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Free Exam
For 2016-18 VCE Study design

Engage
Education
Foundation

Units 3 and 4 Maths Methods (CAS): Exam 2

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Section A – Multiple-choice questions

Question 1
The correct answer is A.

This can be observed by graphing $f(x)$.

Question 2
The correct answer is B.

$f(1) = -6$

$f(3) = -38$

Average rate of change$= \frac{\Delta f(x)}{\Delta x} = \frac{f(3)-f(1)}{3-1} = -16$

Question 3
The correct answer is C.

Let $f(x) = y$. Swap *x* and *y*, then solve for y.

Question 4
The correct answer is A.

$f(-1) = 5$

$f(2) = -1$

The turning point of the function $f(x)$ occurs at$(0,7)$. Hence the range of the function is$[-1,7]$.

Question 5
The correct answer is A.

We can construct a 2x2 matrix and find its determinant. If the determinant is zero, the system of simultaneous equations has either no or infinite number of solutions.

$$\begin{bmatrix} m+2 & 1 \\ 4 & m-1 \end{bmatrix}$$

$Determinant \ = (m+2)(m-1) - 4 = 0$

$m = 2 \ or - 3$

Substituting $m = -3 \ and \ n = 2$ yields the infinite number of solutions.

Question 6
The correct answer is D.

$$-\int_{-1}^{3} (3f(x) - 2)dx = -3\int_{-1}^{3} f(x)dx + \int_{-1}^{3} 2\,dx = 2$$

Question 7
The correct answer is D.

The derivative of a positive parabola yields a positive linear graph. Likewise, a negative linear graph yields a negative horizontal gradient graph.

Question 8
The correct answer is C.

This can be observed from the graph of $f(x)$.

Question 9
The correct answer is B.

$(x, y) \rightarrow (-\frac{x}{2}, y-3) \rightarrow (x', y')$

$x = -2x'$

$y = y' + 3$

$\therefore y' = -4{x'}^2 - 3$

Question 10
The correct answer is B.

$g(x) = -x^3 + \frac{1}{2}x^2 - x + c$

$g(1) = -1 + \frac{1}{2} - 1 + c = \frac{9}{2}$

$c = 6$

$\therefore g(x) = -x^3 + \frac{1}{2}x^2 - x + 6$

$g(-2) = 18$

Question 11
The correct answer is A.

$Average\ value = \frac{2}{5}\int_{\frac{1}{2}}^{3} -\frac{\sin(x)}{x}\,dx = -0.0175$

Question 12
The correct answer is B.

A signed area can have a negative value. Evaluating $\int_{-\infty}^{0} f(x)\,dx$ gives $-\frac{5}{2}$.

Question 13
The correct answer is A.

$\frac{dy}{dx} = 2\cos(2x)$

Hence, the gradient of the tangent is, $2\cos\left(2 * \frac{\pi}{6}\right) = 1$.

$y = \frac{\sqrt{3}}{2}$ when $x = \frac{\pi}{6}$. The equation of the tangent is therefore $y = x - \frac{\pi}{6} + \frac{\sqrt{3}}{2}$.

Question 14
The correct answer is E.

$Var(X) = sd^2 = np(1-p) = 81$

$900p(1-p) = 81$

$p = 0.1 \; or \; 0.9$

$p = 0.1$ since $p < 0.5$

$mean = np = 900 * 0.1 = 90$

Question 15
The correct answer is B.

$Pr(B) = 0.65$ can be obtained by sketching a Venn diagram.

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{0.30}{0.65} = 0.4615$$

Question 16
The correct answer is C.

$X \sim N(21,16)$

$Pr(X < 14) = 0.0401$

Question 17
The correct answer is E.

$\mathrm{Pr}(\mathrm{Z} < \mathrm{z}) = 0.7$

$\mathrm{z} = 0.5244$

$$\mathrm{z} = \frac{\mathrm{x} - \mu}{\sigma}$$

$$0.5244 = \frac{25.6 - 20}{\sigma}$$

$\therefore \sigma = 10.7$

Question 18
The correct answer is D.

