TSSM
Creating VCE Success

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2016 Trial Examination

STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHODS

Written Examination 2

Reading Time:15 minutes
Writing Time: 2 Hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	20	20	20
2	5	5	60
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one approved graphics calculator or CAS (memory DOES NOT have to be cleared) and, if desired, one scientific calculator, one bound reference (may be annotated). The reference may be typed or handwritten (may be a textbook).
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials Supplied

- Question and answer book of 25 pages.
- Working space provided throughout the book.

Instructions

- Print your **name** in the space provided at the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The asymptotes of a function with the rule $y = 2 - \frac{1}{2x-1}$ is/are:

A. $x = 1, \quad y = 2$
B. $x = -\frac{1}{2}, \quad y = 2$
C. $x = \frac{1}{2}, \quad y = -2$
D. $x = \frac{1}{2}, \quad y = 2$
E. $x = \frac{1}{2}$

Question 2

Let $f: R \to R, \; f(x) = -2\cos(4x) - 3$.
The period and range of this function are respectively:

A. $Period = 2\pi, \; Range = [-3, \; 0]$
B. $Period = \frac{\pi}{2}, \; Range = [-5, -1]$
C. $Period = \frac{\pi}{2}, \; Range = R$
D. $Period = \pi, \; Range = [-5, -1]$
E. $Period = \frac{\pi}{2}, \; Range = [-1, \; 5]$

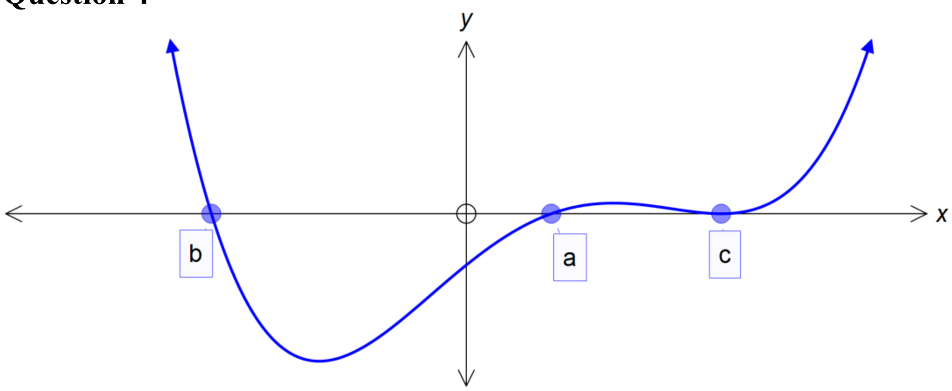
SECTION 1 - continued
TURN OVER

Question 3

The inverse function of $f: [-\infty,\ 2] \to R,\ \ f(x) = \frac{1}{3}\sqrt{2-x}$ is:

A. $f^{-1}: [-\infty,\ 2] \to R,\ \ f^{-1}(x) = 2 - 9x^2$
B. $f^{-1}: R \to R,\ \ f^{-1}(x) = 2 - 9x^2$
C. $f^{-1}: R^+ \cup \{0\} \to R,\ \ f^{-1}(x) = 2 - 9x^2$
D. $f^{-1}: R^+ \cup \{0\} \to R,\ \ f^{-1}(x) = \frac{2}{9} - x^2$
E. $f^{-1}: R^+ \to R,\ \ f^{-1}(x) = 2 - 9x^2$

Question 4



The rule for the function with the graph shown above could be:

A. $y = -2(x+b)(x-a)^2(x-c)$
B. $y = -2(x-b)(x+a)^2(x+c)$
C. $y = 2(x+b)(x-a)(c-x)^2$
D. $y = -2(x+b)(a-x)(x-c)$
E. $y = 2(x-b)(x-a)(x-c)^2$

SECTION 1 - continued

Question 5

The derivative of the function $\frac{1}{\sqrt{f(x)}}$ where $f(x) = ax + b$ is:

A. 0
B. $-\frac{a}{2(ax+b)\sqrt{ax+b}}$
C. $-\frac{a}{2\sqrt{ax+b}}$
D. $\frac{1}{2a(ax+b)\sqrt{ax+b}}$
E. $\frac{2\sqrt{ax+b}}{a}$

Question 6

The tangent to the curve $y = 3x^2 - 4$ at $x = -1$ passes through the point $(a, 2)$. The value of a is:

A. -4
B. $\frac{6}{7}$
C. $-\frac{3}{2}$
D. $\frac{1}{2}$
E. 4

Question 7

The range of the function $f: (-3, 7] \to R,\ f(x) = -x^3 + 4x^2 + x$ is:

A. $(-140,\ 60]$
B. $[0,\ 60)$
C. R
D. $[-140,\ 60)$
E. $(0,\ 60]$

SECTION 1 - continued
TURN OVER

Question 8

For the polynomial $P(x) = x^2 - 2ax + a, \quad P(-2) = -1$. The value of a is:

A. 1
B. -1
C. 0
D. $\frac{5}{3}$
E. $-\frac{5}{3}$

Question 9

The transformation that maps the graph of $y = \frac{1}{2x-1}$ onto the graph of $y = \frac{2}{x-1}$

A. Dilation of ½ unit from the x-axis and translation of ½ unit in the positive direction of x-axis
B. Dilation of ½ unit from the y-axis and translation of 1 unit in the positive direction of x-axis
C. Dilation of 4 units from the y-axis and translation of ½ unit in the positive direction of x-axis
D. Dilation of 4 units from the x-axis and translation of ½ unit in the positive direction of x-axis
E. Dilation of 2 units from the y-axis and translation of ½ unit in the positive direction of x-axis

Question 10

If $\int_0^3 f(x)dx = 15$ then $\int_0^3 (2f(x) - 1)dx$ is equal to:

A. 14
B. 27
C. 29
D. 31
E. 89

Question 11

The binomial random variable X, has a mean of 3. If the number of trials is 10, Pr $(X \leq 1)$ is equal to:

A. 0.1493
B. 0.1211
C. 0.0282
D. 0.0001
E. 0.9718

SECTION 1 - continued

Question 12

Three marbles are selected from a box containing six yellow marbles, three red marbles and five black marbles. The probability of selecting at least one yellow marble is:

A. $\frac{2}{13}$
B. $\frac{7}{13}$
C. $\frac{2}{7}$
D. $\frac{15}{91}$
E. $\frac{11}{13}$

Question 13

Consider the following discrete probability distribution for the random variable *X*.

x	0	1	2	3	4	5
$\Pr(X=x)$	0.10	0.05	p	0.25	$2p$	p

The mean of this distribution is:

A. 2.50
B. 3.00
C. 3.50
D. 3.05
E. 4.10

Question 14

The average value of the function $f(x) = \frac{1}{x^2}$ over the interval $[1, 4]$ is:

A. $\frac{1}{6}$
B. $\frac{1}{3}$
C. $\frac{1}{2}$
D. $\frac{1}{4}$
E. $\frac{3}{4}$

SECTION 1 - continued
TURN OVER

Question 15

The simultaneous equations $2x - my = m\ \ and\ \ (1-m)x + y = 2$, have a unique solution for:

A. $m \in R\backslash\{2,\ 1\}$
B. $m \in R\backslash\{-1\}$
C. $m = -1,\ 2$
D. $m = 1$
E. $m \in R\backslash\{-1,\ 2\}$

Question 16
The transformations required to transform the graph of $y = 2\cos(3x) - 1$ to the graph of $y = \cos(2x) + 4$ are:

A. Dilation of ½ from the x-axis and $\frac{1}{6}$ from the y-axis, translation of +4 units parallel to the y-axis.
B. Dilation of ½ unit from the x-axis and 3/2 from the y-axis, translation of +4.5 units parallel to the y-axis.
C. Dilation of ½ from the y-axis and translation of +4 units parallel to the y-axis.
D. Dilation of 2 from the x-axis and 1/3 from the y-axis, translation of -5 units parallel to the y-axis.
E. Dilation of ½ unit from the x-axis and 3/2 from the y-axis, translation of +4 units parallel to the y-axis

Question 17
Out of 300 students in the school, 265 passed an exam. In a sample of 10 of these students, the standard error of the sample proportion, correct to four decimal places, is:

A. 0.1015
B. 0.8833
C. 0.0103
D. 0.1103
E. 0.0104

SECTION 1 - continued

Question 18
The function f is a probability density function with rule

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ a - x, & 1 < x < 2 \\ 0, & otherwise \end{cases}$$

The value E(X) is:

A. 1.5
B. 1
C. 2
D. 1.75
E. 3

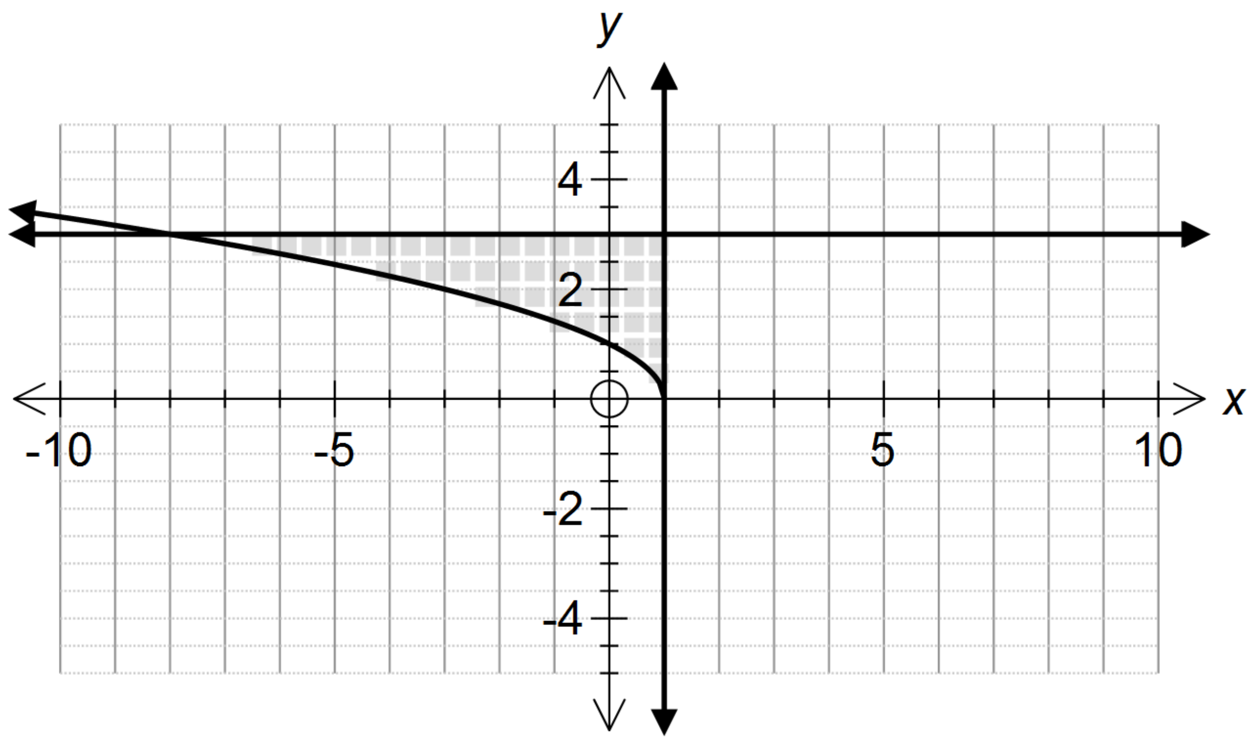
Question 19
A researcher was interested in knowing how many people in the city supported a new tax. She sampled 100 people from the city and found that 60% of these people supported the tax. The upper limit of the 95% confidence interval on the population proportion is:

A. 0.5040
B. 0.60
C. 0.6960
D. 0.4960
E. 0.0490

SECTION 1 - continued
TURN OVER

Question 20

The graph of $y = \sqrt{-x+1}$ *and* $y = 3$ is shown below.



The shaded area in the above graph can be found by calculating:

A. $24 - \int_{-8}^{1} \sqrt{-y+1}\, dy$

B. $\int_{-8}^{1} \sqrt{-x+1}\, dx$

C. $\int_{0}^{3} \sqrt{1-y^2}\, dy$

D. $\int_{0}^{3} \sqrt{1-y^2}\, dy - \int_{0}^{3} 3\, dy$

E. $24 + \int_{-8}^{1} \sqrt{-x+1}\, dx$

END OF SECTION 1

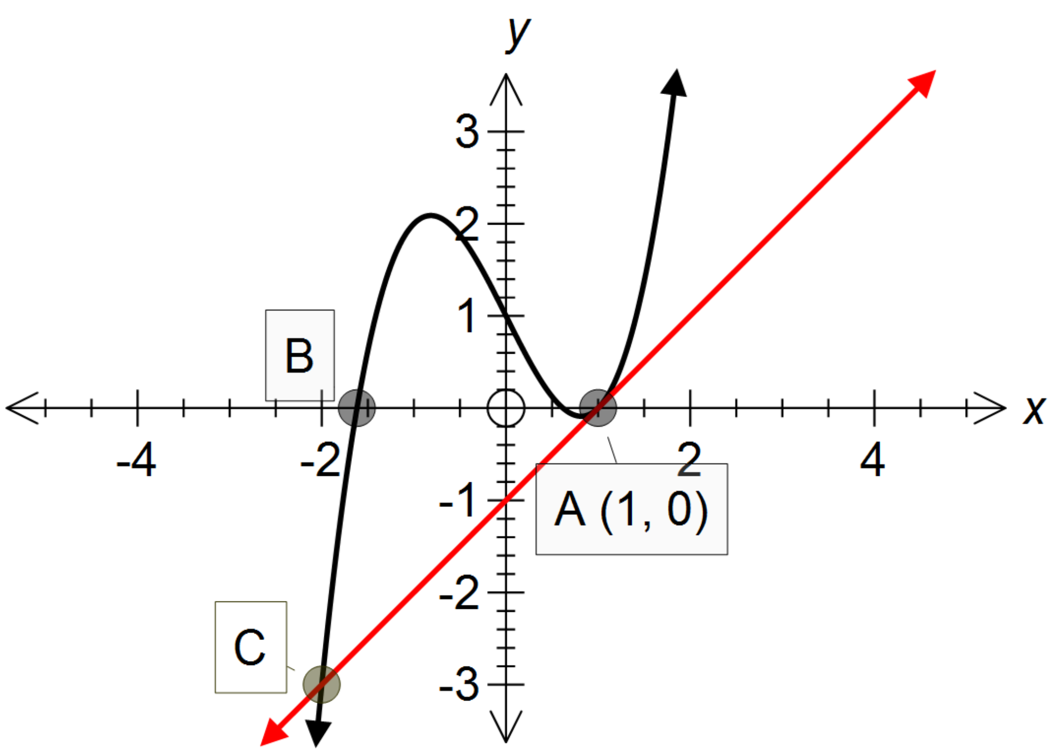
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (10 marks)

Let $f: R \to R,\ f(x) = (x-1)(x^2 + x - a)$. The curve passes through the point (0, 1) and the tangent at the point (1, 0) passes through the curve at point C as shown below.



a. Show that $a = 1$.

1 mark

SECTION 2 - continued
TURN OVER

b. Find the derivative $f'(x)$.

1 mark

c. Find the equation of the tangent at $x = 1$.

2 marks

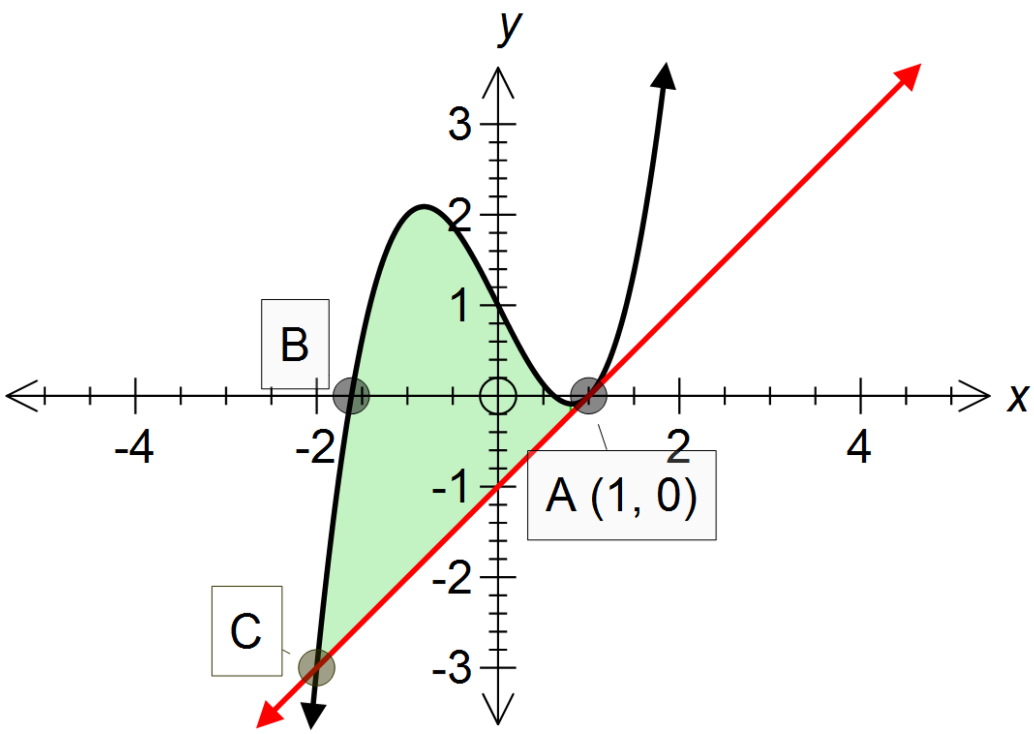
d. Find the coordinates of the point C.

