TSSM
Creating VCE Success

2017 Trial Examination

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STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHODS

Written Examination 2

Reading Time: 15 minutes
Writing Time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	20	20	20
2	4	4	60
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one approved graphics calculator or CAS (memory DOES NOT have to be cleared) and, if desired, one scientific calculator, one bound reference (may be annotated). The reference may be typed or handwritten (may be a textbook).
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials Supplied

- Question and answer book of 25 pages.
- Working space provided throughout the book.

Instructions

- Print your **name** in the space provided at the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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SECTION 1

Instructions for Section 1
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1
The range of the function $f:(0,\infty)\to R,\ where\ f(x)=-2+\frac{1}{x+4}$ is

A. $R\backslash\{-4\}$
B. $R\backslash\{-2\}$
C. $(-2,-1.75)$
D. $(-2,0)$
E. $(0,\ \infty)$

Question 2
The turning point of the function with the rule $y=2x^2-5x+1$ is

A. $\left(-\frac{5}{4},-\frac{17}{8}\right)$
B. $\left(\frac{5}{4},-\frac{17}{8}\right)$
C. $\left(\frac{5+\sqrt{17}}{4},\ \frac{5-\sqrt{17}}{4}\right)$
D. $(1,\ 0)$
E. $(5,1)$

Question 3
The number of x-intercepts of the function $f:[-\pi,\ \pi]\to R,\ where\ \ f(x)=2\sin(\pi x)-1$ is

A. 2
B. 3
C. 4
D. 5
E. 6

SECTION 1 – continued
TURN OVER

Question 4
The period and range of the function $f: R \to R,\ where\ \ f(x) = -3 - \cos\left(\frac{2x}{\pi}\right)$ are respectively

A. $\pi^2,\ \ [-4,\ -2]$
B. $\pi,\ \ [-4,\ -2]$
C. $\pi^2,\ \ R$
D. $4\pi,\ \ [-4,\ -2]$
E. $\pi^2,\ \ [-1,\ 1]$

Question 5
The average rate of change of the function f with rule $f(x) = \sqrt{x+4} - 5x$, between $x = 0$ and $x = 5$, is

A. $-\frac{299}{30}$
B. -24
C. $-\frac{24}{5}$
D. $-\frac{22}{5}$
E. $\frac{299}{6}$

Question 6
Which one of the following is the inverse function of g : $(-\infty, -1] \to R,\ \ g(x) = x^2 + 2x + 4$?

A. $g: [-1, \infty) \to R,\ \ g^{-1}(x) = -1 - \sqrt{x-3}$
B. $g: [3, \infty) \to R,\ \ g^{-1}(x) = -1 - \sqrt{x-3}$
C. $g: R \to R,\ \ g^{-1}(x) = -1 - \sqrt{x-3}$
D. $g: (-\infty, -1] \to R,\ \ g^{-1}(x) = -1 - \sqrt{x-3}$
E. $g: [3, \infty) \to R,\ \ g^{-1}(x) = -1 + \sqrt{x-3}$

SECTION 1 – continued

Question 7

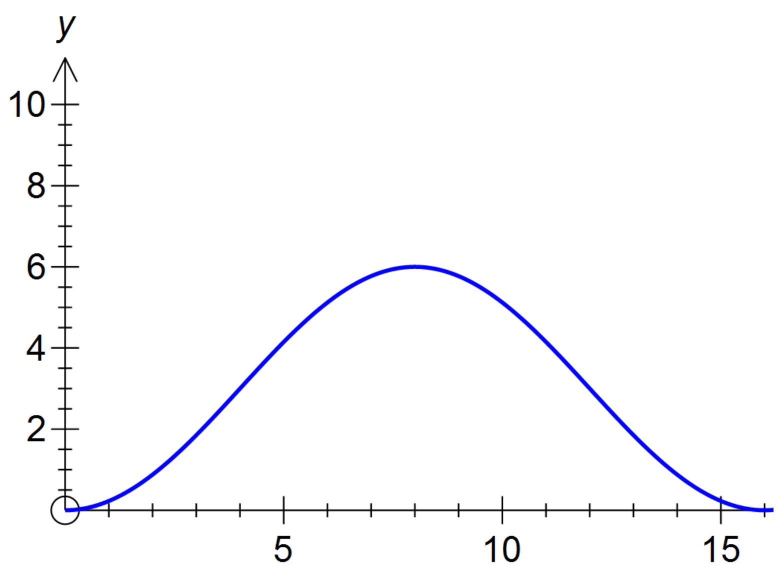
The number of cars, X, owned by each employee in a particular department of a company is a random variable with the following discrete probability distribution

x	0	1	2	3
$\Pr(X = x)$	0.2	0.25	0.45	0.1

If two employees are selected at random, the probability that they do not own the same number of cars is

