MATHEMATICAL METHODS

Written Examination 2

TSSM™
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2016 Trial Examination

SOLUTIONS

Question 1

Answer: D

Explanation:

$vertical\ translation\ of\ 2\ and\ horizontal\ translation\ of\ \frac{1}{2}$

Question 2

Answer: B

Explanation:

$Period = \frac{2\pi}{4} = \frac{\pi}{2} \quad and \quad Range = [-3-2, -3+2]$

Question 3

Answer: C

Explanation:

$solve\left(x = \frac{1}{3}\sqrt{2-y}, y\right), \quad x \geq 0$

Question 4

Answer: E

Explanation:

Turning point at c so a repeated factor.

Question 5

Answer: B

Explanation:

Find the derivative on CAS.

Question 6

Answer: C

Explanation:

Equation of tangent is $y = -6x - 7$. Substitute $y = 2$.

Question 7

Answer: D

Explanation:

Sketch on CAS over restricted domain.

Question 8

Answer: B

Explanation:

$4 + 4a + a = -1$

Question 9

Answer: D

Explanation:

$y = \frac{1}{2\left(x-\frac{1}{2}\right)} \rightarrow y = \frac{4}{2\left(x-\frac{1}{2}\right)} = \frac{2}{x-\frac{1}{2}} \rightarrow y = \frac{2}{x-\frac{1}{2}-\frac{1}{2}} = \frac{2}{x-1}$

Question 10

Answer: B

Explanation:

$2 \times 15 - (3 - 0) = 27$

Question 11

Answer: A

Explanation:

Binomcdf (10, 0.3, 0, 1)

Question 12

Answer: E

Explanation:

$1 - \frac{C(8,3)}{C(14,3)}$

Question 13

Answer: D

Explanation:

x	0	1	2	3	4	5
$\Pr(X = x)$	0.10	0.05	0.15	0.25	0.30	0.15
$x.\Pr(X = x)$	0	0.05	0.30	0.75	1.20	0.75

Mean = 3.05

Question 14

Answer: D

Explanation:

$\frac{1}{3}\int_1^4 \frac{1}{x^2}\,dx$

Question 15

Answer: E

Explanation:

$m^2 - m - 2 \neq 0$

Question 16

Answer: B

Explanation:

$y = 2\cos(3x) - 1 \rightarrow y = \frac{1}{2}(2\cos(3x) - 1) \rightarrow y = \cos\left(3x \times \frac{2}{3}\right) - \frac{1}{2} \rightarrow y = \cos(2x) + 4$

Question 17

Answer: A

Explanation:

$\sqrt{\frac{p(1-p)}{n}}, \ where \ p = \frac{265}{300}$

Question 18

Answer: B

Explanation:

$a = 2, \ E(X) = \int_0^1 x^2 \, dx + \int_1^2 (2 - x)x \, dx$

Question 19

Answer: C

Explanation:

$\sqrt{0.6 \times \frac{0.4}{100}} = 0.0490, \quad 0.6 + 1.96 \times 0.0490$

Question 20

Answer: A

Explanation:

$Area = 24 - \int_{-8}^{1} \sqrt{-x + 1} \, dx = 24 - \int_{-8}^{1} \sqrt{-y + 1} \, dy$

SECTION 2

Question 1

a. $(0-1)(0+0-a)=1 \to a=1$

1 mark

b. $f'(x)=3x^2-2$

1 mark

c. $y-0=1(x-1) \to y=x-1$

2 marks

d. $x^3-2x+1=x-1 \to x=-2 \quad (-2,-3)$

2 marks

e. $f(x)=0 \to \left(\frac{-1-\sqrt{5}}{2},\ 0\right)$

1 mark

f. $Area=\int_{-2}^{1}\left(f(x)-(x-1)\right)dx=\frac{27}{4}$ square units

3 marks

Question 2

a. $k=5000\left(1-\frac{0}{40}\right)^2 \to k=5000$

1 mark

b. $\frac{x}{40}+\frac{y}{5000}=1 \to y=5000-125x$

1 mark

c. $AB=\sqrt{(0-5000)^2+(40-0)^2} \approx 5000.16\text{m}$

2 marks

d. $PQ=1250 \to 1250=5000\left(1-\frac{x}{40}\right)^2 \to x=20$

Distance = 20 – 4 = 16m

2 marks

e. $Area = \int_4^{20} 5000\left(1-\frac{x}{40}\right)^2\, dx \approx 40266.67m^2$

2 marks

f. $Distance = \sqrt{\left(5000\left(1-\frac{x}{40}\right)^2 - 0\right)^2 + (x-0)^2}$