2 marks

e. Find the coordinates of the point B.

1 mark

SECTION 2 - continued



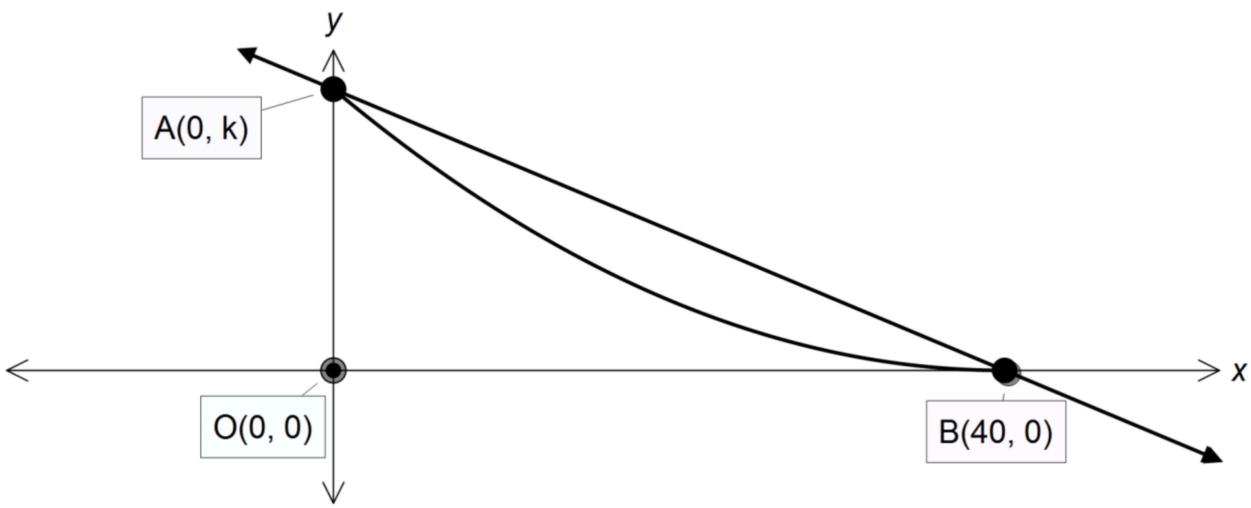
f. Find the shaded area shown above.

3 marks

SECTION 2 - continued
TURN OVER

Question 2 (11 marks)

Tourists take a cable car to travel from point B to A along the curve $y = 5000\left(1 - \frac{x}{40}\right)^2$ where x is the distance from where they park their car (at O) and y is the distance above the ground. Both distances are measured in metres.



a. Find the value of k.

1 mark

b. Find the equation of the straight line joining the points A and B.