The area under the graph $f(x)$ between $x = 0$ and $x = a$ must be 1.

$$\int_0^a e^{-2x} + 3 \, dx = 1$$
$$-\frac{1}{2}e^{-2a} + 3a + \frac{1}{2} = 1$$
$a \approx 0.26$

Question 19
The correct answer is A.

Year 11 Male: 248

Year 11 Female: 248

Year 12 Male: 209

Year 12 Female: 171

Total number of Year 11 and 12 students: 876

Year 11 and 12 boys to be selected:

$\frac{248}{876} * 100 \approx 28$ Year 11 boys

$\frac{209}{876} * 100 \approx 24$ Year 12 boys

The values should be rounded to whole numbers due to the nature of the sample.

Question 20

The correct answer is B.

$\hat{p} = 0.17$

$\mathrm{n} = 20$

z value for 99% confidence interval = 2.58

Substitute the values into the confidence interval formula$\left(\hat{p} - \mathrm{z}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + \mathrm{z}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$.

Section B – Short-answer questions

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a i

Amplitude = 2 [1]

Question 1a ii

Period = $\frac{2\pi}{\frac{\pi}{6}} = 12$ [1]

Question 1b

A sinusoidal graph with 2 complete cycles should be drawn over the domain, $t \in [0,24]$ [2]

End points are at $(0,4-\sqrt{2})$ and $(24,4-\sqrt{2})$ [1]

Maximum and minimum at $\left(\frac{3}{2},2\right),\left(\frac{15}{2},6\right),\left(\frac{27}{2},2\right),(\frac{39}{2},6)$ [1]

Question 1c

$h'(t) = \frac{\pi}{3}\cos(\frac{\pi t}{6} - \frac{3\pi}{4})$ [1]

$h'(2.5) = \frac{\pi}{6}$ metre/hour [1]

Question 1d i

$$h'(t) = \frac{\pi}{3}\cos\left(\frac{\pi t}{6} - \frac{3\pi}{4}\right) = 0$$

$$t = \frac{27}{2}, \frac{39}{2}, t \in [12,24]$$

Lowest depth at $t = \frac{27}{2}$ can be determined using double derivative or graph [1]

Question 1d ii

Highest point at $t = \frac{39}{2}$ [1]

Question 1e

$h(t) = 2.6$ [1]

$t = 3.02, 11.98, 15.02, 23.98$

The ferry operates between 6 am and 8 pm, $t \in [6,20]$ [1]

Hence the ferry can enter the water when $t \in [6,11.98]$ and $t \in [15.02,20]$.

$(11.98-6)+(20-15.02) = 10.96$ hours [1]

45 minutes = 0.75 hours

The number of round trip = $\frac{10.96}{0.75} = 14.62$.

Round down to a whole number = 14 round trips per day [1]

Question 2a

$$f(x) = -\frac{1}{3}(x^3 - 12x + 16)$$

Perform long division $(x^3 - 12x + 16) \div (x - 2)$ [2] for showing long division.

$f(x) = -\frac{1}{3}(x-2)^2(x+4)$ [1]

Question 2b

$f'(x) = -x^2 + 4$ [1]

$f'(x) = 0$

$x = -2 \text{ } or \text{ } 2$ [1]

$f(-2) = -\frac{32}{3}$

$f(2) = 0$

$\therefore (2,0) \text{ local maximum}, \left(-2, -\frac{32}{3}\right) \text{ local minimum}$ [1]

Question 2c

The graph should be a negative cubic with local maximum and x-intercept at$(2,0)$, local minimum at $\left(-2, -\frac{32}{3}\right)$. The other x-intercept is located at $(-4,0)$ and the y-intercept at $(0, -\frac{16}{3})$ [3]

Question 2d

$a = 2$ for an inverse function to exist.