A. 0.645
B. 0.45
C. 0.3
D. 0.315
E. 0.685

Question 8



The graph above is most likely to be the graph of

A. $y = 3\cos\left(\frac{\pi x}{16}\right)$
B. $y = 3 - 3\sin\left(\frac{\pi x}{8}\right)$
C. $y = 3 + 3\cos\left(\frac{\pi x}{8}\right)$
D. $y = 3 - 3\cos\left(\frac{\pi x}{8}\right)$
E. $y = 3 - 3\cos\left(\frac{\pi x}{16}\right)$

SECTION 1 – continued
TURN OVER

Question 9

Given that $\frac{d}{dx}\left(x^2 log_e(kx)\right) = x\left(1 + 2log_e(kx)\right)$ then $\int x log_e(kx)dx$ is equal to

A. $\frac{x^2}{2} log_e(kx) - \frac{x}{2} + c$

B. $\frac{x^2}{2} log_e(kx) - \int \frac{x}{2} dx + c$

C. $\frac{x^2}{2} log_e(kx) - \int \frac{x^2}{4} dx + c$

D. $\int \frac{x^2}{2} log_e(kx)\, dx - \int \frac{x^2}{4} dx$

E. $\int \left(\frac{x^2}{2} log_e(kx) - \frac{x}{2}\right) dx$

Question 10

The tangent to the curve $y = cx^2 - 5$ at $x = -1$ passes through the point (-4, -1).
The value of c is equal to

A. $\frac{4}{7}$

B. $\frac{7}{4}$

C. 1

D. -1

E. $\frac{\sqrt{7}}{4}$

Question 11

The graph of the function $g(x) = e^{2x} - 4$ is obtained from the graph of a function *f* by a reflection in the x-axis followed by a dilation by a factor of ½ from the y-axis.
The rule for the graph of *f* is

