



Trial Examination 2016

# VCE Mathematical Methods Units 3&4

Written Examination 1

## Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: \_\_\_\_\_

Teacher's Name: \_\_\_\_\_

### Structure of Booklet

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

#### Materials supplied

Question and answer booklet of 10 pages.

Formula sheet.

Working space is provided throughout the booklet.

#### Instructions

Write your **name** and **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2016 VCE Mathematical Methods Units 3&4 Written Examination 1.

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**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

**Question 1** (3 marks)

Let  $f(x) = \sin(3x^2 - 4)$ .

a. Find  $f'(x)$ .

2 marks

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b. Evaluate  $f'\left(\frac{2\sqrt{3}}{3}\right)$ .

1 mark

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**Question 2** (3 marks)

Given that  $\frac{d}{dx}(x \cos(4x)) = \cos(4x) - 4x \sin(4x)$ , find an antiderivative of  $x \sin(4x)$ .

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**Question 3** (4 marks)

- a. Simplify  $\frac{\log_z(p^2 - 1)}{\log_z(p + 1)}$ ,  $p > -1$ , expressing your answer in the form  $k + \log_m(n)$ . 2 marks

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- b. Solve the following equation for  $m$  in terms of  $n$ , given  $m > -1$  and  $n < 0$ .

$$2\log_e(m + 1) - \log_e(4) = \log_e(n)^2 \quad 2 \text{ marks}$$

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**Question 4** (3 marks)

For the function  $f(x) = 2x^3 \tan(x)$ ,  $f'(x) = \frac{ax^2}{\cos(x)}(b \sin(x) + cx \sec(x))$ .

Find the values of  $a$ ,  $b$  and  $c$ .

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**Question 5** (3 marks)

Solve the equation  $4\sin^2(2x) = 3$  for  $x \in [-\pi, \pi]$ .

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**Question 6** (4 marks)

Consider  $f: (-1, \infty) \rightarrow \mathbb{R}$ , where  $f(x) = \log_e(x + 1)$ , and  $g: \left(\frac{2}{3}, \infty\right) \rightarrow \mathbb{R}$ , where  $g(x) = 3x - 2$ .

- a.** Find the rule and domain of  $f(g(x))$ . 2 marks

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- b.** Find the rule for  $h^{-1}(x)$ , where  $h(x) = f(g(x))$ . 2 marks

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**Question 7** (5 marks)

- a. Show the expressions  $\frac{3x+2}{x-1}$  and  $3 + \frac{5}{x-1}$  are equivalent. 1 mark

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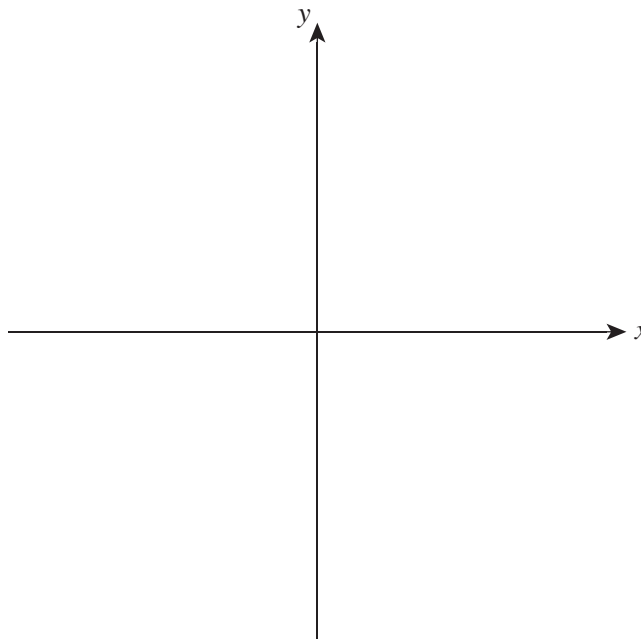


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- b. Hence, sketch the graph  $f(x) = \frac{3x+2}{x-1}$  on the axes provided. Include the coordinates of any intercepts and the equations of any asymptotes. 2 marks



- c. By considering  $g(x) = f(x+2) - 3$ , state the relationship between the graph of  $g(x)$  and the graph of  $h(x) = \frac{1}{x+1}$ . 2 marks