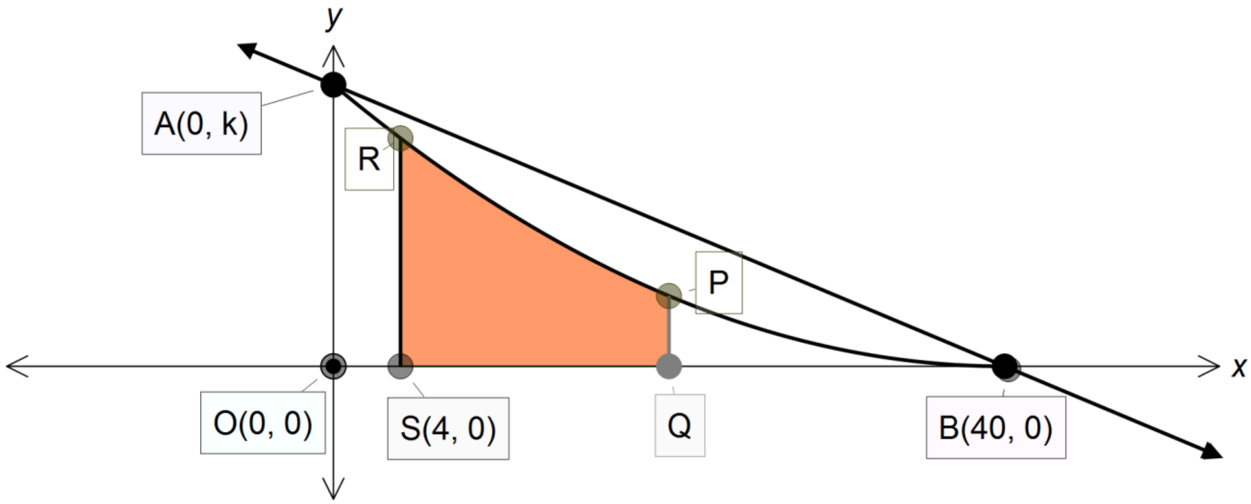
1 mark

c. Find the straight line distance AB. Give your answer to the nearest **centimetre**.

2 marks

SECTION 2 - continued

Two vertical columns PQ and RS are supporting the thick rope as shown below. The smaller column is 1250m high.



d. Find the horizontal distance between these two columns.

2 marks

SECTION 2 - continued
TURN OVER

e. Find the area between the two columns bounded by the rope. Give your answer correct to two decimal places.

2 marks

f. Find the point on the rope where the distance from the car park will be minimum. Give your answer correct to two decimal places.

3 marks

SECTION 2 - continued

Question 3 (12 marks)

Let $A(x) = e^{x}\sin^{2}(x),\ 0 \le x \le 2\pi$.

a. Find $A(0)$ *and* $A(2\pi)$.

1 mark

b. Use calculus to find the exact value of x for which $A(x)$ has an absolute maximum.

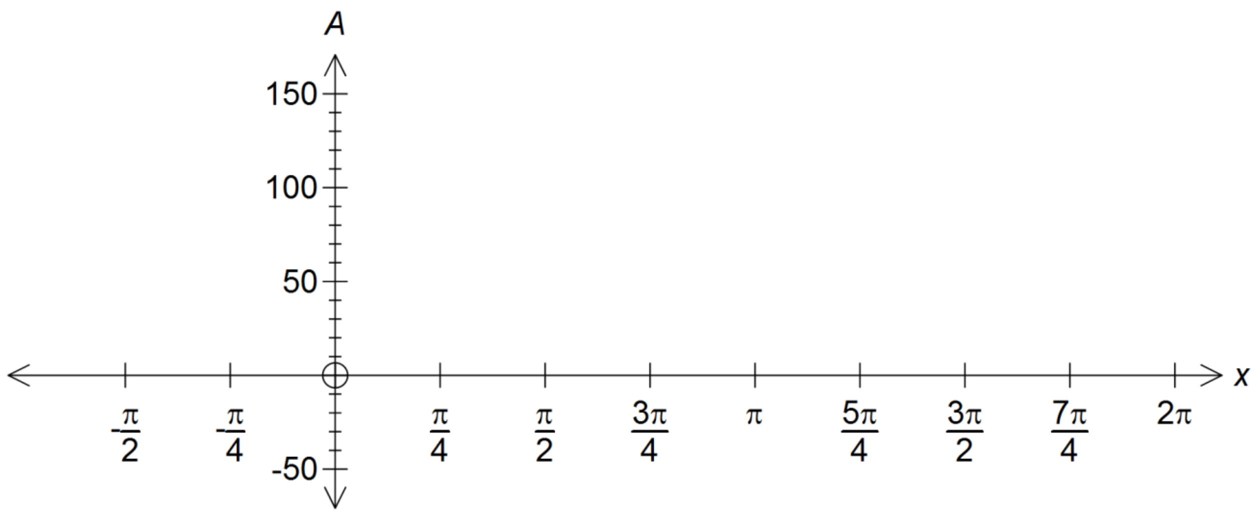
3 marks

c. Find the average rate of change of $A(x)$ between $\left[\pi, \frac{4\pi}{3}\right]$.

2 marks

SECTION 2 - continued
TURN OVER

d. On the axes below sketch the graph of $y = A(x)$ for $0 \le x \le 2\pi$. Label the end points and coordinates of all turning points correct to two decimal places.



3 marks

e. If $V(x) = e^{-x}\sin^2(x)$ state the transformation that changes $A(x)$ to $V(x)$.

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1 mark

f. State the domain of $V(x)$ in exact form and the range of $V(x)$ correct to two decimal places.