A. $f(x) = -\frac{1}{2}e^x + 4$

B. $f(x) = -e^x + 4$

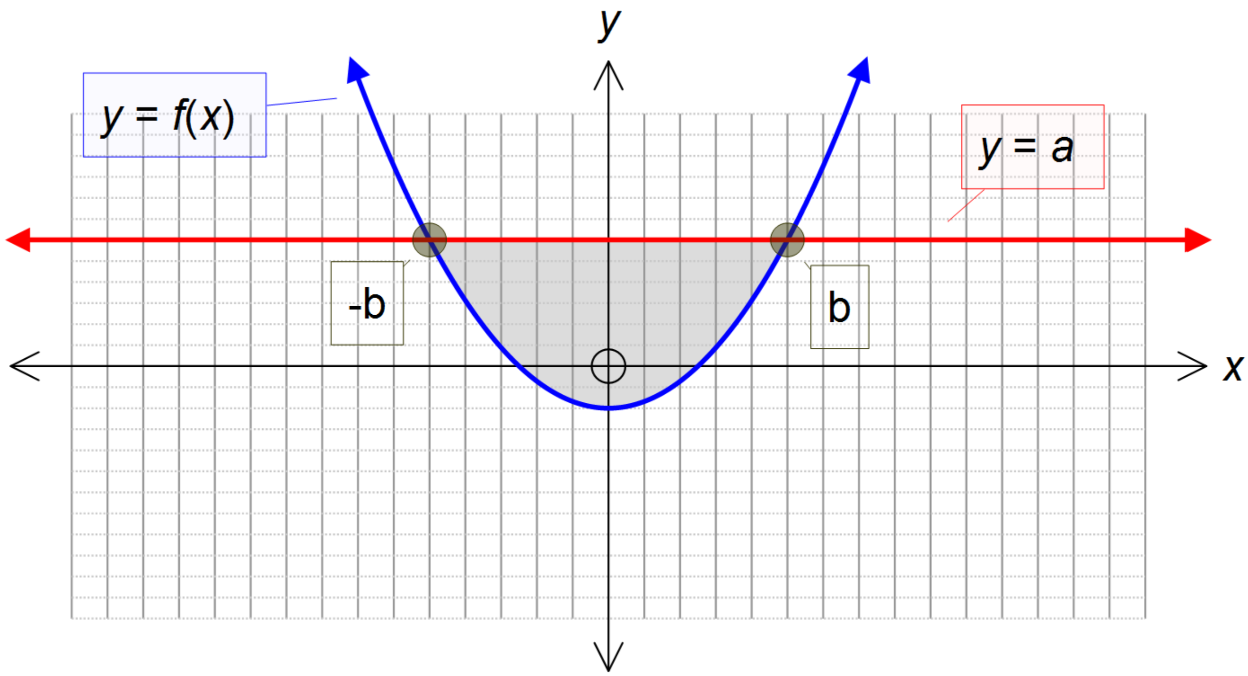
C. $f(x) = -\frac{1}{2}e^{2x} + 4$

D. $f(x) = 2e^{2x} - 4$

E. $f(x) = 2e^{-2x} - 4$

SECTION 1 – continued

Question 12
Consider the graphs of the functions *f* and *g* shown below.



The area of the shaded region above could be represented by

A. $\int_{-b}^{b}(a - f(x))dx$
B. $2a - 2\int_{0}^{b} f(x)dx$
C. $2\int_{0}^{b}(f(x) - a)dx$
D. $2\int_{0}^{b} f(x)dx$
E. $\int_{b}^{a} f(x)\, dx$

Question 13
Let $f: (0, 7) \to R,\ f(x) = \sin\left(\frac{x}{2}\right) + \cos(x)$
Which of the following is true for the graph of f?

A. $f(7) = 0.40$
B. $f'(x) < 0\ \ over\ \ (0.51, 3.14) \cup (5.78, 7)$
C. $f'(x) = 0\ \ at\ \ x = 0.51\ \ only$
D. $f'(x) > 0\ \ over\ \ (0, 5.78)$
E. $f(x)\ is\ \ increasing\ \ over\ \ (0, 7)$

SECTION 1 – continued
TURN OVER

Question 14
The random variable, X, has a normal distribution with mean 24 and standard deviation 0.35
If the random variable, Z, has the standard normal distribution, then the probability that X is greater than 23.3 is equal to

A. $\Pr(Z \geq -1.5)$
B. $\Pr(Z < -2)$
C. $\Pr(Z < 1)$
D. $\Pr(Z < 2)$
E. $\Pr(Z > 3)$

Question 15

The continuous random variable, X, has a probability density function given by

$$f(x) = \begin{cases} k\sqrt{x}(2-x) & 0 \leq x \leq 1 \\ 0 & elsewhere \end{cases}$$

The value of a for which $\Pr(X > a) = \frac{256-11\sqrt{2}}{256}$ is

A. $\frac{2}{3}$
B. $\frac{1}{2}$
C. 1
D. 0.94
E. $\frac{1}{8}$

SECTION 1 – continued

Question 16
Consider the transformation T, defined as

$$T: R^2 \rightarrow R^2, \qquad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The transformation T maps the graph of $y = f(x)$ onto the graph of $y = g(x)$.
If $f(x) = \cos(2x)$, then the rule for g is

A. $g(x) = 1 - \sin(3x)$
B. $g(x) = 1 + \cos(3x)$
C. $g(x) = 1 - 3\cos(x)$
D. $g(x) = -\cos\left(\frac{3x}{2}\right)$
E. $g(x) = 1 - \sin(\pi - 3x)$

Question 17
A machine produces 10 000 coloured counters in one day. It is known that 20% of the counters are white. A sample of 32 counters is taken from these 10 000 counters. For samples of 32 counters, $\hat{P}$ is the random variable of the distribution of sample proportions of white counters. (Do not use a normal approximation).
$\Pr\left(\hat{P} \geq \frac{5}{32}\right)$ is closest to

A. 0.3602
B. 0.0003
C. 0.1558
D. 0.7956
E. 0.6143

SECTION 1 – continued
TURN OVER

Question 18
Consider the discrete probability distribution with random variable X shown in the table below.

