VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2017 VCE Mathematical Methods examination 1 (NHT) examination report

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Question 1a.

$$\frac{dy}{dx} = 2e^{2x}\cos\left(\frac{x}{2}\right) - \frac{1}{2}e^{2x}\sin\left(\frac{x}{2}\right)$$

Students are reminded to take care with notation, especially with the placement of negative signs and brackets.

Question 1b.

$$f'(x) = \frac{\cos(x)}{\sin(x)}$$

$$f'\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

Question 2a.

$$\int \cos(1-x)dx = -\sin(1-x)$$

Including an arbitrary constraint, ‘+c’, was also correct as any value that is a constant was correct.

Question 2b.

$$\int_1^2 \left(3x^2 + \frac{4}{x^2}\right)dx$$

$$= \int_1^2 \left(3x^2 + 4x^{-2}\right)dx$$

$$= \left[x^3 - \frac{4}{x}\right]_1^2$$

$$= 9$$

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Question 2c.

$$\int\left(\frac{3}{8}x^2-10x^{-\frac{1}{2}}+1\right)dx=\frac{x^3}{8}-20x^{\frac{1}{2}}+x+c$$

$$f(4)=8-40+4+c$$
$$=25$$
$$c=53$$
$$f(x)=\frac{x^3}{8}-20x^{\frac{1}{2}}+x+53$$

Question 3a.

$$\tan\left(\frac{3\pi}{4}\right)=-1$$

$$\cos\left(\frac{3\pi k}{4}\right)=-1$$

$$\frac{3\pi k}{4}=(2n-1)\pi,\ n\in Z$$

Hence the smallest positive value of *k* is $\frac{4}{3}$.

Question 3b.

$2\sin^2(x) + 3\sin(x) – 2 = 0$

Let $k = \sin(x)$, substituting gives

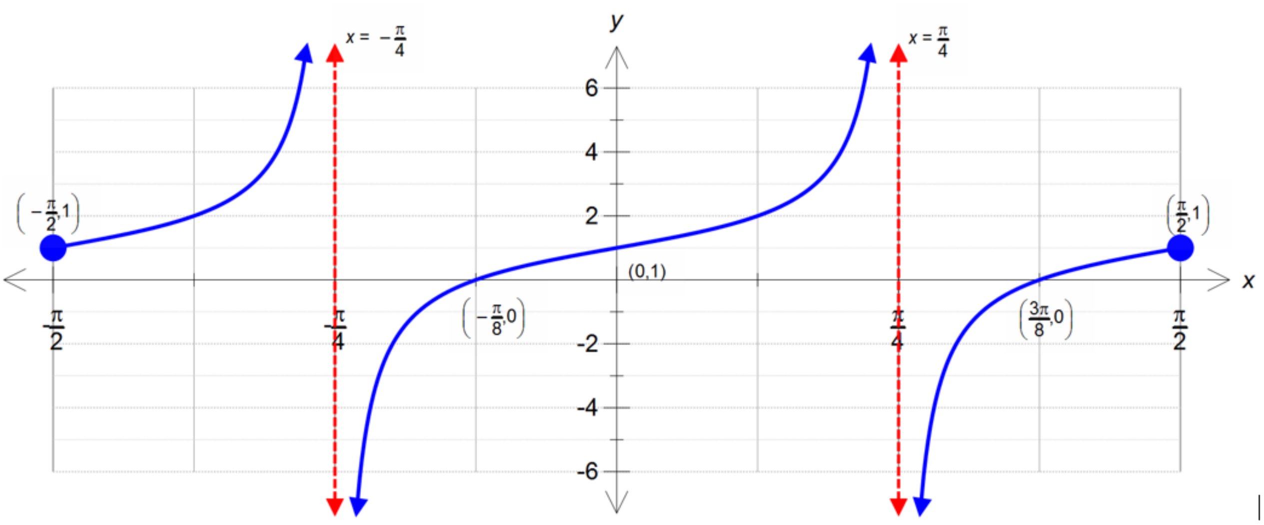
$2k^2 + 3k – 2 = 0$

$(2k – 1)(k + 2) = 0$

$k = –2$, no solutions for x

or $k = \frac{1}{2}$, hence $x=\frac{\pi}{6},\frac{5\pi}{6}$

Question 4a.



