Year 12 *Trial Exam Paper*

2017

MATHEMATICAL METHODS

Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring blank sheets of paper and/or correction liquid/tape into the examination.

Materials provided

- Question and answer book of 27 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the box provided above on this page, and on your answer sheet for multiple-choice questions.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

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SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the multiple-choice answer sheet.

Select the response that is **correct** for the question.

A correct answer scores 1 mark, whereas an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

If more than one answer is selected, no marks will be awarded.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f: R \to R,\ f(x) = -2\sin\left(\frac{\pi x}{2}\right) + 3$.

The period and range of this function are, respectively

A. 4 and $[-5, 1]$

B. 4 and $[-2, 2]$

C. 4 and $[1, 5]$

D. $\frac{\pi}{2}$ and $[-5, 1]$

E. $\frac{\pi}{2}$ and $[1, 5]$

Question 2

The inverse function of $f: (3, \infty) \to R,\ f(x) = \frac{1}{\sqrt{x-3}}$ is

A. $f^{-1}: R^+ \to R \quad f(x) = \frac{1}{x^2} - 3$

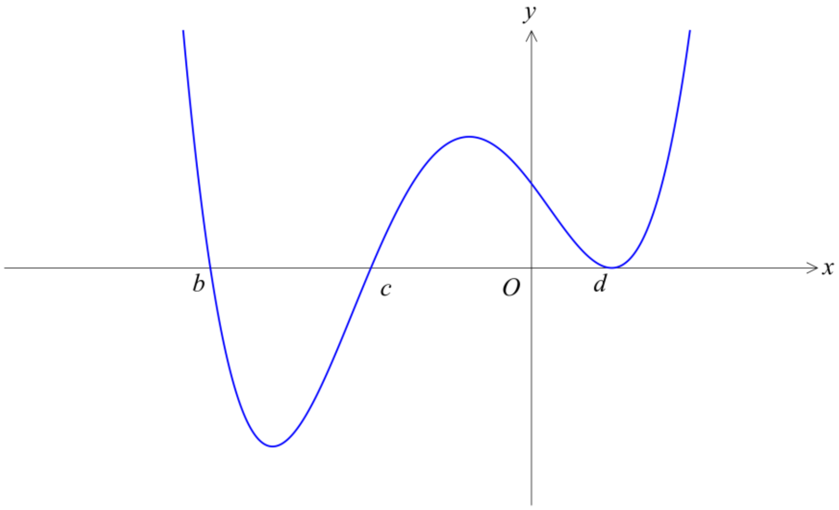
B. $f^{-1}: R^+ \to R \quad f(x) = \frac{1}{x^2} + 3$

C. $f^{-1}: R \backslash \{0\} \to R \quad f(x) = \frac{1}{x^2} + 3$

D. $f^{-1}: R^+ \to R \quad f(x) = x^2 + 3$

E. $f^{-1}: (3, \infty) \to R \quad f(x) = \frac{1}{x^2 + 3}$

Question 3



The rule for a function with the graph shown above could be

A. $y=(x+b)(x+c)(x-d)^2$

B. $y=(x-b)(x-c)(x+d)^2$

C. $y=(x-b)(x-c)(x-d)^2$

D. $y=(x+b)(x+c)(x-d)$

E. $y=(x+b)(x+c)(x+d)^2$

Question 4

Consider the tangent to the graph of $y=x^3$ at the point $(-2,-8)$.

Which of the following points lies on this tangent?

