Neap:

Trial Examination 2016

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (3 marks)

a. $f(x) = h(g(x))$, where $h(x) = \sin(x)$ and $g(x) = 3x^2 - 4$.

$f'(x) = h'(g(x))g'(x)$ *use of chain rule* M1

$= \cos(3x^2 - 4) \times 6x$

$= 6x\cos(3x^2 - 4)$ A1

b. $f'\left(\frac{2\sqrt{3}}{3}\right) = 6 \times \frac{2\sqrt{3}}{3} \times \cos\left(3 \times \left(\frac{2\sqrt{3}}{3}\right)^2 - 4\right)$

$= 4\sqrt{3} \times \cos\left(3 \times \left(\frac{12}{9}\right) - 4\right)$

$= 4\sqrt{3} \times \cos(0)$

$= 4\sqrt{3}$ A1

Question 2 (3 marks)

$$\int\left(\frac{d}{dx}(x\cos(4x))\right)dx = \int(\cos(4x) - 4x\sin(x))dx$$

$$x\cos(4x) = \int(\cos(4x))dx - \int(4x\sin(x))dx$$ *correct set-up* M1

$$x\cos(4x) = \frac{1}{4}\sin(4x) - 4\int(x\sin(x))dx$$ *correct antiderivative* M1

$$4\int(x\sin(x))dx = \frac{1}{4}\sin(4x) - x\cos(4x)$$

$$\int(x\sin(x))dx = \frac{1}{4}\left(\frac{1}{4}\sin(4x) - x\cos(4x)\right)$$

$$\int(x\sin(x))dx = \frac{1}{16}\sin(4x) - \frac{x}{4}\cos(4x)$$ A1

Question 3 (4 marks)

a. $\log_z(p^2 - 1) = \log_z((p+1)(p-1))$

$= \log_z(p+1) + \log_z(p-1)$ M1

Therefore, $\frac{\log_z(p-1)}{\log_z(p+1)} = \frac{\log_z(p+1) + \log_{p+1}(p-1)}{\log_z(p+1)}$

$= 1 + \frac{\log_z(p-1)}{\log_z(p+1)}$

$= 1 + \log_{p+1}(p-1)$ A1

b. $\log_e(m+1)^2 - \log_e(4) = \log_e(n)^2$

$$\log_e\left(\frac{(m+1)^2}{4}\right) = \log_e(n)^2$$ M1

$$\frac{(m+1)^2}{4} = n^2$$

$$m+1 = 2n,\ -2n$$

$$m = -2n - 1 \text{ as } m > -1 \text{ and } n < 0.$$ A1

Question 4 (3 marks)

$f(x) = 2x^3\tan(x)$ *use of rule* M1

$f'(x) = 6x^2 \times \tan(x) + 2x^3 \times \sec^2(x)$ *correct derivative* A1

$$= 2x^2(3\tan(x) + x\sec^2(x))$$

$$= 2x^2\left(3 \times \frac{\sin(x)}{\cos(x)} + x \times \frac{1}{\cos^2(x)}\right)$$

$$= \frac{2x^2}{\cos(x)}\left(3 \times \sin(x) + x \times \frac{1}{\cos(x)}\right)$$

$$= \frac{2x^2}{\cos(x)}(3\sin(x) + x\sec(x))$$

Since $f'(x) = \dfrac{ax^2}{\cos(x)}(b\sin(x) + cx\sec(x))$, $a = 2$, $b = 3$, $c = 1$. A1

Question 5 (3 marks)

$4\sin^2(2x) = 3$ for $x \in [-\pi, \pi]$

$$\sin^2(2x) = \frac{3}{4}$$

$$\sin(2x) = \pm\frac{\sqrt{3}}{2}$$

$$2x = \sin^{-1}\left(\pm\frac{\sqrt{3}}{2}\right) \text{ for } 2x \in [-2\pi, \pi]$$

base angle $= \dfrac{\pi}{3}$ M1

$2x = -\dfrac{5\pi}{3}, -\dfrac{4\pi}{3}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ (positive square root) and $-\dfrac{2\pi}{3}, -\dfrac{\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$ (negative square root)

correct solutions for positive (or negative) square root only M1

$$x = -\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$$ A1

Question 6 (4 marks)

a. $f(g(x)) = \log_e(3x - 2 + 1)$

$= \log_e(3x - 1)$ A1

domain $f(g(x))$ = domain $g(x)$

$= \left(\frac{2}{3}, \infty\right)$ A1

b. Let $y = h(x)$.