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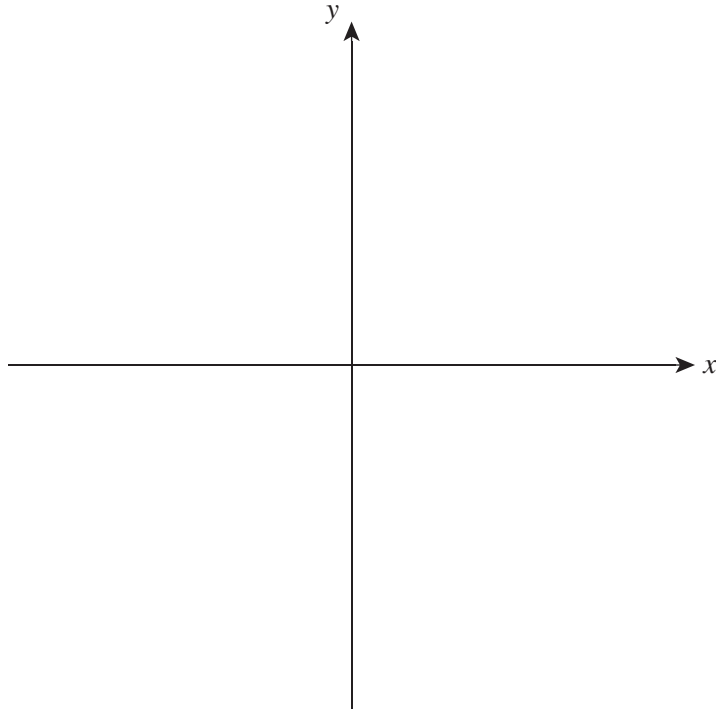
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**Question 8** (4 marks)

- a. On the axes below, sketch the graph of the function  $f: [-2, 2] \rightarrow \mathbb{R}$ , where  $f(x) = \frac{1}{3}(x-2)(x+1)^2$ . Label all intercepts and endpoints with their coordinates. 2 marks



- b. Find the area of the region bounded by the  $x$ -axis and the curve  $f(x) = \frac{1}{3}(x-2)(x+1)^2$ . 2 marks

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**Question 9** (6 marks)

Jenny loves birdwatching. She lives near a national park and each day has a variety of birds coming to her verandah to eat seed, which she leaves out for them.

Jenny decides to observe the birds diligently for one month. Each morning after putting out new seed, Jenny watches and keeps a tally of the numbers of each type of bird that comes to her verandah to eat the seed.

- a.** Jenny thinks this will give her a good sample, which will be representative of the proportions of those types of birds at her local national park.

Is Jenny right? Give reasons.

1 mark

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On a particular day, Jenny notices 4 king parrots and 6 crimson rosellas in a nearby gumtree.

- b.** What is  $p$ , the proportion of king parrots in the tree?

1 mark

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- c.** Three of these birds come to eat the seed from the feeder at the same time and Jenny uses this as her sample.

What are the possible values of the sample proportion,  $\hat{p}$ , of king parrots in this sample?

1 mark

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- d.** Given  $\Pr(\hat{p} = 0) = \frac{1}{6}$  and  $\Pr(\hat{p} = 1) = \frac{1}{30}$ , complete a probability distribution table which summarises the sampling distribution of the sample proportion of king parrots when samples of size 3 are taken of the birds in the tree, without replacement.

2 marks

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- e. Hence or otherwise, determine the probability that the proportion of king parrots in the sample is more than 0.2.

1 mark

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**Question 10** (5 marks)

The distribution of a function  $f$  is modelled by a continuous random variable,  $X$ , with probability density function

$$f(x) = \begin{cases} a \cos\left(x - \frac{\pi}{3}\right) & 0 \leq x \leq \frac{2\pi}{3} \\ 0 & \text{elsewhere} \end{cases}$$

- a.** Find the value of  $a$ . 2 marks

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- b.** Find  $m$  such that  $\Pr(X < m) = \frac{1}{2}$ . 2 marks

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- c.** Find the mode of  $f(x)$ . 1 mark

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**END OF QUESTION AND ANSWER BOOKLET**