MATHEMATICAL METHODS

Written Examination 2

TSSM™
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2017 Trial Examination

<u>SOLUTIONS</u>

SECTION 1

Question 1

Answer: C

Explanation:

$Sketch\ on\ CAS\ over\ the\ restricted\ domain$

Question 2

Answer: B

Explanation:

$Sketch\ on\ CAS\ or\ solve\left(\frac{dy}{dx}=0\right)$

Question 3

Answer: E

Explanation:

$solve\ on\ CAS\ over\ restricted\ domain$

Question 4

Answer: A

Explanation:

$Period=\frac{2\pi}{2/\pi},\ \ range:[-3-1,-3+1]$

Question 5

Answer: C

Explanation:

$\frac{f(5)-f(0)}{5}\ on\ CAS$

Question 6

Answer: B

Explanation:

Find inverse rule on CAS. $Domain\ of\ g^{-1} = Range\ of\ g = [3, \infty)$

Question 7

Answer: E

Explanation:

$1 - \Pr(same) = 1 - (0.2^2 + 0.25^2 + 0.45^2 + 0.1^2)$

Question 8

Answer: D

Explanation:

$Period = 16 \rightarrow n = \frac{\pi}{8},\ amp = 3$

Question 9

Answer: B

Explanation:

$\frac{1}{2}\frac{d}{dx}\left(x^2 log_e(kx)\right) - \frac{x}{2} = xlog_e(kx)$

$\frac{x^2}{2} log_e(kx) - \int \frac{x}{2}\ dx + c = \int xlog_e(kx)\ dx$

Question 10

Answer: A

Explanation:

$tangentline(cx^2 - 5, x, -1)\ on\ CAS\ and\ then\ sub\ in\ (-4, -1)$

Question 11

Answer: B

Explanation:

$x' = \frac{1}{2}x,\ y' = -y \rightarrow -y = e^{2\left(\frac{1}{2}x\right)} - 4$

Question 12

Answer: A

Explanation:

$Area = \int_{-b}^{b} (y1 - y2)dx$

$= \int_{-b}^{b} \left(a - f(x)\right)dx$

Question 13

Answer: B

Explanation:

Sketch on CAS

Note that option A is incorrect because $x = 7$ is outside the domain of the function.

Question 14

Answer: D

Explanation:

$\Pr(X > 23.3) = \Pr(Z > -2) = \Pr(Z < 2)$

Question 15

Answer: E

Explanation:

$\int_0^1 f(x)dx = 1 \rightarrow k = \frac{15}{14},\ \ solve\ \int_a^1 f(x)dx = \frac{256-11\sqrt{2}}{256}\ \ for\ \ a$

Question 16

Answer: C

Explanation:

$x' = 2x\ \ and\ \ y' = -3y + 1$

$\rightarrow x = \frac{x'}{2}\ \ and\ \ y = \frac{y'-1}{-3}$

Substituting into $f(x) = \cos(2x)$ *gives*

$g(x) = 1 - 3\cos(x)$

Question 17

Answer: D

Explanation:

$\Pr\left(\hat{P} \geq \frac{5}{32}\right) = \Pr(X \geq 5) = binomcdf(32, 0.2, 5, 32)$

Question 18

Answer: A

Explanation:

Check for solution that satisfies both equations- $0.15 + a = 0.58\ \ and\ \ a + b = 0.55$

Question 19

Answer: B

Explanation:

$(x-2)(3-x)=\frac{k}{2}-x,\ \Delta>0$

Question 20

Answer: D

Explanation:

$\frac{d}{du}\left(\sqrt{u^2+(u^2+1-2)^2}\right)=0\rightarrow u=0,\pm0.5\sqrt{2}$

When u = 0 the distance is 1, when u = $0.5\sqrt{2}$ the distance is $\frac{\sqrt{3}}{2}$ that is <1.

Minimal distance occurs when $x=0.5\sqrt{2}$, distance = $\frac{\sqrt{3}}{2}$

SECTION 2

Question 1

a. $Period = \frac{2\pi}{\frac{\pi}{24}} = 48,$ 1 mark

$Range: [-1, 11]$

1 mark

b. $f'(x) = -\frac{\pi}{4}\sin\left(\frac{\pi}{24}x\right)$

1 mark

c. $y = -\frac{\pi}{4}x + 5(3\pi + 1)$ (on CAS)

1 mark

d. $\frac{\pi}{4} = -\frac{\pi}{4}\sin\left(\frac{\pi}{24}x\right) \rightarrow x = 36, 84$ 1 mark

$tangentline(f(x), x, 36):\ y = \frac{\pi x}{4} - 9\pi + 5$

$tangentline(f(x), x, 84):\ y = \frac{\pi x}{4} - 21\pi + 5$

1 mark

e. $x' = -x + b,\ y' = ay + 5$ 1 mark

$x = -x' + b,\ y = \frac{y' - 5}{a}$

$\frac{y' - 5}{a} = -\frac{\pi}{4}\sin\left(\frac{\pi}{24}(-x' + b)\right)$ 1 mark