$Solve(derivative\ of\ distance = 0)\ on\ CAS\ to\ get\ x = 38.7435$

Point on rope is $(38.74, 4.93)$

3 marks

Question 3

a. $A(0) = 0,\ A(2\pi) = 0$

1 mark

b. $A'(x) = \sin(x)\, e^x(2\cos(x) + \sin(x))$

$\sin(x) = 0 \rightarrow x = 0,\ \pi,\ 2\pi$

$2\cos(x) = -\sin(x) \rightarrow x = \tan^{-1}\frac{1}{2} + \frac{\pi}{2}\ ,\ \ \tan^{-1}\frac{1}{2} + \frac{3\pi}{2}$

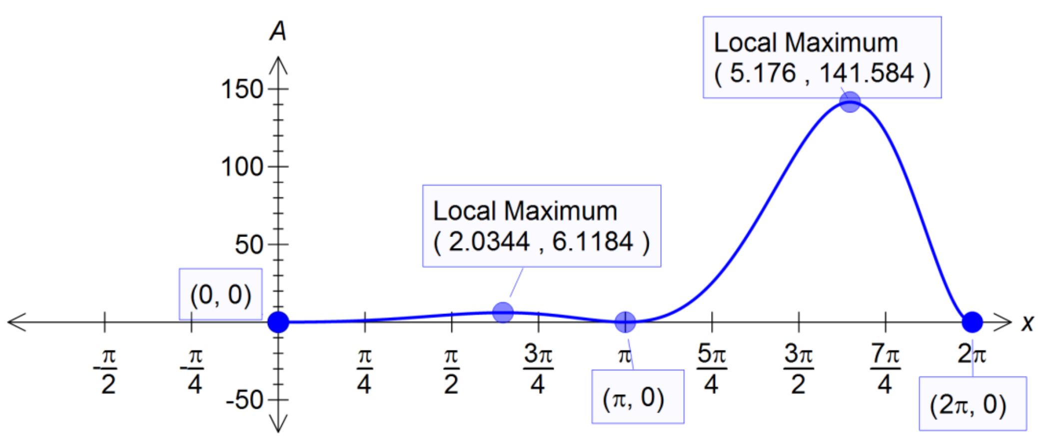
Absolute maximum at $x = \tan^{-1}\frac{1}{2} + \frac{3\pi}{2}$

3 marks

c. $\frac{A\left(\frac{4\pi}{3}\right) - A(\pi)}{\frac{4\pi}{3} - \pi} = \frac{9e^{\frac{4\pi}{3}}}{4\pi}$

2 marks

d.



3 marks

e. $Reflection\ in\ the\ x-axis$

1 mark

f. Domain = $[-2\pi, 0]$, Range = [0,141.58]

2 marks

Question 4

a. $k\int_1^3 (4x - x^2 - 3)dx = 1$

$k\left(2x^2 - \frac{x^3}{3} - 3x\right)\begin{matrix}3\\1\end{matrix} = 1$

$k\left(18 - 9 - 9 - 2 + \frac{1}{3} + 3\right) = 1 \rightarrow k = \frac{3}{4}$

2 marks

b. $\Pr(X > 2.5) = \int_{\frac{5}{2}}^{3} \frac{3}{4}(x-1)(3-x)dx = \frac{5}{32}$

2 marks

c. $\int_1^m \frac{3}{4}(x-1)(3-x)dx = \frac{1}{2} \rightarrow m = 0.27,\ 2,\ \ 3.72$

