VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
Year

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter |

MATHEMATICAL METHODS

Written examination 1

Day Date

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (1 hour)**

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 13 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Version 2 – April 2016

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

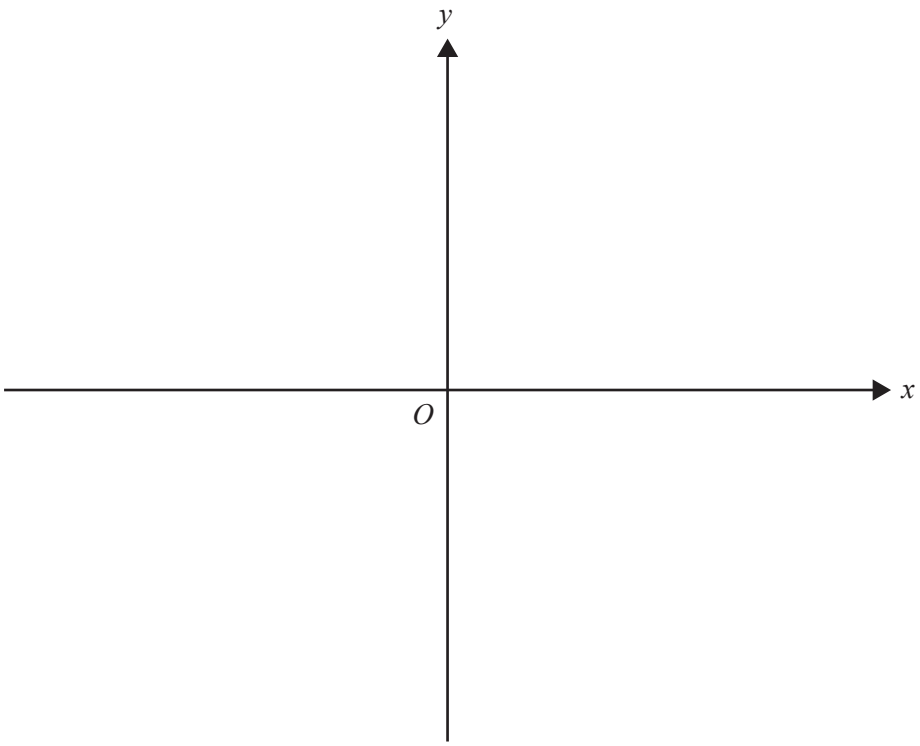
a. Differentiate $\sqrt{4-x}$ with respect to x. 1 mark

b. If $f(x) = \dfrac{x}{\sin(x)}$, find $f'\left(\dfrac{\pi}{2}\right)$. 2 marks

Question 2 (3 marks)

On the axes below, sketch the graph of $f: R\backslash\{-1\} \rightarrow R,\ f(x) = 2 - \dfrac{4}{x+1}$.

Label each axis intercept with its coordinates. Label each asymptote with its equation.



Question 3 (4 marks)

a. Find an antiderivative of $\frac{1}{(2x-1)^3}$ with respect to x. 2 marks

b. The function with rule $g(x)$ has derivative $g'(x) = \sin(2\pi x)$.

Given that $g(1) = \frac{1}{\pi}$, find $g(x)$. 2 marks

Question 4 (3 marks)

Let X be the random variable that represents the number of telephone calls that Daniel receives on any given day with probability distribution given by the table below.

x	0	1	2	3
$\Pr(X = x)$	0.2	0.2	0.5	0.1

a. Find the mean of X. 2 marks

b. What is the probability that Daniel receives only one telephone call on each of three consecutive days? 1 mark

Question 5 (3 marks)

The graphs of $y = \cos(x)$ and $y = a\sin(x)$, where a is a real constant, have a point of intersection at $x = \frac{\pi}{3}$.

a. Find the value of a. 2 marks

b. If $x \in [0, 2\pi]$, find the x-coordinate of the other point of intersection of the two graphs. 1 mark

Question 6 (5 marks)

a. Solve the equation $2\log_3(5) - \log_3(2) + \log_3(x) = 2$ for x. 2 marks

b. Solve $3e^t = 5 + 8e^{-t}$ for t. 3 marks

Question 7 (4 marks)

A student performs an experiment in which a computer is used to simulate drawing a random sample of size n from a large population. The proportion of the population with the characteristic of interest to the student is p.

a. Let the random variable $\hat{P}$ represent the sample proportion observed in the experiment.

If $p = \frac{1}{5}$, find the smallest integer value of the sample size such that the standard deviation of $\hat{P}$ is less than or equal to $\frac{1}{100}$. 2 marks

Each of 23 students in a class independently performs the experiment described above and each student calculates an approximate 95% confidence interval for p using the sample proportions for their sample. It is subsequently found that exactly one of the 23 confidence intervals calculated by the class does not contain the value of p.

b. Two of the confidence intervals calculated by the class are selected at random without replacement.

Find the probability that exactly one of the selected confidence intervals does not contain the value of p. 2 marks

Question 8 (4 marks)

A continuous random variable, X, has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{5}e^{-\frac{x}{5}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The median of X is m.