$y' = -\frac{\pi}{4}a\sin\left(-\frac{\pi}{24}x' + \frac{b\pi}{24}\right) + 5$

$\frac{b\pi}{24} = \frac{\pi}{2} \rightarrow b = 12, \quad -\frac{\pi}{4}a = 6 \rightarrow a = -\frac{24}{\pi}$ 1 mark

f. $Solve\ on\ CAS:\ x = 6,\ 30,\ 54,\ 78$

2 marks

Question 2

a. $A(2, e^{-4} + 3)\ \ and\ \ B(0, 4)$

2 marks

b. $Area = \int_0^2 (e^x - e^{-2x})\,dx$ 1 mark

$= \left(e^x + \frac{e^{-2x}}{2}\right)\begin{matrix}2\\0\end{matrix} = e^2 - 1 + \frac{e^{-4}}{2} - \frac{1}{2} = e^2 + \frac{1}{2e^4} - \frac{3}{2}$ 1 mark

c. $Tangent\ line: y = ex + 3$ 1 mark

$C(0, 3)$

1 mark

d. $\theta = \tan^{-1}(e^1) = 69.8 \approx 70°$

2 marks

e. $Area = \int_1^2 ((e^x + 3) - (ex + 3))\, dx$ 1 mark

$= e^2 - \frac{5e}{2}$

1 mark

f. $Tangent\ passing\ through\ origin: y = ex$ 1 mark

$ea = e^{-2a} + 3$ 1 mark

$a = 1.41$ 1 mark

g. Length = $\sqrt{2^2 + (e^{-4} + 3 - 3)^2}$

= $\sqrt{4 + e^{-8}}$

1 mark

Question 3

a. $f(x) = 2 - \frac{7}{x+4}$ (Use propfrac on CAS or long division)

1 mark for “2”

1 mark for “-7”

b. $Range: R\backslash\{2\}$

1 mark

c. $x = 2 - \frac{7}{y+4} \rightarrow$ 1 mark

$f^{-1}(x) = -4 - \frac{7}{x-2}$

1 mark

d. $solve\ -4 - \frac{7}{x-2} = x \rightarrow x = -1 \pm \sqrt{2}$

$\left(-1 + \sqrt{2}, -1 + \sqrt{2}\right)\ and\ \left(-1 - \sqrt{2}, -1 - \sqrt{2}\right)$

2 marks

e. $Area = \int_{-1-\sqrt{2}}^{-1+\sqrt{2}} \left(\frac{2x+1}{x+4} - x\right) dx$

1 mark for correct integral and 1 mark for correct terminal values

f.

i. $[-5, -4) \cup (-4, \infty)$

1 mark

ii. $\frac{d}{dx}\big(h(x)\big) = 0 \rightarrow x = -4 \pm \sqrt{7}$ 1 mark

$\left(-4-\sqrt{7},\ 2\sqrt{7}+6\right)$ and $\left(-4+\sqrt{7},\ 6-2\sqrt{7}\right)$

1 mark

iii. $Distance = \sqrt{x^2 + \left(\frac{2x+1}{x+4} - x\right)^2}$ 1 mark

$\frac{d}{dx}(distance) = 0 \rightarrow x = -6.13, 0.11$

$Min\ \ distance\ at\ \ x = 0.11 \quad (0.11, 0.19)$ 1 mark

Minimum distance = 0.22

1 mark

Question 4

a. $binomcdf(24, 0.1, 1, 24) = 0.9202$

2 marks

b. $\Pr(X < 3 | X \geq 1) = \frac{binomcdf(24,0.1,1,2)}{binomcdf(24,0.1,1,24)}$ 1 mark

$= 0.5265$

1 mark

c. $normcdf(0, 40, 60, 12) = 0.0478$

2 marks

d. $invnorm(1 - 0.4062, 60, 12) = 62.85$

1 mark

$m = 63$

1 mark

e. $invnorm(0.28, 0, 1) = \frac{46-60}{c}$ 1 mark

$-0.5828 = -\frac{14}{c}$ 1 mark

$c = 24.02$ 1 mark

f. $binompdf(24, 3, 0.28) = 0.0448$

1 mark

g. $\Pr(\hat{P} \geq 0.06 \mid \hat{P} \geq 0.04)$ 1 mark

$= \Pr(X \geq 3 | X \geq 2) = \frac{\Pr(X\geq3)}{\Pr(X\geq2)} = 0.617$

2 marks

h. $\left(\frac{5}{50} - 1.96\sqrt{\frac{\frac{5}{50}\left(1-\frac{5}{50}\right)}{50}},\ \frac{5}{50} + 1.96\sqrt{\frac{\frac{5}{50}\left(1-\frac{5}{50}\right)}{50}}\right) = (0.02, 0.18)$

2 marks

i.

i. $\int_0^\infty f(x)dx = 1 \rightarrow k = \frac{e^{-\frac{1}{5}}}{4}$ 1 mark

$Mean = \int_0^\infty x \times f(x)dx = 4\ minutes$

1 mark

ii. $\int_0^m f(x)dx = \frac{1}{2} \rightarrow k = -1.54,\ 3.36$

$Median = 3.36\ minutes$

1 mark