$\therefore y = \log_e(3x - 1)$

To find rule for inverse, swap $x \leftrightarrow y$ and rearrange.

$x = \log_e(3y - 1)$ M1

$e^x = e^{\log_e(3y - 1)}$

$e^x = 3y - 1$

$e^x + 1 = 3y$

$\frac{e^x + 1}{3} = y$

So the rule for $h^{-1}(x)$ is: $h^{-1}(x) = \frac{e^x + 1}{3}$. A1

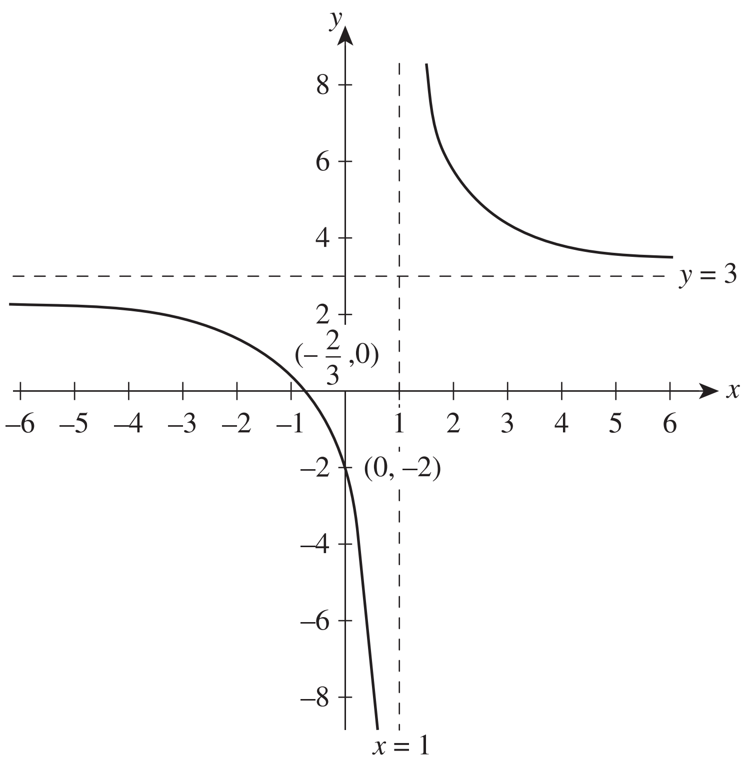
Question 7 (5 marks)

a. $3 + \frac{5}{x - 1} = \frac{3(x - 1) + 5}{x - 1}$ M1

$= \frac{3x - 3 + 5}{x - 1}$

$= \frac{3x + 2}{x - 1}$

b.



correct shape A1
correct intercepts and asymptotes A1

c. $g(x) = 3 + \frac{5}{(x+2-1)} - 3$ M1

$= \frac{5}{(x+1)}$

$= 5 \times h(x)$

Hence $g(x)$ is the graph of $h(x)$ after it has been dilated by a factor of 5 from the x-axis. A1

Question 8 (4 marks)

a. $f(-2) = \frac{1}{3}(-2-2)(-2+1)^2$

$= \frac{1}{3}(-4)(-1)^2$

$= -\frac{4}{3} \quad \therefore \text{endpoint } \left(-2, -\frac{4}{3}\right)$

$f(2) = \frac{1}{3}(2-2)(2+1)^2$

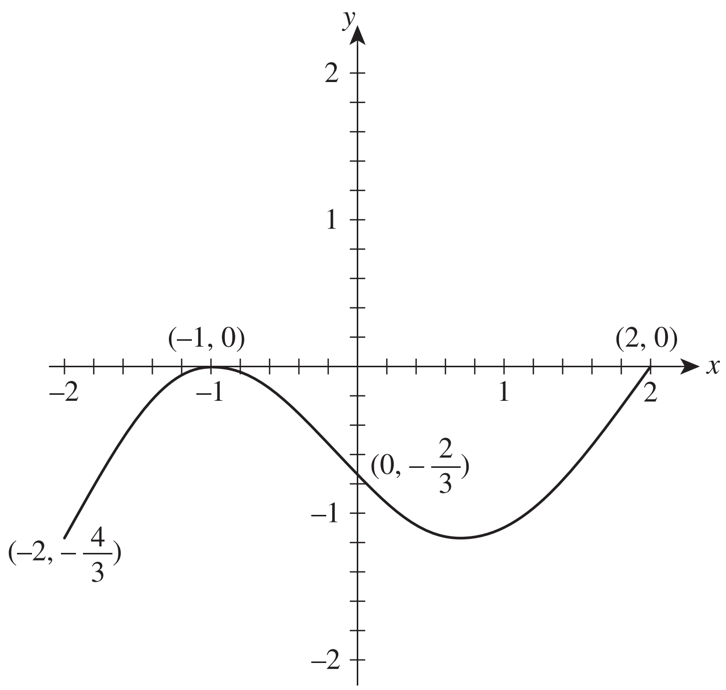
$= 0 \quad \therefore$ endpoint (2, 0) and x-intercept