x	1	2	3	4	5
$\Pr(X = x)$	0.1	0.15	a	b	0.2

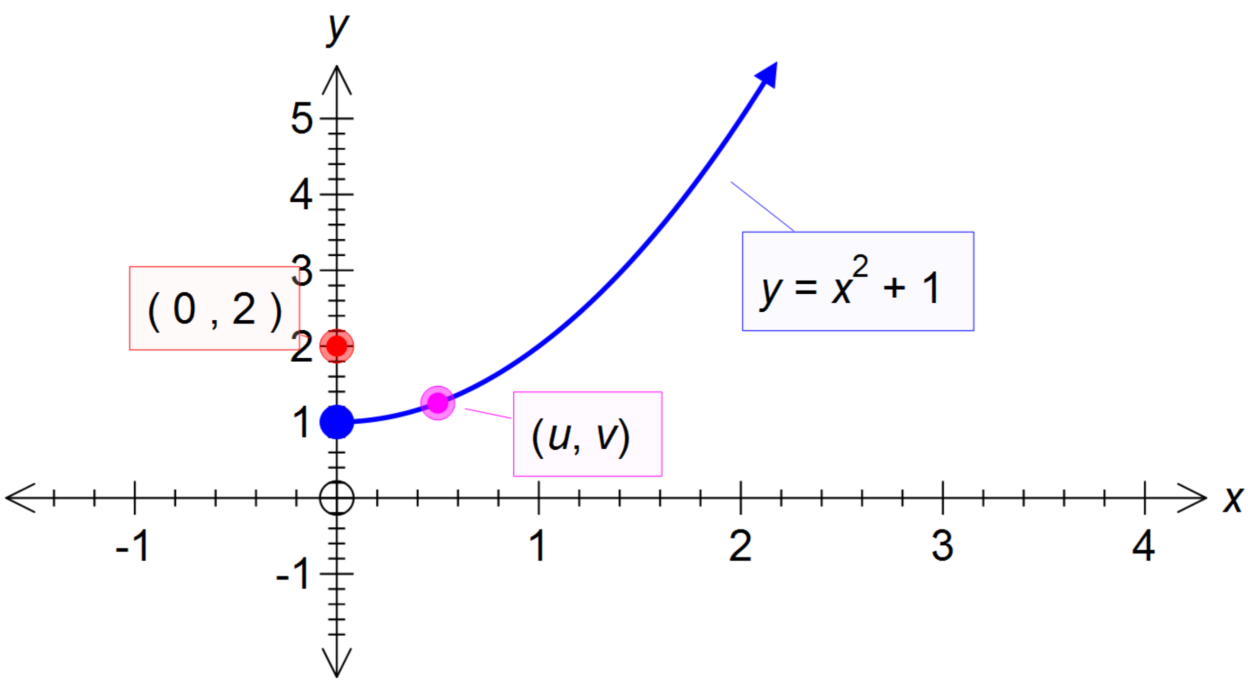
If $\Pr(2 \leq X < 4) = 0.58$, the value of a and b respectively are

A. $0.43 \;\; and \;\; 0.12$
B. $0.1 \;\; and \;\; 0.45$
C. $0.30 \;\; and \;\; 0.25$
D. $0.42 \;\; and \;\; 0.13$
E. $0.13 \;\; and \;\; 0.42$

Question 19
The graph of $y = \frac{k}{2} - x$ intersects the graph of $y = (x - 2)(3 - x)$ at two distinct points for

A. $k > 6$
B. $k < 6$
C. $k < 12$
D. $6 \leq k \leq 12$
E. $k \geq 6$

Question 20
The graph of $y = x^2 + 1$ is shown below.



The point (u, v) is any point on the graph.
The minimum distance from the point (0, 2) to any point on the curve $y = x^2 + 1$ is

A. 1
B. 2
C. $\frac{\sqrt{2}}{2}$
D. $\frac{\sqrt{3}}{2}$
E. $\sqrt{3}$

END OF SECTION 1
TURN OVER

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (11 marks)

Let $f: [0, 100] \to R, \ \ f(x) = 5 + 6\cos\left(\frac{\pi}{24}x\right)$.

a. Find the period and range of f. 2 marks

b. State the rule for the derivative function f'. 1 mark

c. Find the equation of the tangent to the graph of f at $x = 60$. 1 mark

SECTION 2 – Question 1 - continued

d. Find the equations of tangents to the graph of $f:[0, 100] \rightarrow R,\;\; f(x) = 5 + 6\cos\left(\frac{\pi}{24}x\right)$ that have gradient $\frac{\pi}{4}$. 2 marks

e. The rule of f can be obtained from the rule of f' under a transformation T, such that

$$T: R^2 \rightarrow R^2, \qquad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b \\ 5 \end{bmatrix}$$

