



***Online & home tutors*** Registered business name: itute ABN: 96 297 924 083

# ***Mathematical Methods***

## ***2017***

### ***Trial Examination 1 (1 hour)***

## Instructions

Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

### Question 1

A secondary school has 500 tall (defined as over 175 cm in height) students and 1000 short (under 175 cm) students. The school is divided into 30 groups at random. There are 50 students in each group. Assume that each group is a random sample of students at the school. Let  $q$  be the proportion of tall students in a group.

- a. Find the mean of  $q$  of the 30 groups of students. 1 mark

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- b. Find the proportion of the 30 groups having  $q > \frac{2}{5}$ .

Express your answer as a percentage correct to the nearest whole number of percent. 3 marks

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Another secondary school also has 1500 students. A sample is taken at random and it consists of 5 tall students and 20 short students.

- c. Find an approximate 95% confidence interval for the number of tall students at this school. 2 marks  
*Use 2 instead of 1.96 in your calculations. Correct your answer to the nearest whole number.*

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**Question 2** Consider  $f(x) = \frac{x^2 - 3x + 2}{x^2 - 1}$  where  $x > -2$ .

a. Find the value(s) of  $x$  such that  $f(x) = 0$

1 mark

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b. Find  $f'(x)$  in simplest form.

2 marks

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c. State the domain and range of  $f'(x)$ .

2 marks

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**Question 3** Let  $f(x) = \tan\left(\frac{2x}{3}\right)$ .

a. Given  $g(x)$  is a transformation of  $f(x)$ . Find a possible  $g(x)$  such that  $g(x) = f(x)$ . 2 marks

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b. Evaluate  $f'\left(\frac{11\pi}{4}\right)$ . 3 marks

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**Question 4**

The graphs of  $y = e^{2x}$  and  $y = 2(e^x + 1)$  have an intersection. Find the  $y$ -coordinate of the intersection. Express your answer in  $a + b\sqrt{c}$  form where  $a$ ,  $b$  and  $c$  are natural numbers. 3 marks

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**Question 5**

Given  $f(x) = \log_{10}(x) - \log_{10}(x+1)$  and  $f^{-1}(x) = \frac{a}{b10^{-x} + c}$  where  $a$ ,  $b$  and  $c$  are integers.

Find a set of values of  $a$ ,  $b$  and  $c$ .

3 marks

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**Question 6** Consider  $f(x) = \frac{1}{4}(x-2)^4$ .

a. Find the average value of  $f(x)$  in the interval  $[0, 3]$ .

2 marks

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b. Find the average rate of change of  $f(x)$  in the interval  $[0, 3]$ .

2 marks

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**Question 7**

- a. Find  $\frac{d}{dx}(x \cos x)$ . 1 mark

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- b. Hence evaluate  $\int_0^{\pi} (x \sin x) dx$ . 3 marks

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- Question 8** Consider the following set of simultaneous linear equations where parameter  $m \in R$ .

$$mx - (m - 1)y = m + 1$$

$$(m - 1)x + my = m + 1$$

- a. Find the value(s) of  $m$  such that the graphs of the two equations intersect at the y-axis. 2 marks

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- b. Show that the two equations always have a unique solution for any  $m$  value. 2 marks

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**Question 9**

Determine the maximal domain of  $\left(\frac{1}{2} - \log_e\left(\frac{x-1}{2}\right)\right)^{\frac{3}{2}}$ .

3 marks

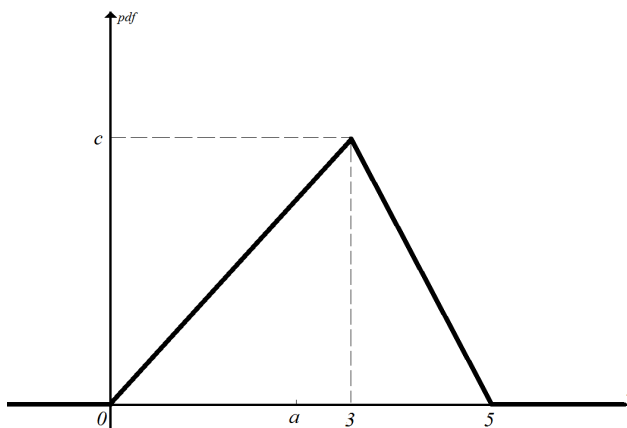
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**Question 10** The following graph shows the probability density function of random variable  $X$ .



a. The maximum value of the probability density function of  $X$  is  $c$ . Find the value of  $c$ . 1 mark

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b. The median of  $X$  is  $a$ . Find the value of  $a$ . 2 marks

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**End of exam 1**