other x-intercept (–1, 0)

$f(0) = \frac{1}{3}(0-2)(0+1)^2$

$= \frac{1}{3}(-2)(1)^2$

$= -\frac{2}{3} \quad \therefore y\text{-intercept } \left(0, -\frac{2}{3}\right)$



correct shape A1
all coordinates correct A1

b. area of region $= -\frac{1}{3}\int_{-1}^{2}\left(\frac{1}{3}(x-2)(x+1)^2\right)dx$

$$= -\frac{1}{3}\int_{-1}^{2}(x^3-3x-2)dx$$

$$= -\frac{1}{3}\left[\frac{x^4}{4}-\frac{3x^2}{2}-2x\right]_{-1}^{2}$$ M1

$$= -\frac{1}{3}\left[\left(\frac{16}{4}-\frac{12}{2}-4\right)-\left(\frac{1}{4}-\frac{3}{2}+2\right)\right]$$

$$= -\frac{1}{3}\left[(-6)-\left(\frac{3}{4}\right)\right]$$

$$= -\frac{1}{3}\times-\frac{27}{4}$$

$$= \frac{9}{4}\text{ units}^2$$ A1

Question 9 (6 marks)

a. No, Jenny is not right. The sample is biased to the birds who prefer the type of seed Jenny offers and those types that are not naturally shy of human habitats. A1

b. $p = \frac{4}{10}$

$= 0.4$ A1

c. 0, 1, 2 or 3 king parrots can come out of 3 birds.

Therefore, $\hat{p}$ can take values: 0, $\frac{1}{3}$, $\frac{2}{3}$ or 1. A1

d.

No. of king parrots	0	1	2	3
Proportion of king parrots ($\hat{p}$)	0	$\frac{1}{3}$	$\frac{2}{3}$	1
Pr($\hat{P} = \hat{p}$)	$\frac{1}{6}$	$\frac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = \frac{1}{2}$	$\frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}} = \frac{3}{10}$	$\frac{1}{30}$

The second of these values can be obtained by subtracting the first from 1.

correct third column of table A1
correct fourth column of table A1

e. $\Pr(\hat{p} > 0.2) = 1 - \Pr(\hat{p} < 0.2)$

$= 1 - \Pr(P = 0)$

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$ A1

Question 10 (5 marks)

a. $$\int_0^{\frac{2\pi}{3}} \left(a\cos\left(x - \frac{\pi}{3}\right)\right)dx = 1$$ M1

$$a\left[\sin\left(x - \frac{\pi}{3}\right)\right]_0^{\frac{2\pi}{3}} = 1$$

$$a\left[\sin\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) - \sin\left(0 - \frac{\pi}{3}\right)\right] = 1$$

$$a\left(\frac{\sqrt{3}}{2} - -\frac{\sqrt{3}}{2}\right) = 1$$

$$a\sqrt{3} = 1$$

$$a = \frac{1}{\sqrt{3}}$$ A1

b. $$\int_0^{m} \left(\frac{1}{\sqrt{3}}\cos\left(x - \frac{\pi}{3}\right)\right)dx = \frac{1}{2}$$ M1

$$\frac{1}{\sqrt{3}}\left[\sin\left(m - \frac{\pi}{3}\right) - \sin\left(0 - \frac{\pi}{3}\right)\right] = \frac{1}{2}$$

$$\sin\left(m - \frac{\pi}{3}\right) - -\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\sin\left(m - \frac{\pi}{3}\right) = 0$$

$$m - \frac{\pi}{3} = \sin^{-1}(0)$$

$$m = \frac{\pi}{3}$$ A1

c. As the graph of $f(x)$ is a translation of $\cos(x)$ by $\frac{\pi}{3}$ units to the right, the highest point is at $x = \frac{\pi}{3}$.

Hence the mode is $\frac{\pi}{3}$. A1