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Student Name……………………………………

MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 1

2018

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 9 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where a numerical answer is required, an exact value must be given unless otherwise directed.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on pages 12 and 13 of this exam.

Question 1 (4 marks)

a. Differentiate $\sqrt{2x-x^3}$ with respect to x. 1 mark

b. Let $f(x)=\frac{\sin(2x)}{e^{2x}}$.

Evaluate $f'\left(\frac{\pi}{2}\right)$. 3 marks

Question 2 (4 marks)

Let $y = x\cos(2x)$.

a. Find $\frac{dy}{dx}$. 2 marks

b. Hence, evaluate $\int_0^{\frac{\pi}{6}} 2x\sin(2x)\,dx$. 2 marks

Question 3 (2 marks)

The Year 12 leadership team at a school is made up of four female and three male students. Three of these students are rostered at random to be present at the Junior assembly each week.
Let $\hat{P}$ represent the sample proportion of female students who are rostered to be present at next week's Junior assembly.

a. Write down the possible values of $\hat{P}$. 1 mark

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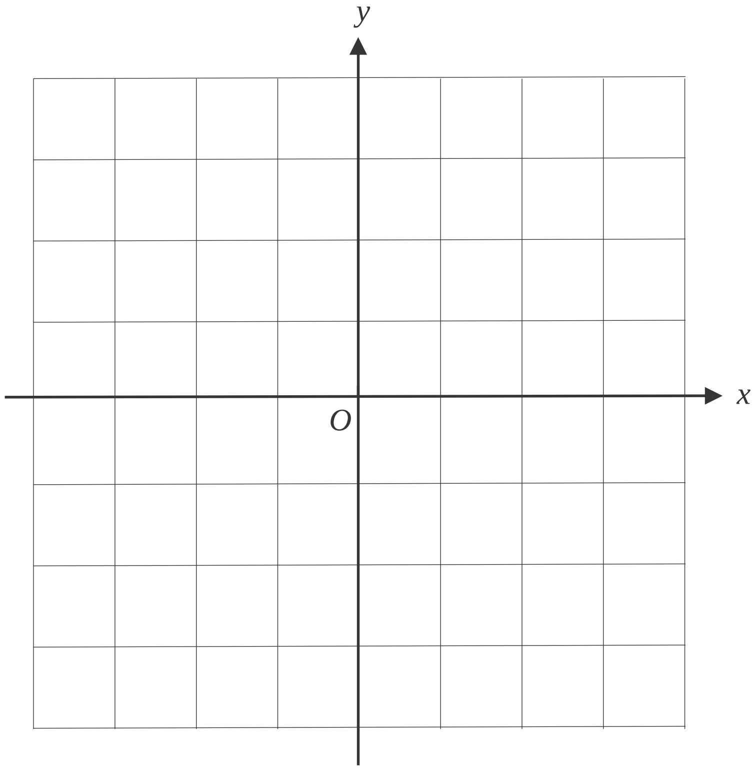
b. Find $\Pr(\hat{P} = 1)$. 1 mark

Question 4 (5 marks)

a. Solve $\left(\sin(\theta)-\frac{1}{\sqrt{2}}\right)\left(\sqrt{3}\sin(\theta)+\cos(\theta)\right)=0$ for θ, where $0 \le \theta \le \pi$. 2 marks

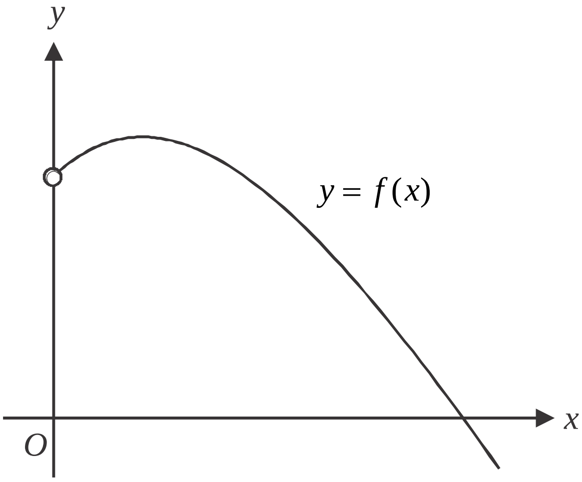
b. Let $f:\left[-\frac{\pi}{8},\frac{\pi}{4}\right) \to R, f(x)=\tan(2x)$.