A. $(-1,-4)$

B. $(-1, 4)$

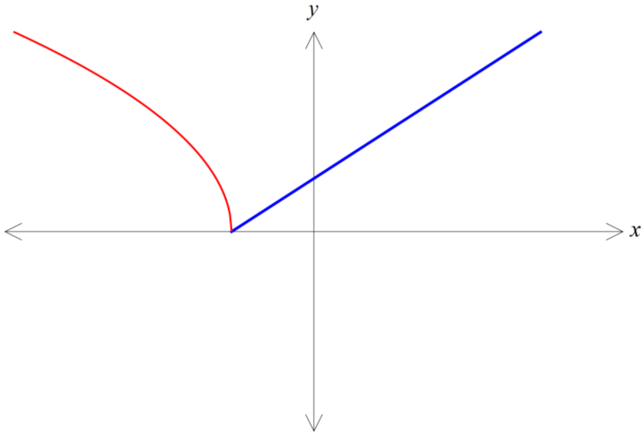
C. $(2,-40)$

D. $(1, 8)$

E. $(2,-16)$

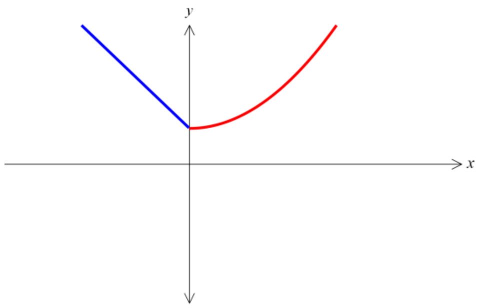
Question 5

The graph of the function with the equation $y = f(x)$ is shown below.

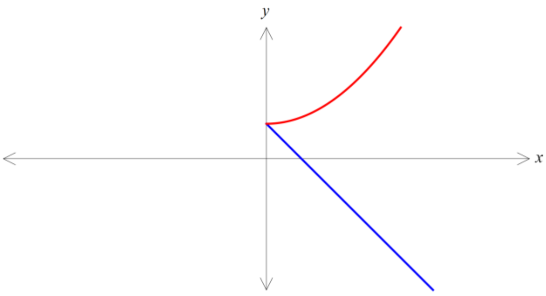


Which of the following is most likely to be the graph of $y = -f^{-1}(x)$?

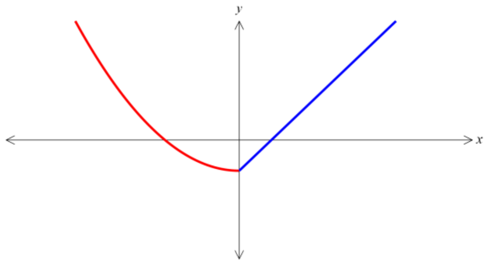
A.



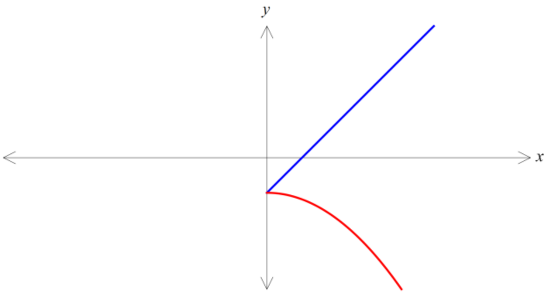
B.



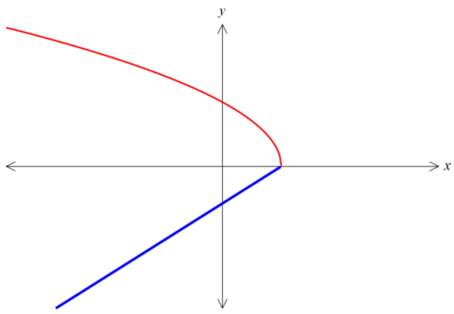
C.



D.



E.



Question 6

For the polynomial $P(x) = x^3 - ax^2 + 6x - 7$ with $P(2) = -3$, the value of a is