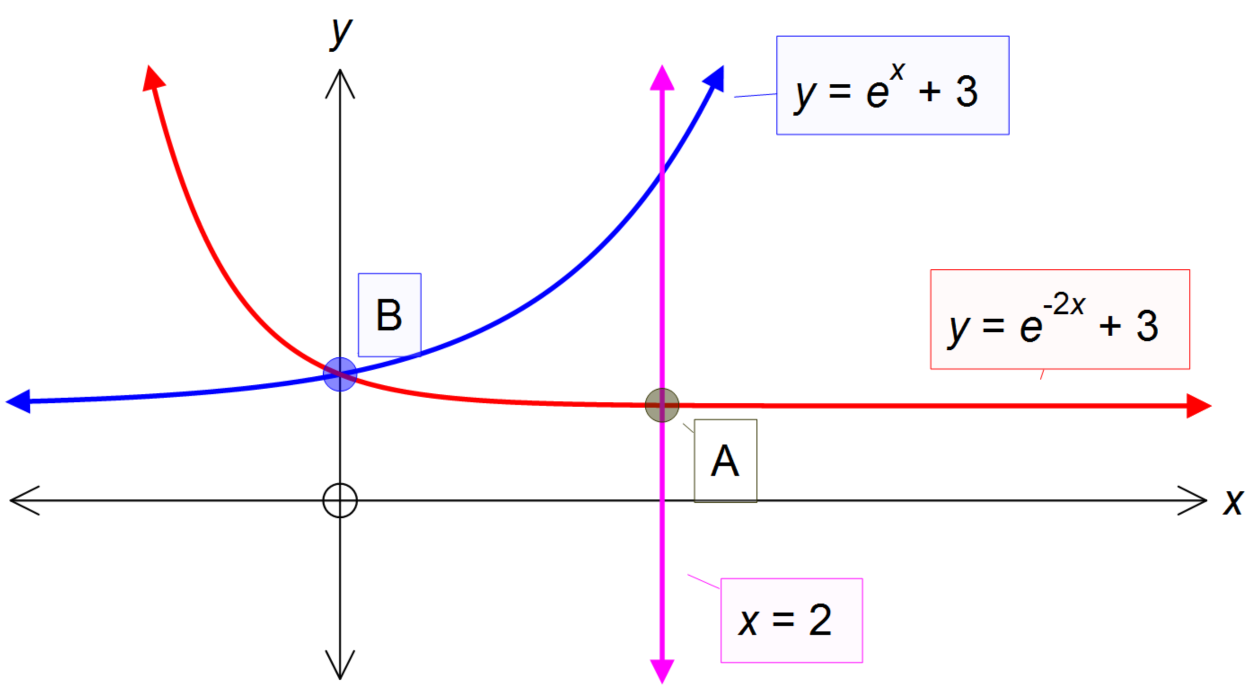
Find the value of a and value of b. 3 marks

SECTION 2 – Question 1 - continued
TURN OVER

f. Find the values of x, $0 \le x \le 100$, such that $f(x) = -\frac{24}{\pi} f'(x) + 5$ 2 marks

Question 2 (14 marks)

The diagram below shows part of the graphs $y = e^x + 3$, $y = e^{-2x} + 3$ and $x = 2$.