$Median = 2\ kg$

2 marks

d. $p = \int_1^{1.5} \frac{3}{4}(x-1)(3-x)dx = 0.15625$

$\Pr(at\ least\ one\ small\ bag) = 1 - \Pr(no\ small\ bag)$

$= 1 - C(20,0)(0.15625)^0(1 - 0.15625)^{20} = 0.9666$

3 marks

e. $invnorm(0.1, 0, 1) = \frac{5-5.1}{\sigma}$

$-1.28155 = -\frac{0.1}{\sigma}$

$\sigma = 0.0780$

2 marks

f. $normcdf(5.3, \infty, 5.1, 0.0780) = 0.005172$
Percentage is 0.5%

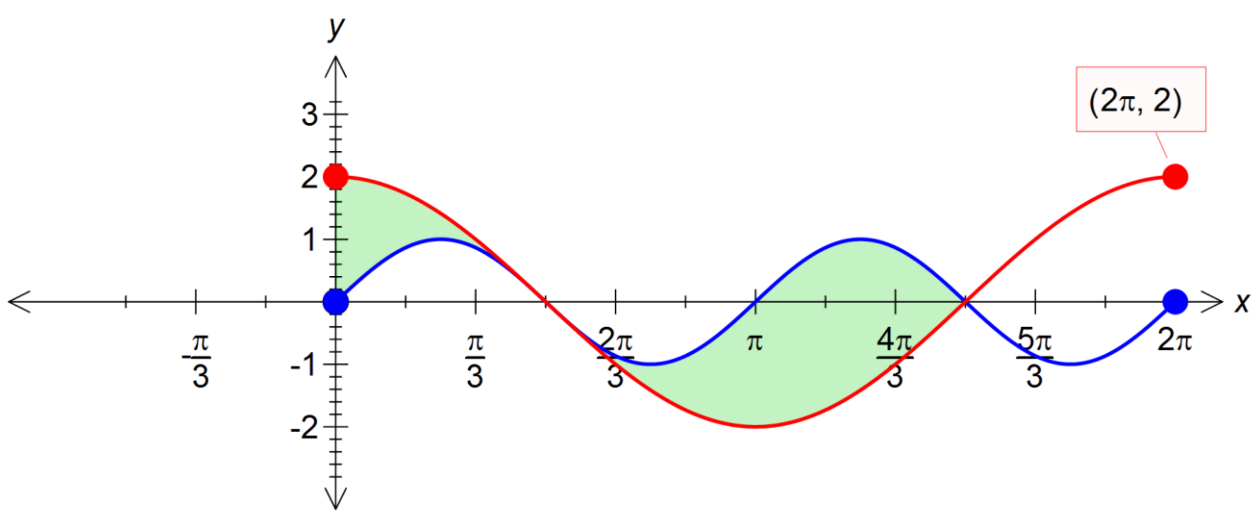
2 marks

g. $SD(\hat{p}) = \sqrt{0.1 \times \frac{0.9}{200}} = 0.0212$
$\Pr\left(Z < \frac{0.05-0.1}{0.0212}\right) = \Pr(Z < -2.3585) = 0.0092$

3 marks

Question 5

a.



1 mark

b. $\int_0^{\frac{\pi}{2}}(2\cos(x) - \sin(2x))dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(\sin(2x) - 2\cos(x))\,dx$

3 marks

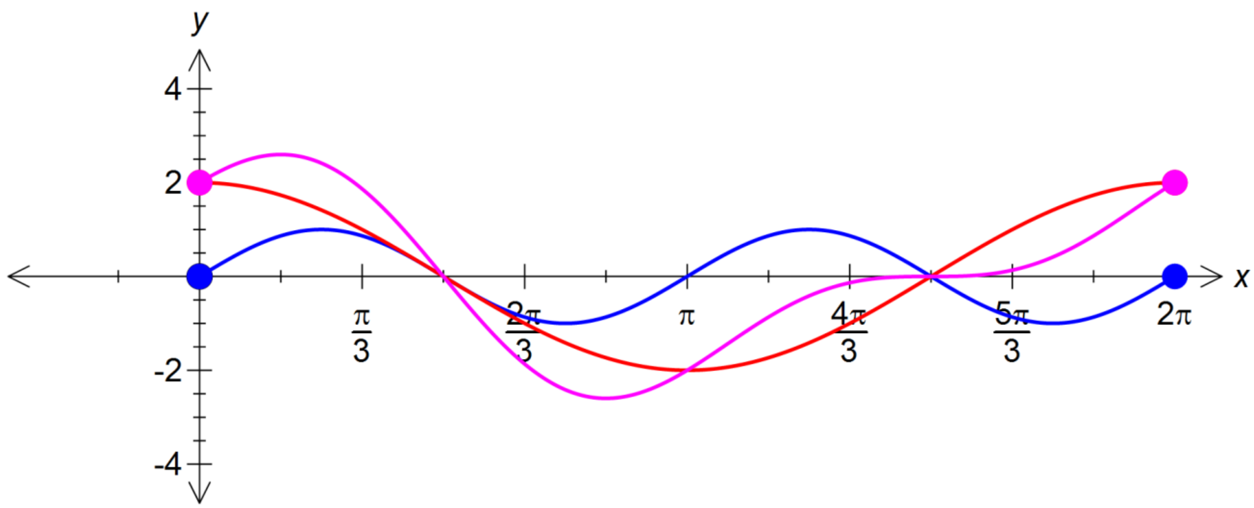
c. Dilations of 2 units from the y-axis, dilation of 2 units from the x-axis, translation of $\pi/2$ units in the negative direction of x-axis.

OR

Dilations of 2 units from the y-axis, dilation of 2 units from the x-axis, reflection in the y-axis and translation of $\pi/2$ units in the positive direction of x-axis.

3 marks

d.



2 marks

e. Solve $\int_0^{\frac{\pi}{2}}(2\cos(x) - a\sin(x))\,dx = 4$ for a

$a = -2$

2 marks