A. 4

B. –4

C. –3

D. 3

E. –1

Question 7

The range of the function $f : (-3, 5] \to R,\ f(x) = -x^2 - 3x - 5$ is

A. [–45, –5)

B. R

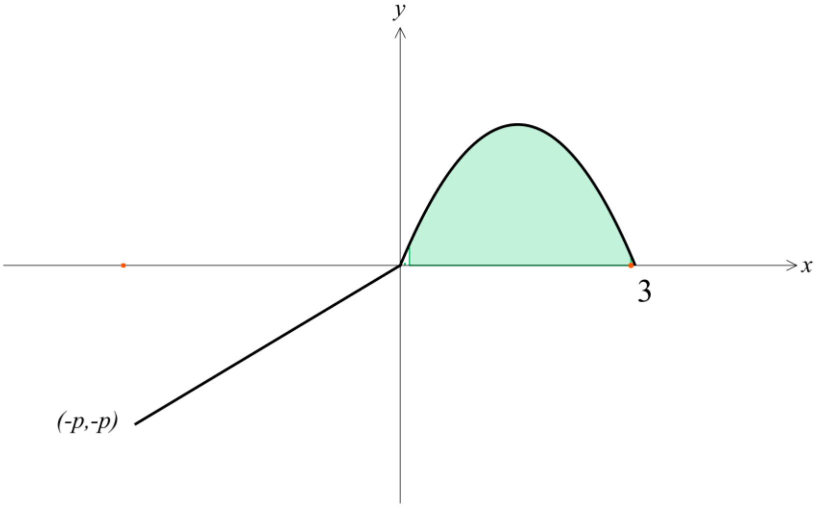
C. [–45, –5]

D. [–45, –2.75]

E. [–45, –1.5]

Question 8

The graph of the function $f:[-p, 3] \rightarrow R$ is shown below.



The average value of f over the interval $[-p, 3]$ is zero.

The area of the shaded region is $\frac{81}{8}$ square units.

If the graph is a straight line for $x \in [-p, 0]$, then the value of p is

A. 3

B. 9

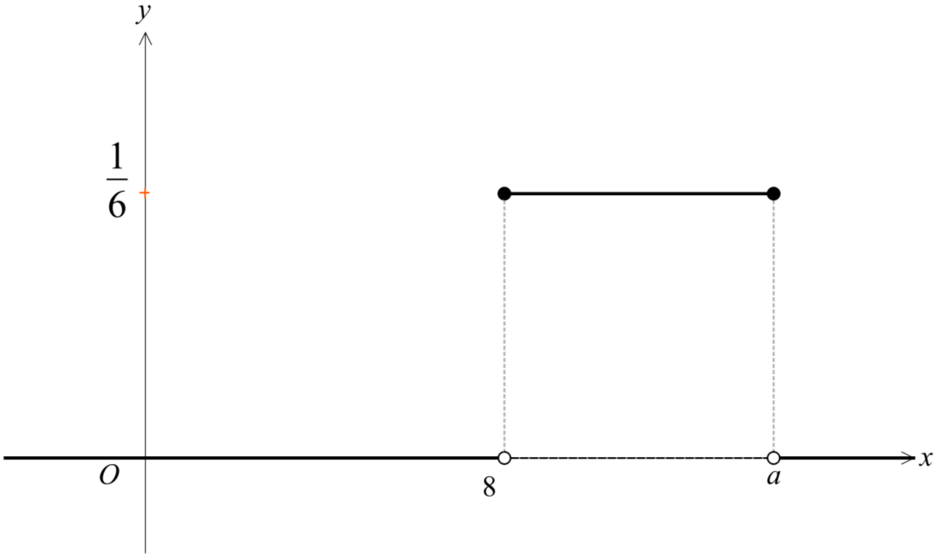
C. $\frac{81}{2}$

D. $\frac{9}{2}$

E. $-\frac{9}{2}$

Question 9

The graph of the probability density function of a continuous random variable, *X*, is shown below.



If $a > 8$, then $E(X)$ is equal to

A. 6

B. 14

C. 10

D. 11

E. 12

Question 10

Let the random variable $\hat{P}$ represent a sample proportion observed in an experiment.

If $p = \frac{1}{5}$, what is the smallest integer value of the sample size such that the standard deviation of $\hat{P}$ is less than or equal to $\frac{1}{625}$?

A. 25

B. 6250

C. 125

D. 325

E. 62 500

Question 11

An exit poll of 2000 voters found that 1240 favoured candidate A.