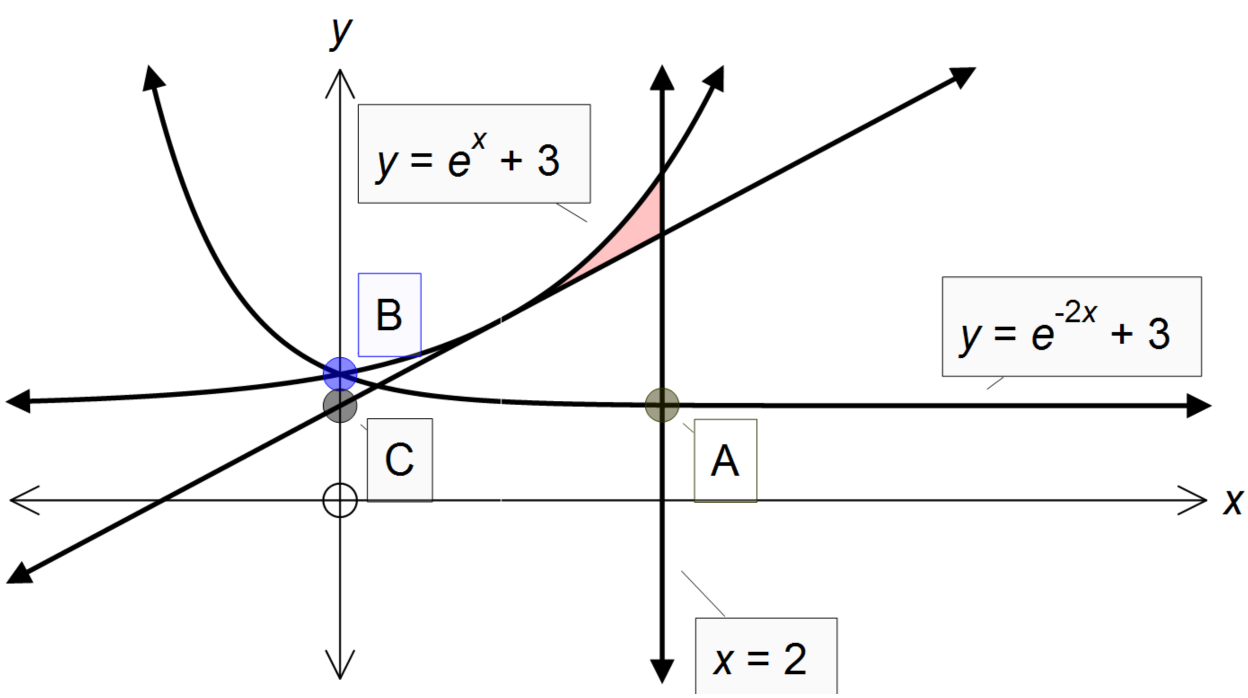


a. Find the coordinates of A and B. 2 marks

SECTION 2 – Question 2 - continued

b. Show that the area bounded between the graphs of $y = e^x + 3$ and $y = e^{-2x} + 3$ between $x = 0$ and $x = 2$ is $e^2 + \frac{1}{2e^4} - \frac{3}{2}$ 2 marks

A tangent is drawn to the graph of $y = e^x + 3$ at $x = 1$. This tangent cuts the y-axis at the point C as shown below.



SECTION 2 – Question 2 - continued
TURN OVER

c. Find the coordinates of C. 2 marks

d. Find the angle, that the tangent line at $x = 1$ makes with the positive direction of the $x - axis$. Write your answer to the nearest degree.

2 marks

e. Find the area of the shaded region in the diagram above. 2 marks

SECTION 2 – Question 2 - continued

f. A line parallel to the tangent to $y = e^{x} + 3$ at $x = 1$ cuts the graph of $y = e^{-2x} + 3$ at $x = a$ and passes through the origin.
Find the value of a, correct to two decimal places. 3 marks

g. Find the exact length of AC. 1 mark

SECTION 2 – continued
TURN OVER

Question 3 (15 marks)

Let $f: R\backslash\{-4\} \to R,\ f(x) = \frac{2x+1}{x+4}$.

a. Express $f(x)$ in the form $a + \frac{b}{x+4}$, where a and b are non-zero integers. 2 marks

