



**Instructions**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1**

Let  $f(x) = ke^x - 1 + ke^{-x}$

- a. For what value(s) of  $k$  will the equation  $f(x) = 1$  have two solutions?

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2 marks

- b. Find the solution(s) to the equation  $f(x) = 1$  in terms of  $k$ .

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2 marks

Total 4 marks

**Question 2**

Consider  $f(x) = 2x\sqrt{1-x}$

a. Find  $f'(x)$ .

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2 marks

b. Hence, find the area between the curve  $y = \frac{x}{\sqrt{1-x}}$ , the  $x$ -axis and the lines  $x = -2$  and  $x = 0$ .

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3 marks

Total 5 marks

**TURN OVER**

**Question 3**

Consider the function  $g(x) = \sin(x) + \cos(x)$

- a. Find the general solution for the equation  $g(2x) = 0$ .

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2 marks

- b. Hence, find all solutions to the equation  $g(2x)=0$  for  $x \in (-\pi, \pi)$ .

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2 marks

Total 4 marks

**Question 4**

- a. Find the image,  $g(x)$ , of the equation  $f(x) = \frac{1}{2x}$  under the matrix transformation

$$T = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}.$$

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3 marks

b. State the transformations made to the function  $f(x)$  to obtain  $g(x)$ .

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2 marks

c. Find the inverse,  $g^{-1}(x)$ , of the transformed function  $g(x)$ .

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3 marks

d. State the domain and range of  $g^{-1}(x)$ .

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2 marks

Total 10 marks

**TURN OVER**

**Question 5**

A random variable  $X$  has probability density function given by

$$f(x) = \begin{cases} 0.3p, & 0 \leq x \leq 4 \\ xp, & 4 \leq x \leq 5 \\ 0, & \text{elsewhere} \end{cases}$$

**a.** Find the exact value of  $p$ .

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3 marks

**b.** Find the exact value of  $\Pr(1 < x < 2.5)$ .

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2 marks

Total 5 marks

**Question 6**

An approximate 95% confidence interval for the population proportion is given by the interval  $(r, s)$ .

- a. Find an expression for the sample proportion  $\hat{p}$  in terms of  $r$  and  $s$ .

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2 marks

- b. Find an expression for the margin of error for this confidence interval in terms of  $r$  and  $s$ .

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2 marks

Total 4 marks

**TURN OVER**

**Question 7**

Find the equation of the tangent(s) to the curve  $y = -x^2 - 8x$  such that the tangent(s) cross the  $y$  axis at  $(0,3)$ .

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4 marks

**Question 8**

Consider the function  $f(x) = \sin\left(\frac{4x}{3}\right)$

a. Find the value of  $\int_0^{\frac{\pi}{8}} f(x) dx$ .

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2 marks



- b. Find the average value of  $f(x)$  over the interval  $\left[0, \frac{\pi}{4}\right]$ .

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2 marks

Total 4 marks

**END OF QUESTION AND ANSWER BOOK**