An approximate 95% confidence interval for the proportion of voters from the total population in favour of candidate A is

A. $\left(0.62-\sqrt{\frac{0.62\times0.38}{2000}},\ 0.62+\sqrt{\frac{0.62\times0.38}{2000}}\right)$

B. $\left(0.62-1.65\sqrt{\frac{0.62\times0.38}{2000}},\ 0.62+1.65\sqrt{\frac{0.62\times0.38}{2000}}\right)$

C. $\left(0.62-2.58\sqrt{\frac{0.62\times0.38}{2000}},\ 0.62+2.58\sqrt{\frac{0.62\times0.38}{2000}}\right)$

D. $\left(0.62-1.96\sqrt{\frac{0.62\times0.38}{2000}},\ 0.62+1.96\sqrt{\frac{0.62\times0.38}{2000}}\right)$

E. $\left(0.62-\sqrt{\frac{0.62\times0.38}{620}},\ 0.62+\sqrt{\frac{0.62\times0.38}{620}}\right)$

Question 12

The binomial random variable, X, has $E(X)=3$ and $\text{Var}(X)=\frac{9}{4}$.

$\Pr(X=1)$ is equal to

A. $\left(\frac{1}{4}\right)^{12}$

B. $\left(\frac{3}{4}\right)^{12}$

C. $\left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{11}$

D. $12\left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{11}$

E. $12\left(\frac{1}{4}\right)^{11}\left(\frac{3}{4}\right)^{1}$

Question 13

The continuous random variable, *X*, has a normal distribution with mean 60 and standard deviation 7. If the random variable, *Z*, has the standard normal distribution, then the probability that *X* is less than 46 is equal to

A. $\Pr(Z > 2)$

B. $\Pr(Z > -2)$

C. $\Pr(Z > 74)$

D. $\Pr(X > 2)$

E. $\Pr(X > -2)$

Question 14

The transformation $T : R^2 \to R^2$, which maps the curve with the equation $y = e^x$ to the curve with the equation $y = e^{2x+4} - 3$, could have the rule

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

Question 15

If $y = \log_e(\sqrt{f(2x)})$, then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{\sqrt{f(2x)}}$

B. $\frac{1}{2\sqrt{f(2x)}}$

C. $\frac{f'(2x)}{2\sqrt{f(2x)}}$

D. $\frac{f'(2x)}{f(2x)}$

E. $\frac{1}{2f(2x)}$

Question 16

If $k = \int_2^5 \frac{2}{x}\, dx$, then $e^{\frac{k}{2}}$ is equal to

A. $\frac{25}{4}$

B. $e^{\frac{5}{2}} - e^1$

C. 5

D. $e^5 - e^2$

E. $\frac{5}{2}$

Question 17

If $\int_{-1}^{2} f(x)\,dx = 4$, then $\int_{-1}^{2} (3 - f(x))\,dx$ is equal to

A. -1

B. 5

C. 1

D. -4

E. 7

Question 18

The area of the region bounded by the *x*-axis, the *y*-axis and the curve $y = e^{2x} - 2e^{x} - 3$ is

A. $4\log_e(3)$

B. 3

C. $3\log_e(3)$

D. $4 - \log_e(3)$

E. $4 - 3\log_e(3)$

Question 19

If $y = 2e(e^{x} - 1)$, then the rate of change of *y* with respect to *x* when $x = 0$ is

A. $2e^{2}$

B. $2e - 1$

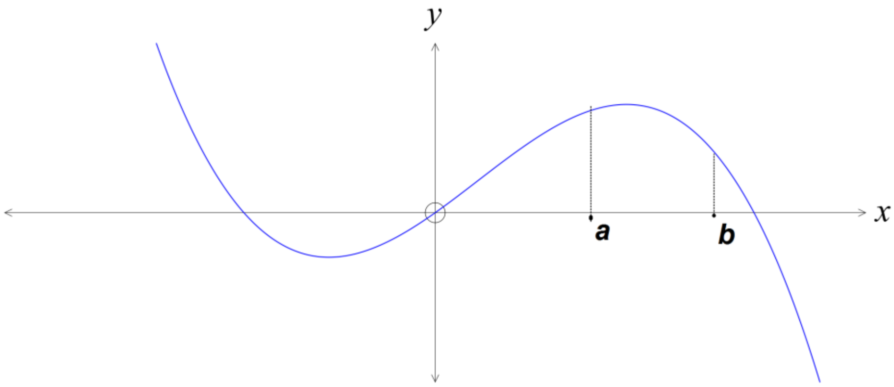
C. 0

D. $2e$

E. $2e^{2} - 2e$

Question 20

The graph of the function with equation $y = f(x)$ is shown below.