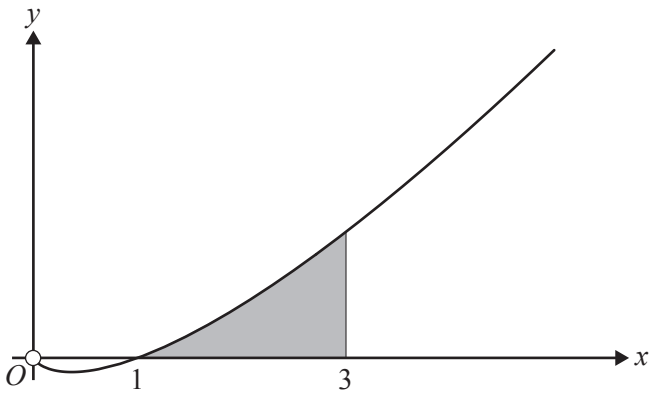
a. Determine the value of m. 2 marks

b. The value of m is a number greater than 1.

Find $\Pr(X < 1 | X \leq m)$. 2 marks

Question 9 (4 marks)

Part of the graph of $f: R^+ \rightarrow R, f(x) = x \log_e(x)$ is shown below.

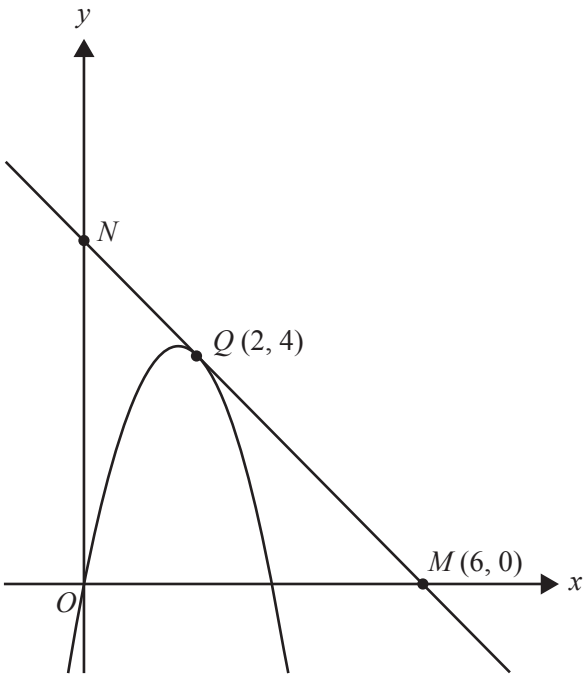


a. Find the derivative of $x^2 \log_e(x)$. 1 mark

b. Use your answer to **part a.** to find the area of the shaded region in the form $a \log_e(b) + c$, where a, b and c are non-zero real constants. 3 marks

Question 10 (7 marks)

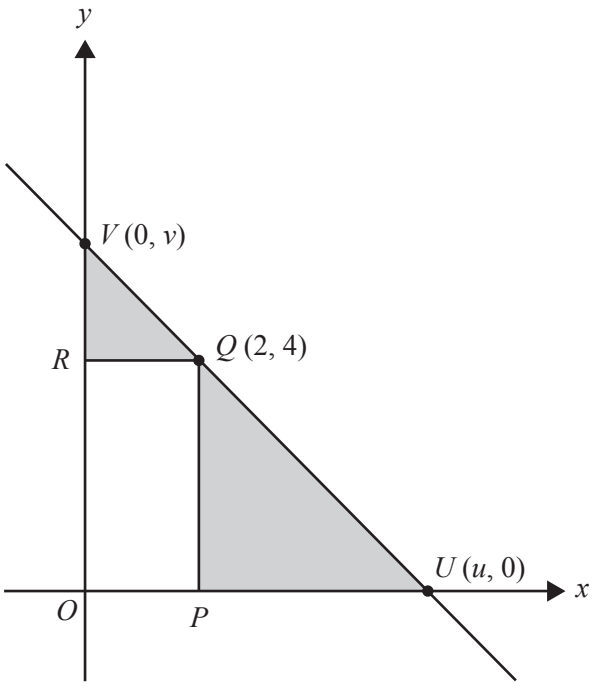
The line containing the points M and N intersects the x-axis at the point M with coordinates (6, 0). The line is also a tangent to the graph of $y = ax^2 + bx$ at the point Q with coordinates (2, 4), as shown below.



a. If a and b are non-zero real numbers, find the values of a and b. 3 marks

b. The line containing the points U and V intersects the coordinate axes at the points U and V with coordinates $(u, 0)$ and $(0, v)$, respectively, where u and v are positive real numbers and $\frac{5}{2} \leq u \leq 6$, as shown below.

The rectangle $OPQR$ has a vertex at Q on the line. The coordinates of Q are $(2, 4)$, as shown below.



i. Find an expression for v in terms of u. 1 mark

ii. Find the **minimum** total shaded area and the value of u for which the area is a minimum. 2 marks

iii. Find the **maximum** total shaded area and the value of u for which the area is a maximum. 1 mark

END OF QUESTION AND ANSWER BOOK