Engage Education Foundation

Units 3 and 4 Maths Methods (CAS): Exam 1

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 1 hour writing time

Structure of book:

Number of questions	Number of questions to be answered	Number of marks
11	11	40
	Total:	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:
- This question and answer booklet of 11 pages including a formula sheet.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

a. Let $\mathrm{y} = x^2 \sin(3x)$. Find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

2 marks

b. For $f(x) = \frac{x}{e^x}$, find $f'(2)$.

2 marks

Total: 4 marks

Question 2

Evaluate $\int_1^2 \frac{1}{\sqrt{2x-1}} dx$.

2 marks

Question 3

Solve $\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right)=\sqrt{3}$, where $\theta \in [0,5\pi]$.

4 marks

Question 4

Solve $\boldsymbol{e^{2x}-6e^{x}=-8}$, for x.

2 marks

Question 5

Consider the functions:

$$f(x) = -4x^3$$

$$g(x) = \sqrt{2x - 1}$$

a. Assuming maximal domain for both $f(x)$ and $g(x)$, state whether $g(f(x))$ exists and briefly explain why.

1 mark

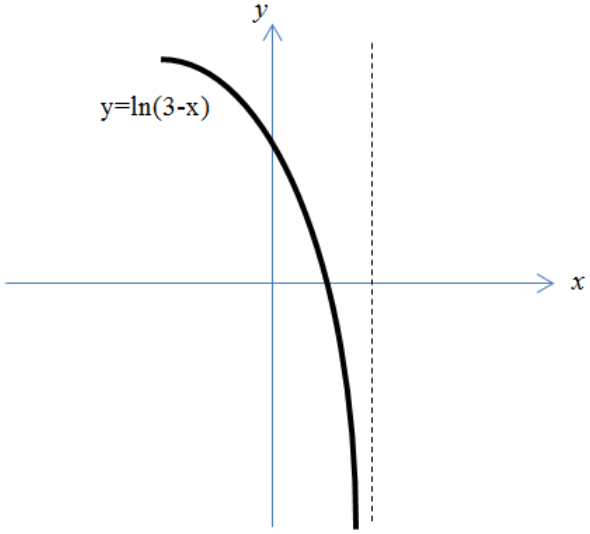
b. Write the equation for $\boldsymbol{f(g(x))}$ and state the domain and range.

3 marks

Total: 4 marks

Question 6

a. Find the exact area enclosed by the coordinate axes and $y = \ln(3 - x)$.



5 marks

b. Find the image of the curve $y = log_e(3 - x)$ after applying the following series of transformations:

- Reflection in the x-axis
- Dilation from the x-axis of factor 2
- Translation of 1 unit in the positive direction of x-axis

2 marks

Total: 7 marks

Question 7

The number of flowers, X, that Jenny sells in a given hour is a random variable with probability distribution:

X	4	5	6	7
$Pr(X = x)$	0.2	0.4	0.3	0.1

a.

i. Find the number of flowers that Jenny expects to sell in an hour.

1 mark

ii. Find the probability that Jenny sells more than the expected number of flowers.

1 mark

b. Given that variance of the distribution is 0.81, find the value of sd(2X-1).

2 marks

Total: 4 marks

Question 8

Consider the function:

$f(x) = 3x^2 - 4x$

Find the equation of normal to the curve f(x) at x = 2.

3 marks

Question 9

Find the sample size if the distribution of sample proportion $\hat{p}$ has $p = 0.1$ and standard deviation of $\hat{p}$ is 0.2.

2 marks

Question 10

A random variable X is normally distributed with mean 36 and variance 9.

a. Find $\Pr(33<X<42)$.

2 marks

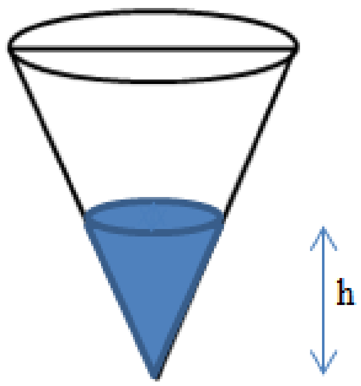
b. Find $\Pr(X<39|X>36)$.

1 mark

Total: 3 marks

Question 11

An inverted cone container with height 20 cm and diameter 10cm is shown below. Oil is poured into a container at a constant rate of $5cm^3/s$. The height of oil at any given time is h.



a. Express the volume ($V\ cm^3$) of oil in terms of h.

2 marks

b. What is the volume of oil inside the cone when the height of the oil is h = 15 cm?

1 mark

c. Determine the height of the oil when $\frac{dV}{dh} = \pi$.

2 marks

Total: 5 marks

Formula Sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}(\sin(ax)) = a\ \cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}(\cos(ax)) = -a\ \sin(ax)$		$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

End of Booklet