Question 2e

$f'(x) = -x^2 + 4$

$f'(-3) = -5$ [1]

$f(-3) = -\frac{25}{3}$ [1]

$y - \left(-\frac{25}{3}\right) = -5\big(x - (-3)\big)$

$y = -5x - \frac{70}{3}$ [1]

Question 2f

$y = -5x - \frac{70}{3}$

$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + (-5x - \frac{70}{3})^2}$ [1]

$\frac{dL}{dx} = \frac{\sqrt{2}(39x+175)}{\sqrt{117x^2+1050x+2450}} = 0$ for minimum distance [1].

$\therefore x = a = -\frac{175}{39}$ and $y = b = -\frac{35}{39}$

Hence, $a = -4.487, b = -0.897$ [1]

$L = \sqrt{(\frac{-175}{39})^2 + (-\frac{35}{39})^2} = 4.576$ to 3 decimal places [1]

Question 2g

$y = -5x - \frac{70}{3}$

$f(x) = -\frac{x^3}{3} + 4x - \frac{16}{3}$

$-5x - \frac{70}{3} = -\frac{x^3}{3} + 4x - \frac{16}{3}$ [1]

$x = -3 \ or \ 6$

$Area = \int_{-3}^{6} \left(-\frac{x^3}{3} + 4x - \frac{16}{3}\right) - \left(-5x - \frac{70}{3}\right) dx$ [1]

$Area = 182.25$ [1]

Question 3a

Let X = the number of times that Jen won the race.

$\Pr(X > 3.5)$ [1]

$= \Pr(X = 4) + \Pr(X = 5) + \Pr(X = 6) + \Pr(X = 7)$

$= \binom{7}{4}(0.48)^4(0.52)^3 + \binom{7}{5}(0.48)^5(0.52)^2 + \binom{7}{6}(0.48)^6(0.52)^1 + \binom{7}{7}(0.48)^7(0.52)^0$ [1]

$= 0.4563$ [1]

Question 3a ii

$\Pr(X = 29 | X > 25)$

$= \frac{\Pr(X=29)}{\Pr(X>25)}$ [1]

$= \frac{0.0791}{0.5569}$

$= 0.1421$ [1]

Question 3b i

$M = 60$ [1]

Question 3b ii

$\int_{60}^{k} -\frac{1}{800}(x - 60)^2 + \frac{1}{10} \ dx = \frac{1}{2}$ [1]

$k = 42.440, 65.822, 71.737$

$\therefore k = 65.822, k \in (60, 68.944)$ [1]

$\therefore a = 65.822 - 60 = 5.822$ [1]

Question 3b iii

$\mu = \int_{54.178}^{65.822} -\frac{x}{800}(x - 60)^2 + \frac{1}{10} dx$ [1]

$= 59.997$ seconds [1]

Question 3b iv

The lower limit for the probability density function: $M - a = 54.178$ [1]

$\int_{54.178}^{56} f(x)\,dx = 0.127$ [1]

Question 4a
$\mathrm{N}(0) = 215$ [1]

Question 4b
$\mathrm{N}\left(\frac{1}{3}\right) \approx 221$ [1]

Question 4c
$N_{max} = 215 * 100 = 21500$

$15 * e^{t} + 200 = 21500$

$t = 7.258$ hours [1]

$D(T) = ae^{-\frac{1}{100}T} + 500$

$21500 = ae^{-\frac{1}{100}*7.258} + 500$ [1]

$a = 22581$ [1]

Question 4d
Criteria for a large population:

$np \geq 10$

$nq \geq 10$

$10n \leq N$

Let $np = 10$ for the smallest sample size.

$p = 0.02$

$\therefore \mathrm{n} = 500$ [1]

Question 4e
$\mu_{\hat{p}} = p = 0.02$ [1]

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.02*0.98}{5000}} = 0.002$ [2]

Question 4f
$\mathrm{n} = 100\ 000, \hat{p} = 0.035$ [1]

For the 90% confidence interval, the area under the curve to the left of z value is 0.95.

Hence the relevant z-value is 1.64 using the inverse normal [1]

$(\hat{p} - \mathrm{z}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + \mathrm{z}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}) = \ (0.0340, 0.0360)$ [1]

The microbiologist can be 90% confident that a 3.4-3.6% of the *E.coli* population is pathogenic [1]