Let g be a function such that $g'(x) = f(x)$.

Over the interval (a, b), the graph of g will have a

A. minimum turning point.

B. maximum turning point.

C. positive gradient.

D. negative gradient.

E. stationary point of inflection.

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

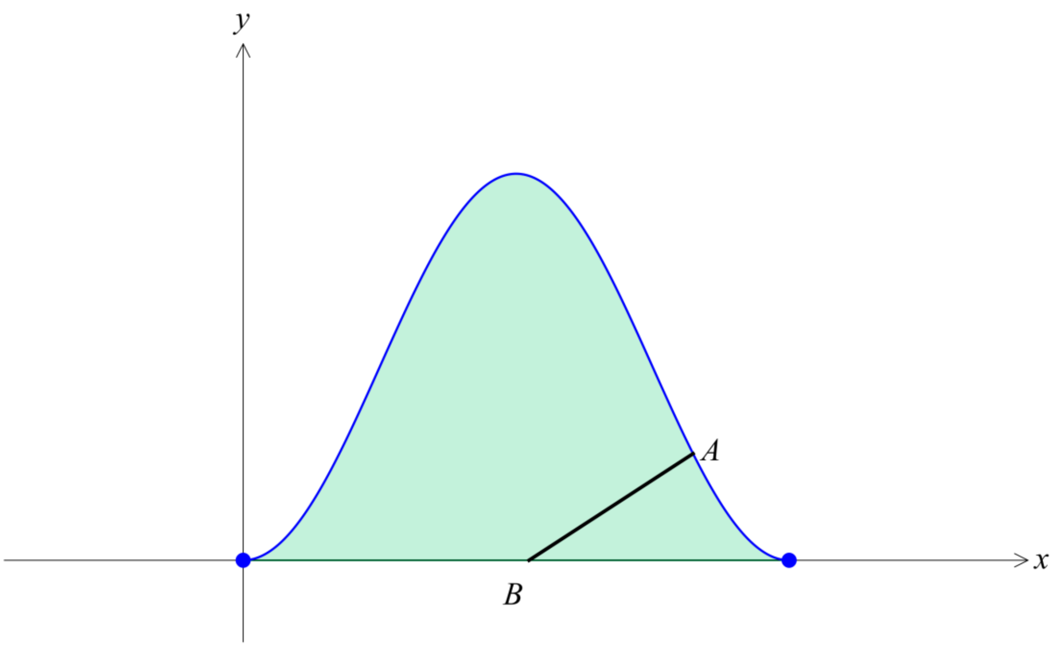
For questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise stated, diagrams are **not** drawn to scale.

Question 1 (13 marks)

Jordan has built a BMX bike ramp.

A cross-section of the ramp is shown in the diagram below, with coordinate axes as shown.



The curve has the equation $y = 2 - 2\cos\left(\frac{\pi x}{3}\right)$, $0 \le x \le 6$. All measurements are in metres.

a. State the maximum height of the ramp.

1 mark

b. Show that the gradient of the ramp is never greater than $\frac{2\pi}{3}$.

1 mark

c. Find the area of the shaded region.

1 mark

d. Find the value of c, correct to 2 decimal places, such that the area under the curve for $x \in [3-c, 3+c]$ is equal to two-thirds of the shaded area in **part c.**

2 marks

There is a supporting strut, *AB*, on the bike ramp, as shown in the original diagram. *A* is a point, as shown on the curve, that is 1 metre above the *x*-axis. *B* is a point on the *x*-axis such that *AB* is normal to the curve at *A*.

e. Find the equation of the line that passes through *A* and *B*.

2 marks

f. It is decided that another supporting strut be put in place. This particular supporting strut is placed at a point $x = a$ on the curve, where $3 < a \leq 6$. The strut is normal to the curve at $x = a$ and connects to the point $(3, 0)$.

Find the value of *a*, correct to three decimal places.

3 marks

g. Additional supporting struts are placed at other points along the curve over the interval $[3, 6]$. Each of these struts is normal to the curve and passes through different points along the *x*-axis.