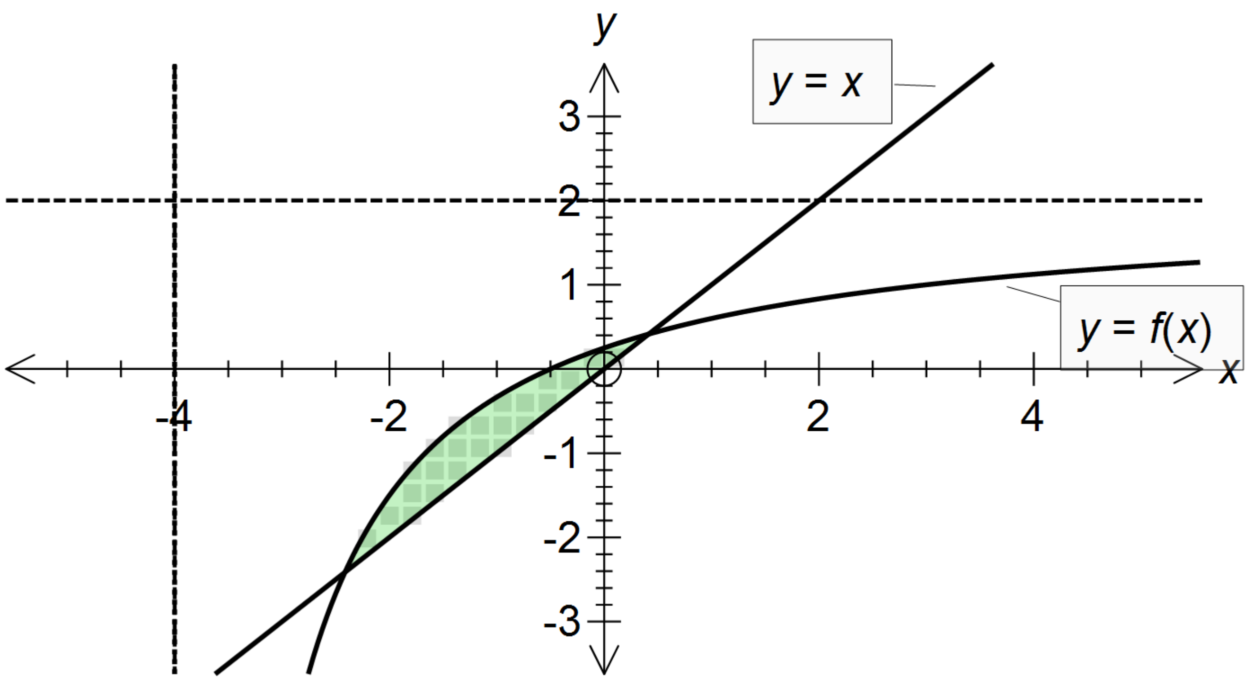
b. State the range of f. 1 mark

c. Find the rule of f^{-1}, the inverse function of f. 2 marks

SECTION 2 – Question 3 - continued

d. Find the point(s) of intersection of f and f^{-1}. 2 marks

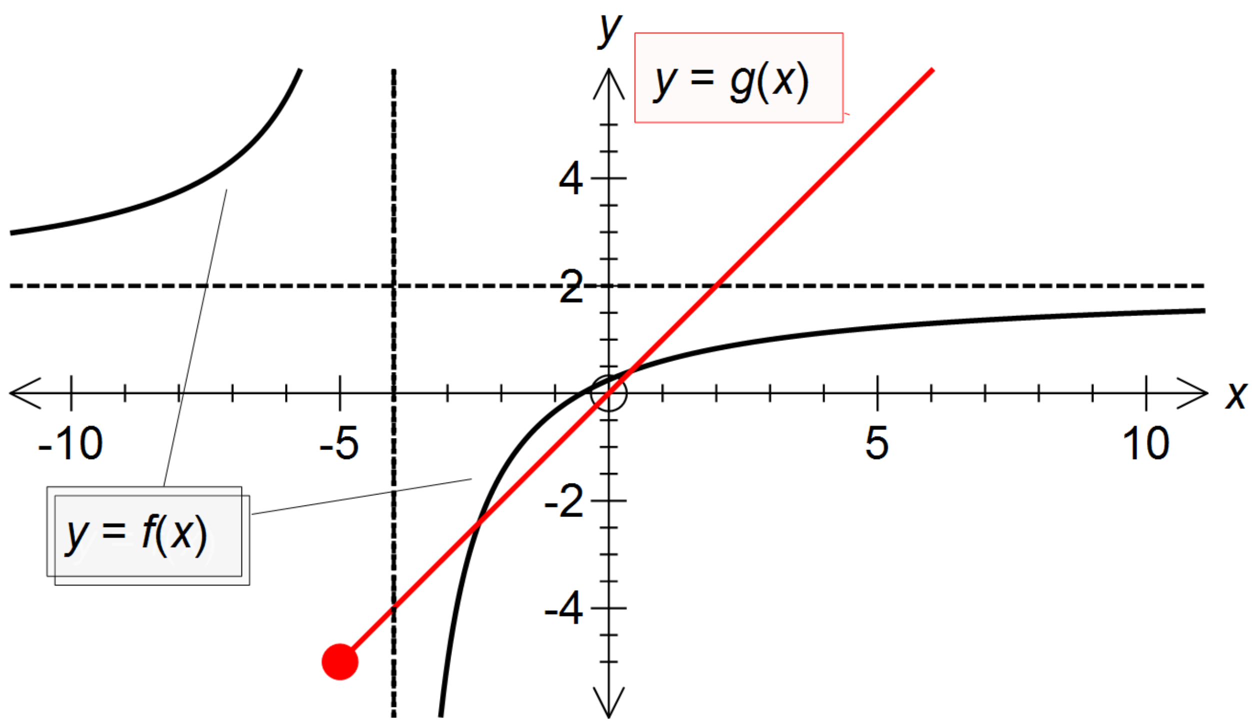
Part of the graph of f and the line $y = x$ is drawn below.