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2 marks

SECTION 2 - continued

Question 4 (16 marks)

A meat seller sells remnants of meat in 5kg bags. The amount, in kg, of remnants is a continuous random variable, X, with probability density function

$$f(x)=\begin{cases} k(x-1)(3-x), & 1<x<3 \\ 0, & otherwise \end{cases}.$$

a. Show that $k=\frac{3}{4}$.

2 marks

SECTION 2 - continued
TURN OVER

b. Find the probability that one of these bags has weight greater than 2.5kg. Express your answer as a fraction.

2 marks

c. What is the median weight of these remnant bags?

2 marks

SECTION 2 - continued

At the end of a particular day, the wholesaler is left with 20 bags of meat remnants. He labels the bags as small, medium and large on the basis of their weights. A bag is labelled small if it has at most 1.5kg remnants in it.

d. Find the probability of at least one small bag left at the end of the day. Give your answer correct to four decimal places.

3 marks

The meat wholesaler claims that he sells meat in bags of 5kg each. The weights of these bags are approximately normally distributed. The mean weight of these bags is 5.1kg and it is known that 10% of these bags have less than 5kg meat in them.

e. Find the standard deviation of the weights of these bags correct to four decimal places.

2 marks

SECTION 2 - continued
TURN OVER

f. Find the percentage of bags that weigh at least 5.3kg. Give your answer correct to one decimal place.

2 marks

The proportion of all bags that weigh at least 5.2kg is 0.1. A sample of 200 bags represents the whole population.

g. What is the probability that the proportion of bags that weigh at least 5.2kg is less than 0.05? Give your answer correct to four decimal places.

3 marks

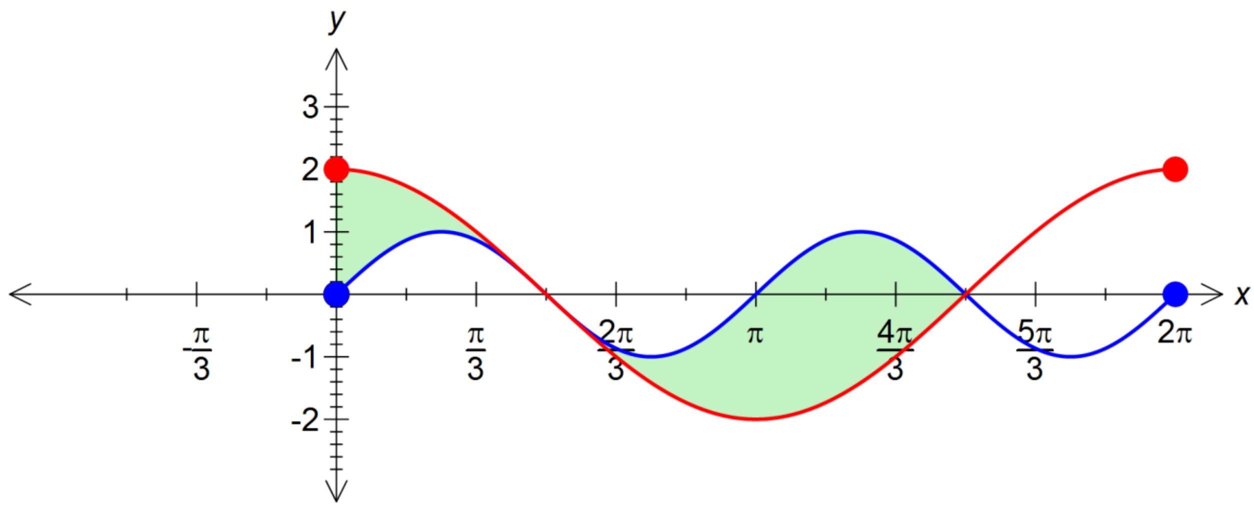
SECTION 2 - continued

Question 5 (11 marks)

The graphs of two functions

$f:[0,\ 2\pi] \to R,\ f(x) = \sin(2x)$ *and* $g:[0,\ 2\pi] \to R,\ g(x) = 2\cos(x)$

are shown below.



a. Label the end-point of the function $y = g(x)$ on the graph above.

1 mark

b. Write down an integral that will find the shaded area.