To assist with the construction of the ramp it is necessary to know both the length of the longest strut and where the longest strut will be placed. If each strut measures strictly from the *x*-axis to the point on the curve, find the position on the curve that locates the longest strut and the length of the longest strut.

Give your answers correct to four decimal places.

3 marks

Question 2 (14 marks)

Susie is gathering data on two particular species of ants, jumping jack ant and red fire ant. They are very difficult to tell apart and both species are equally likely to be caught.

a. Let X be the random variable with values equal to the distance, in metres, of the jumping jack ant from an old log. The probability density function of X is

$$f(x) = \begin{cases} \dfrac{2x}{a^2} & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

i. It is known that the mean distance of a jumping jack ant from the old log is 120 metres. Show that the value of a is 180.

2 marks

ii. Find the probability that a jumping jack ant is more than 150 metres from the old log.

2 marks

One technique for distinguishing between the two types of ant is to measure the length of their bodies.

For red fire ants it is known that their body lengths are normally distributed with a mean of 25 mm and a standard deviation of 5 mm.

b. Find the probability that a randomly chosen red fire ant has a body length that is shorter than 18 mm. Give your answer correct to four decimal places.

2 marks

c. It is also known that 10% of jumping jack ants have a body length shorter than 20 mm and 10% of jumping jack ants have a body length longer than 28 mm. Assuming that the body length of a jumping jack ant is normally distributed, find the mean and the standard deviation of the body length of a jumping jack ant.

Give your answer correct to two decimal places.

3 marks

d. During her studies, Susie finds a particular site near the coast that has these two types of ants in abundance. 70% of them are jumping jack ants and 30% are red fire ants.

i. Susie examines a single ant from this site and finds its body to be shorter than 18 mm. What is the probability that it is a jumping jack ant?

Give your answer correct to three decimal places.

3 marks

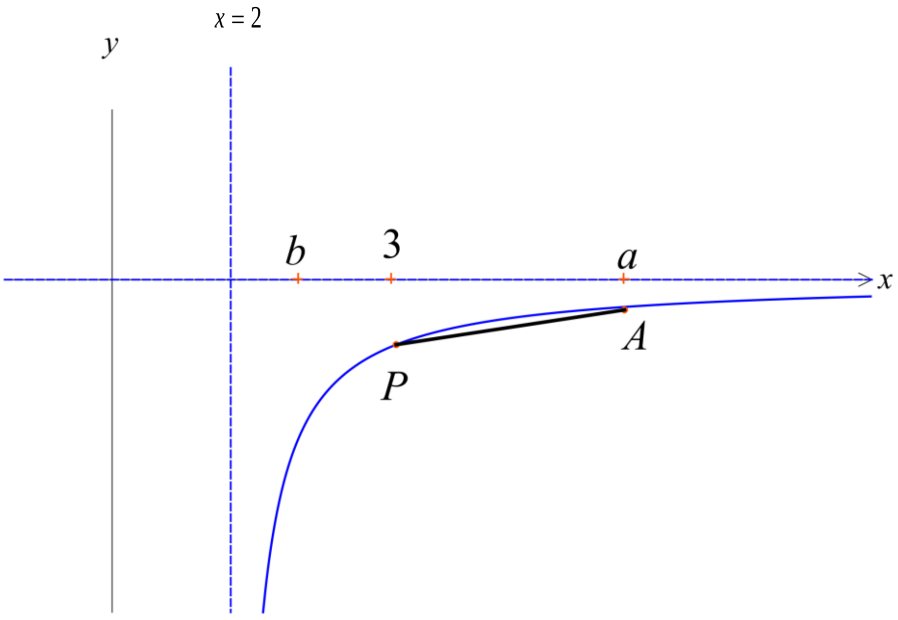
ii. Susie selects a sample of 500 ants from the site. Let $\hat{P}$ be the proportion of jumping jack ants in the sample. Use normal approximation to find the probability that the proportion of jumping jack ants in the sample is greater than 0.75.

Give your answer correct to four decimal places.