Students are advised to show asymptotes with dashed lines, so as to ensure that the curve is clearly distinguishable from the asymptote.

Question 4b.

$$\text{Average value } = \frac{1}{\frac{\pi}{4}} \times \frac{1}{2} \times \frac{\pi}{4} \times 2$$
$$= 1$$

or by observation of symmetry of graph.

Question 5a.

$p = 1 - (0.1 + 0.4 + 0.3)$
$p = 0.2$

Question 5b.

$\text{E}(X) = -0.1 + 0.3 + 0.4$
$= 0.6$

Question 5c.

$\text{Var}(X) = \text{E}(X^2) - (\text{E}(X))^2$
$= 1.2 - 0.6^2$
$= 0.84$

Question 5d.

$$\Pr(X = 4) = {}^5C_4 \times 0.1^4 \times 0.9$$
$$= 5 \times .0001 \times 0.9$$
$$= 0.00045 \text{ or } \frac{9}{20000} \text{ or } 4.5 \times 10^{-4}$$

Question 6ai.

Number of men working: 0, 1, 2, 3, 4

Corresponding proportions: $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$

Question 6aii.

$$\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$$
$$= \frac{1}{126}$$

Question 6b.

$$sd(\hat{P}) = \sqrt{\frac{\frac{5}{9} \times \frac{4}{9}}{2000}}$$
$$= \frac{1}{90}$$

Question 7ai.

$$g(f(x)) = 4 - 2(2x^3 + 1)$$
$$= 2 - 4x^3$$

Question 7aii.

$$f(g(x)) = 2(4 - 2x)^3 + 1$$
$$= 2(2(2 - x))^3 + 1$$
$$= 1 - 16(x - 2)^3$$

Question 7b.

$g(f(x)) = 2 - 4x^3$ to $f(g(x)) = 1 - 16(x - 2)^3$

$$x' = x + b, \; y' = ay + c$$
$$\frac{y' - c}{a} = 2 - 4\left(x' - b\right)^3$$
$$y = 2a - 4a\left(x' - b\right)^3 + c \quad \text{equate to} \quad y = 1 - 16(x - 2)^3$$

$$a = 4, \; b = 2, \; c = -7$$

Alternatively, dilation of 4 from the *x*-axis: $4(2 - 4x^3) = 8 - 16x^3$,

translation of 2 in the positive direction of the *x*-axis:

$8 - 16(x - 2)^3$,

translation of 7 in the negative y direction: $1 - 16(x-2)^3$

Matrix = $\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 2 \\ -7 \end{bmatrix}$

So $a = 4$, $b = 2$, $c = -7$

Question 8a.

$f(x) = \sqrt{2x+3} - 1$

let $x = \sqrt{2y+3} - 1$

$x + 1 = \sqrt{2y+3}$

$y = \frac{(x+1)^2}{2} - \frac{3}{2}$

Hence $f^{-1}(x) = \frac{(x+1)^2 - 3}{2}$

Dom $f^{-1} = [-1, \infty)$

Question 8b.

Solve a suitable equation and check; for example,

$\sqrt{2x+3} - 1 = x$

$2x + 3 = (x+1)^2$

$x = \pm\sqrt{2}$

$x = \sqrt{2}$ as $x \geq -1$

Question 8ci.

$\sqrt{2x+c} - 1 = x$

$2x + c = (x+1)^2$

$x^2 = c - 1$

No solutions if $c - 1 < 0$

$c < 1$

Question 8cii.

$x = \pm\sqrt{c-1}$ and must be in domain of g and g^{-1}. Exactly one solution for $c = 1$ or for $c > 2$.