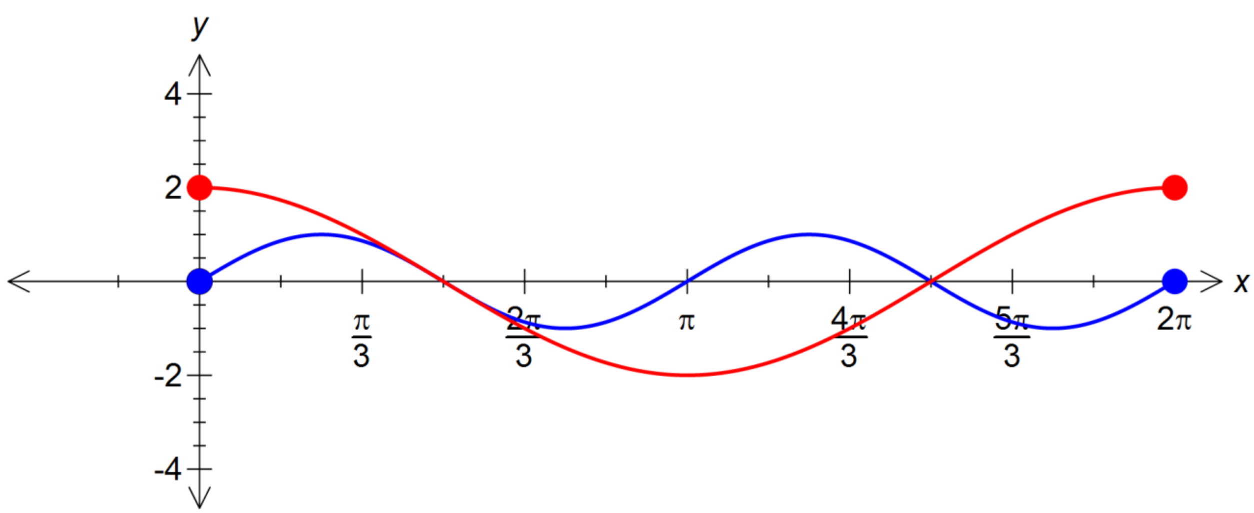
3 marks

SECTION 2 - continued
TURN OVER

c. State the transformations that are required to transform $y = f(x)$ *to* $y = g(x)$.

3 marks

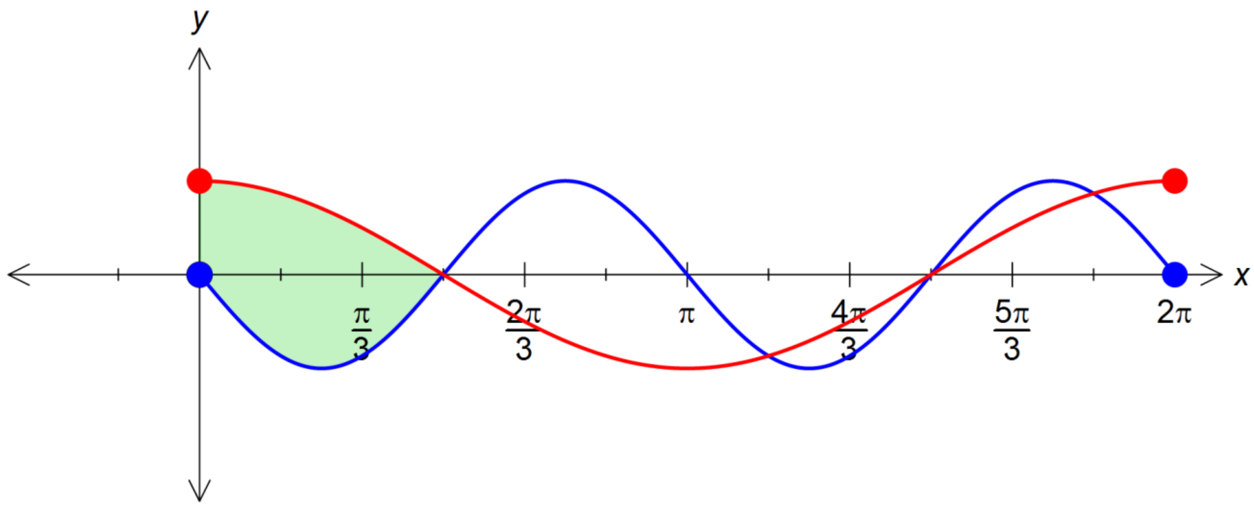
d. Sketch the graph of $y = f(x) + g(x)$ on the axes below.



2 marks

SECTION 2 - continued

The area bounded by the graphs of $y = 2\cos(x)$ *and* $y = a\sin(x)$, is shown below.



e. If the shaded area shown above is 4 square units, find the value of a.

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2 marks

END OF QUESTION AND ANSWER BOOK