Sketch the graph of *f* on the axes below. Label any asymptotes with the appropriate equation, and label any endpoints with their coordinates. 3 marks



Question 5 (4 marks)

Let $f:(0,\infty)\to R,\quad f(x)=1-x\log_e(x)$.
The graph of *f* is shown below.



a. Find the coordinates of the stationary point of *f*, expressed in simplest form. 3 marks

b. Find the values of *x* for which *f* is strictly decreasing. 1 mark

Question 6 (5 marks)

Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$ where *f* and *g* have maximal domains.

a. **i.** Find the rule for $f(g(x))$. 1 mark

ii. State the domain and range of $f(g(x))$. 2 marks

The domain of *f* is to be restricted to $x \in [a, \infty)$ where *a* is the smallest possible value such that $g(f(x))$ exists.

b. Find the value of *a*. 2 marks

Question 7 (5 marks)

In a sample space with two events A and B, $\Pr(A) = 3q$, $\Pr(A' \cap B') = q$ and $\Pr(A' \mid B') = \frac{1}{3}$.

a. Find $\Pr(B)$ in terms of q. 1 mark

b. Find $\Pr(A \cup B)$ in terms of q. 2 marks

c. If $\Pr(A \cap B') \le \frac{1}{2}$, find the possible values for q. 2 marks

Question 8 (4 marks)

A continuous random variable *X* has a probability density function given by

$$f(x)=\begin{cases}\frac{1}{3}e^{-\frac{x}{3}} & x\geq 0\\ 0 & \text{elsewhere}\end{cases}$$

The median of *X* is *m*.

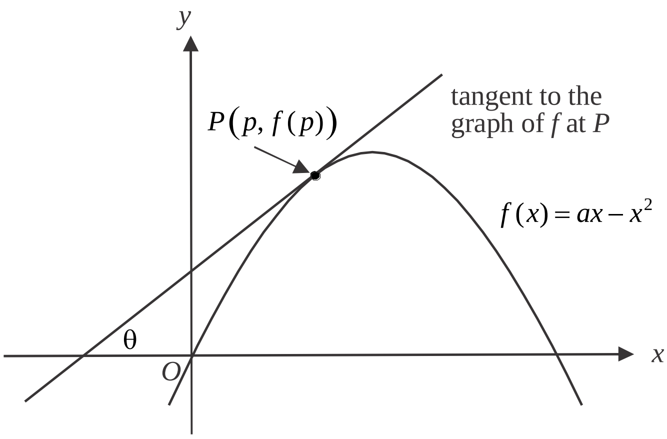
a. Find the value of *m*. 2 marks

b. Given that $m>2$, find $\Pr(X<2|X\leq m)$. 2 marks

Question 9 (7 marks)

Let $f: R \to R, f(x) = ax - x^2$, where a is a positive constant.

A tangent is drawn to the graph of f at the point $P(p, f(p))$, where $0 \le p < \frac{a}{2}$, as shown below.



The tangent makes an angle of θ with the positive direction of the horizontal axis as shown.

a. Show, without using calculus, that the turning point of the graph of f is located at the point where $x = \frac{a}{2}$. 1 mark

b. Find, in terms of a, the maximum value of θ. 2 marks

c. If $p = \frac{a}{4}$, find in terms of a, the area enclosed by the tangent, the graph of f and the y-axis. 4 marks

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$	$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e(x)+c,\ \ x>0$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x - \mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2\, f(x)\, dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$