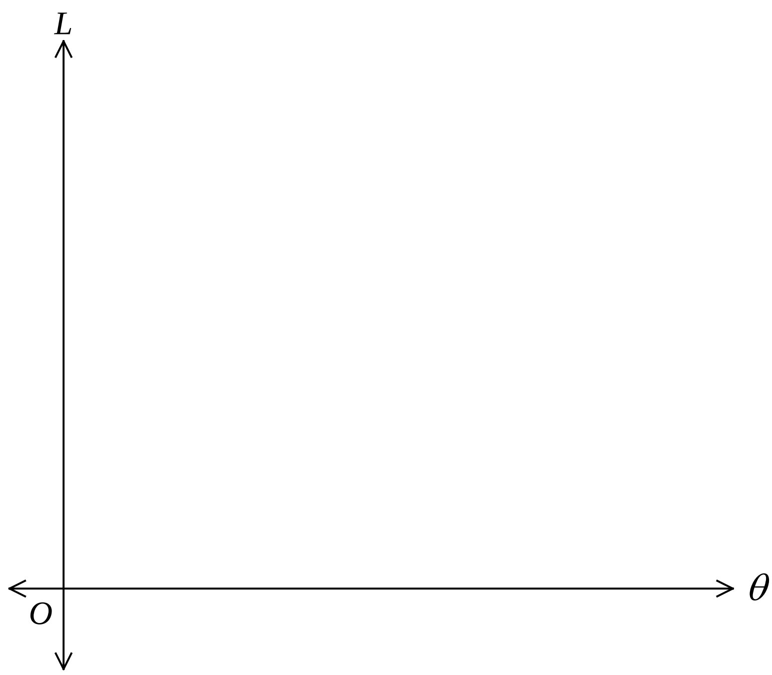
c. Hence, show that $L = \dfrac{3\cos(\theta) + 5.5\sin(\theta) - w}{\cos(\theta)\sin(\theta)}$.

1 mark

d. Suppose that a particular rectangle has a width of 1.5 units and a length of *L* units.

i. Sketch the graph of *L* as a function of θ for this rectangle. Label all stationary points with their coordinates, correct to four decimal places, and all asymptotes with their equations.

3 marks



ii. What is the widest rectangle that can fit within the figure?

1 mark

iii. Briefly explain whether or not a rectangle of width 1.5 units and length 4.4 units can be rotated 90º within the figure.

1 mark

Question 3 (20 marks)

The amount of homework completed on a given weeknight by a Year 9 student is a normally distributed random variable with a mean of 60 minutes and a standard deviation of 25 minutes.

a. **i.** Find the probability that a randomly selected Year 9 student will complete less than 68 minutes of homework. Give your answer correct to four decimal places.

1 mark

ii. The probability that a Year 9 student will complete more than *a* minutes of homework is equal to 0.65.

Find *a*, correct to one decimal place.

1 mark

A student is considered to have done insufficient homework if less than 20 minutes is done on a weeknight. A typical student completes between 20 minutes and b minutes of homework. The probability that a student is typical is 0.9.

A Year 9 student who spends more than b minutes on homework is spending an excessive amount of time on homework.

b. **i.** Find the value of b, correct to one decimal place.

2 marks

ii. Find the probability that a typical Year 9 student completes less than 60 minutes of homework, correct to four decimal places.

2 marks

c. **i.** Find the probability that three students in a random group of nine Year 9 students will each complete between 40 and 60 minutes of homework on a given weeknight, correct to four decimal places.

2 marks

ii. The probability that more than one student completes an insufficient amount of homework in a random group of Year 9 students is greater than 0.4.

Find the smallest size of the group.

3 marks

d. Let $\hat{P}$ be the random variable of the distribution of sample proportions of Year 9 students who complete more than 90 minutes of homework, in a sample of size 60.

Find the probability $\Pr(\hat{P} \geq 0.1 \mid \hat{P} \leq 0.15)$, correct to four decimal places. Do **not** use a normal approximation.

4 marks

The amount of time required for a Year 9 student to complete all of their homework is a normally distributed random variable with a mean of 85 minutes and a standard deviation of 20 minutes.

e. Find the percentage of Year 9 students who will be able to complete all of their homework without spending an excessive amount of time on homework, correct to two decimal places.

2 marks

The amount of time required for a Year 7 student to finish their homework is a normally distributed random variable with a mean of 30 minutes. The probability that more than 25 minutes is required is equal to 0.83.

f. Find the standard deviation of the amount of time required for a Year 7 student to finish their homework. Give your answer in minutes, correct to two decimal places.

2 marks

In order to estimate the number of Year 7 students who require more than 30 minutes to complete their homework, 80 Year 7 students were surveyed.

It is found that 57 students require more than 30 minutes to complete their homework.

g. Determine a 90% confidence interval for the population proportion from this sample. Give values correct to two decimal places.

1 mark

Question 4 (11 marks)

Let the function $f:(-\infty,\frac{5}{4}],\ f(x)=\sqrt{5-4x}$.

a. The transformation $T:R^2 \to R^2,\ T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} c \\ d \end{bmatrix}$ maps the graph of $y=f(x)$ onto the graph of $y=\sqrt{x},\ x \in R$. Find the values of a, c and d.

2 marks

Let the function $g:D_g \to R,\ g:R \to R,\ g(x)=x^2+\beta x-1$, where $\beta \in R$.

b. If $\beta > 0$ and $D_g=\left\{x:\ -\beta \le x \le -\frac{\beta}{2}\right\}$, find in terms of β the area between the graph of $y=g^{-1}(x)$ and the line $y=1$.

2 marks

c. Find all values of β for which the equation $f(x) = g(x)$ has exactly one solution.

2 marks

Let the function $h = f \cdot g$ and let the domain of g be such that h has its maximal domain.

d. Find the x-coordinate of the maximum turning point of h when:

i. $\beta = 0.$ If no maximum turning point exists, briefly explain why.

1 mark

ii. $\beta = -1.$ If no maximum turning point exists, briefly explain why.

1 mark

e. Find all values of β for which the graph of h has exactly one turning point.

3 marks

END OF QUESTION AND ANSWER BOOK