SECTION 2 – Question 3 - continued
TURN OVER

e. Write down an integral that will calculate the area of the shaded region. 2 marks

f. Let $g: [-5, \infty) \to R,\ g(x) = x$. The graphs of f and g are drawn below.



i. Write down the domain of h where $h(x) = f(x) - g(x)$. 1 mark

SECTION 2 – Question 3 - continued

ii. Find the stationary point(s) of h, leaving the coordinates in exact form. 2 marks

iii. Find the coordinates of the point on h, with coordinates correct to two decimal places, which is at a minimum distance from the origin. Find this minimum distance correct to two decimal places. 3 marks

SECTION 2 – continued
TURN OVER

Question 4 (20 marks)

A school has a policy of each student placing their chair on the table at the end of the day. Every classroom in the school has 24 chairs in the room. On a particular day 24 students are present in a class and each student is expected to place their chair on the table. The probability that a student does **not** put their chair on the table at the end of the day is 10%. The expectation from one student of placing the chair on the table at the end of the day, is completely independent from another student.

a. Determine the probability that at least one of the chairs is **not** placed on the table at the end of the lesson. Give your answer correct to four decimal places. 2 marks

b. A teacher observes that at least one of the chairs is not placed on the table at the end of the lesson.
Given this, find the probability that fewer than three chairs are **not** placed on the table. Give your answer correct to four decimal places. 2 marks

SECTION 2 – Question 4 - continued

The time it takes for students to place the chairs on the tables is approximately normally distributed with a mean of one minute and standard deviation of 12 seconds.

c. Find the probability that a particular student takes at most 40 seconds to place the chair on the table. Give your answer correct to four decimal places. 2 marks

d. The probability that a particular student takes at least *m* seconds to place the chair on the table is 0.4062.
Find the value of *m*, correct to the nearest second. 2 marks

SECTION 2 – Question 4 - continued
TURN OVER

In another class of 24 students of the same school, the time it takes for students to place the chairs on the tables is approximately normally distributed with a mean of one minute and standard deviation of c seconds. In this class, the probability that a student takes fewer than 46 seconds is 28%.

e. Find the value of c correct to two decimal places. 3 marks

f. Find the probability that exactly three students of this class take fewer than 46 seconds to place the chairs on tables. Give your answer correct to four decimal places. 1 mark

The principal of the school decides to take a sample of 50 students from a number of different classes. For samples of size 50 from the population of students with a mean time of placing chairs on tables of 1 minute and standard deviation of 12 seconds, $\hat{P}$ is the random variable of the distribution of sample proportions of students with a mean time of less than 40 seconds.

g. Find the probability that $\Pr(\hat{P} \geq 0.06 \mid \hat{P} \geq 0.04)$. Give your answer correct to three decimal places. Do not use a normal approximation. 3 marks

SECTION 2 – Question 4 - continued

The principal finds that, in a particular sample of 50 students, five of them take less than 40 seconds to place chairs on tables.

h. Determine the 95% confidence interval for the principal's estimate of the proportion of interest. Give values correct to two decimal places. 2 marks

i. The probability density function for lateness to class, *x* minutes, is

$$f(x) = \begin{cases} kxe^{-\frac{1}{2}(x-0.4)} & x \geq 0 \\ 0 & elsewhere \end{cases}$$

i. Find the mean time for lateness to class. 2 marks

ii. Find the median time for lateness to class, correct to two decimal places. 1 mark

END OF QUESTION AND ANSWER BOOK