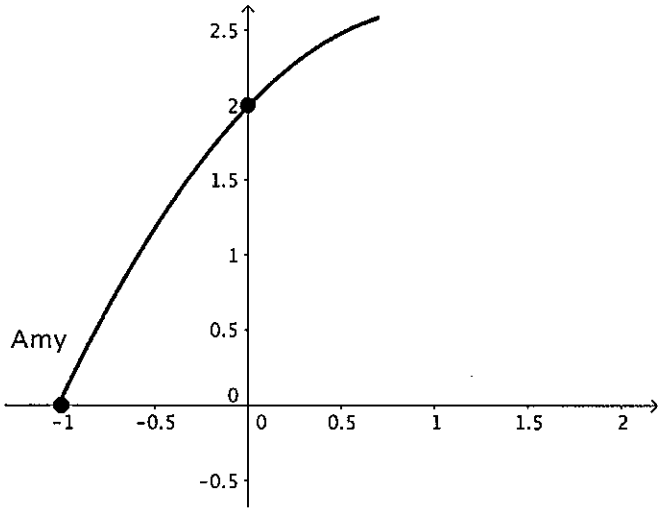
Analysis Task

Fire when ready

ANSWERS.

Name:

Two countries that share a common border are at war with each other. Take the line along the ground between the two armies to be the x -axis and the y -axis to be vertically above the border between the two countries (see graph). The army that is 1 km to the left of the border fires a missile towards the enemy. The equation of the path of the missile is $y_{M1} = -x^2 + bx + c$ and the missile will pass 2 km directly above the border.



Part 1- Ready, aim, fire!

(a) Find the values of b and c .

$y_{M1} = -x^2 + bx + c$

sub (0,2) $2 = c$

sub (-1,0) $0 = -1 - b + 2$

$b = 1$

① substitution

① $c = 2$

① $b = 1$

(b) How far into enemy territory, in kilometres, will the missile land?

$y_{M1} = -x^2 + x + 2$

Let $-x^2 + x + 2 = 0$

$x = -1$ or 2

missile lands 2km into enemy territory. ①

(c) What is the greatest height in kilometres reached by the missile?

2·25 km. ①

Part 2-Oops!

Moments before the first missile was fired, peace was declared between the two countries. As a result, just after the time that the first missile was fired, a second missile was fired to destroy the first. The equation of the path of the second missile is $y_{M2} = -\frac{1}{5}(x^2 - 3x - 4)$.

(d) Show that the point at which the missiles collide is given by one of the solutions of

$$2x^2 - x - 3 = 0.$$

$-\frac{1}{5}(x^2-3x-4) = -x^2+x+2$ ① method

$x^2-3x-4 = 5x^2-5x-10$

$4x^2-2x-6 = 0$ ①

$2x^2-x-3 = 0$

(e) Use algebra to find the coordinates of the point at which the missiles collide.

$2x^2-x-3=0$

$(2x-3)(x+1)=0$ ① method

$x = 3/2$ or -1 ①

Missiles collide at $(3/2, 5/4)$ ①

(f) Assume that the second army is positioned where the first missile would have landed if it were not destroyed. How far from this army does the collision of the missiles take place:

i. exactly?

ii. correct to the nearest tenth of a kilometre?

$d = \sqrt{(2-3/2)^2+(0-5/4)^2}$ ① method

$= \sqrt{\frac{29}{16}}$

$= \frac{\sqrt{29}}{4}$ km ①

(ii) 1.3 km. ①

Part 3 Oops again!

It is known that anything within 1.5 km of the collision of the missiles will be destroyed. As a result, shortly after the second missile was fired, a third missile was fired. The third missile was aimed to destroy the first missile before the second missile reached it, so that the second missile passed safely over the army and into the sea. The third missile was much faster than the other missiles and travelled in a straight line. It was fired at an angle of 45° to the ground.

(g) What is the gradient of the path of the third missile?

$m = 1$ ①

(h) What is the equation of the path of the third missile?

$y_{M_3} = x + 1$ ①

(i) Use algebra to find the coordinates of the point at which the first and third missiles collide.

$-x^2 + x + 2 = x + 1$ ① method.

$x^2 - 1 = 0$

$x = \pm 1$ ①

Missiles collide at $(1, 2)$ ①

(j) Exactly how far from the army on the right does the collision occur?

$(1, 2)$

$(2, 0)$

$d = \sqrt{(2-1)^2 + (0-2)^2}$ ① method.

$= \sqrt{5}$ km. ①