2 marks

Question 3 (12 marks)

The diagram below shows part of the graph of the function $f:(2,\ \infty)\to R,\ f(x)=\dfrac{6}{2-x}$.



a. **i.** Find, in terms of a, the gradient of the line segment PA.

1 mark

ii. Find the value of x over the interval $(2,\ a]$ such that the gradient of PA is equal to the gradient of the tangent to the graph of $y=f(x)$.

2 marks

b. **i.** Find $-\int_3^{e+2} \frac{6}{2-x}\, dx$.

1 mark

ii. Find the value of b for $b \in (2,3)$ such that $-\int_b^3 \frac{6}{2-x}\, dx = 6$.

2 marks

c. **i.** Find, in terms of a, the area bounded by the line segment PA, the x-axis and the lines $x = 3$ and $x = a$.

2 marks

ii. For what value of a does this area equal 6?

2 marks

iii. Hence, explain why $e > \sqrt{2} + 1$.

2 marks

Question 4 (14 marks)

Consider the function $f: R \to R,\ f(x) = x(x-4)^3 + 1$.

a. If $f'(x) = a(x-1)(x-4)^2$, where a is a constant, show that $a = 4$.

2 marks

b. The coordinates of the stationary points of the graph of $y = f(x)$ are $(u, -26)$ and $(4, v)$. Find the values of u and v.

2 marks

c. Find the values of x, correct to three decimal places, for which both $f'(x) \leq 0$ and $f(x) \leq 0$.

2 marks

d. Find the real values of k, correct to three decimal places, for which both solutions to the equation $f(x-k) = 2$ are positive.

1 mark

The function $f(x)$ can also be written as $f(x)=x^4-12x^3+48x^2-64x+1$.

e. Show that the area bounded by the graph of $y=f(x)-1$ and the x-axis is 51.2 square units.

3 marks

f. **i.** Describe a sequence of transformations that maps the graph of $y=f(x)$ on to the graph of $y=f(2x)-1$.

2 marks

ii. Hence, find the x-axis intercepts of the graph of $y=f(2x)-1$.

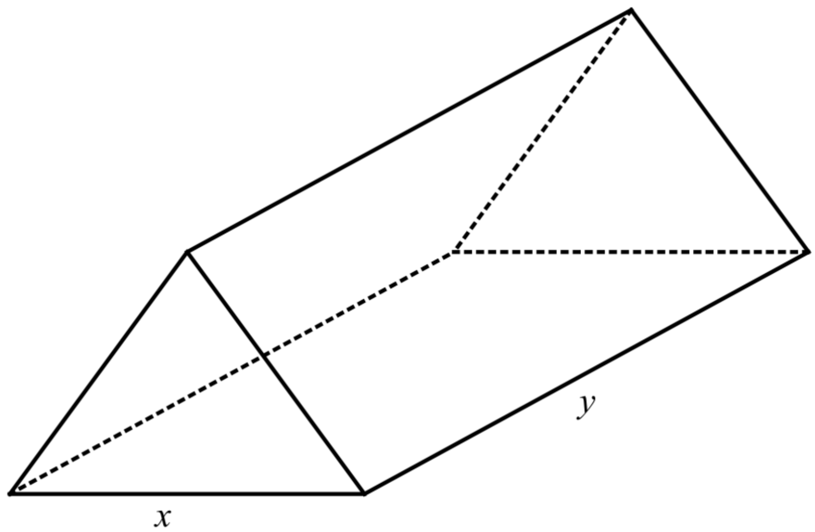
1 mark

iii. Use the answers to **part e.** and **part f.** to find the area of the region bounded by the graph of $y=f(2x)-1$ and the x-axis.

1 mark

Question 5 (7 marks)

A solid triangular prism is shown.



The cross-section of the prism is an equilateral triangle, and the sum of the edges of the prism is E cm.

a. **i.** Show that the volume of the prism is given by $V = \dfrac{\sqrt{3}Ex^2}{12} - \dfrac{\sqrt{3}x^3}{2}$.

3 marks

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ii. Find the side length, x, of the triangle that gives maximum volume.

2 marks

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b. Find the value of x for which the prism's surface area is a maximum.

2 marks

END OF QUESTION AND ANSWER BOOK