



Trial Examination 2016

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

Neap Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

Question 1 (3 marks)

a. $f(x) = h(g(x))$, where $h(x) = \sin(x)$ and $g(x) = 3x^2 - 4$.

$$f'(x) = h'(g(x))g'(x)$$

use of chain rule M1

$$= \cos(3x^2 - 4) \times 6x$$

$$= 6x \cos(3x^2 - 4)$$

A1

b. $f'\left(\frac{2\sqrt{3}}{3}\right) = 6 \times \frac{2\sqrt{3}}{3} \times \cos\left(3 \times \left(\frac{2\sqrt{3}}{3}\right)^2 - 4\right)$

$$= 4\sqrt{3} \times \cos\left(3 \times \left(\frac{12}{9}\right) - 4\right)$$

$$= 4\sqrt{3} \times \cos(0)$$

$$= 4\sqrt{3}$$

A1

Question 2 (3 marks)

$$\int \left(\frac{d}{dx}(x \cos(4x)) \right) dx = \int (\cos(4x) - 4x \sin(x)) dx$$

$$x \cos(4x) = \int (\cos(4x)) dx - \int (4x \sin(x)) dx$$

correct set-up M1

$$x \cos(4x) = \frac{1}{4} \sin(4x) - 4 \int (x \sin(x)) dx$$

correct antiderivative M1

$$4 \int (x \sin(x)) dx = \frac{1}{4} \sin(4x) - x \cos(4x)$$

$$\int (x \sin(x)) dx = \frac{1}{4} \left(\frac{1}{4} \sin(4x) - x \cos(4x) \right)$$

$$\int (x \sin(x)) dx = \frac{1}{16} \sin(4x) - \frac{x}{4} \cos(4x)$$

A1

Question 3 (4 marks)

a. $\log_z(p^2 - 1) = \log_z((p+1)(p-1))$

$$= \log_z(p+1) + \log_z(p-1)$$

M1

Therefore, $\frac{\log_z(p-1)}{\log_z(p+1)} = \frac{\log_z(p+1) + \log_{p+1}(p-1)}{\log_z(p+1)}$

$$= 1 + \frac{\log_z(p-1)}{\log_z(p+1)}$$

$$= 1 + \log_{p+1}(p-1)$$

A1

b. $\log_e(m+1)^2 - \log_e(4) = \log_e(n)^2$

$$\log_e\left(\frac{(m+1)^2}{4}\right) = \log_e(n)^2$$

M1

$$\frac{(m+1)^2}{4} = n^2$$

$$m+1 = 2n, -2n$$

$$m = -2n - 1 \text{ as } m > -1 \text{ and } n < 0.$$

A1

Question 4 (3 marks)

$$f(x) = 2x^3 \tan(x)$$

use of rule M1

$$f'(x) = 6x^2 \times \tan(x) + 2x^3 \times \sec^2(x)$$

correct derivative A1

$$= 2x^2(3 \tan(x) + x \sec^2(x))$$

$$= 2x^2 \left(3 \times \frac{\sin(x)}{\cos(x)} + x \times \frac{1}{\cos^2(x)} \right)$$

$$= \frac{2x^2}{\cos(x)} \left(3 \times \sin(x) + x \times \frac{1}{\cos(x)} \right)$$

$$= \frac{2x^2}{\cos(x)} (3 \sin(x) + x \sec(x))$$

Since $f'(x) = \frac{ax^2}{\cos(x)}(b \sin(x) + cx \sec(x))$, $a = 2$, $b = 3$, $c = 1$.

A1

Question 5 (3 marks)

$$4 \sin^2(2x) = 3 \text{ for } x \in [-\pi, \pi]$$

$$\sin^2(2x) = \frac{3}{4}$$

$$\sin(2x) = \pm \frac{\sqrt{3}}{2}$$

$$2x = \sin^{-1}\left(\pm \frac{\sqrt{3}}{2}\right) \text{ for } 2x \in [-2\pi, \pi]$$

base angle = $\frac{\pi}{3}$

M1

$$2x = -\frac{5\pi}{3}, -\frac{4\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3} \text{ (positive square root) and } -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \text{ (negative square root)}$$

correct solutions for positive (or negative) square root only M1

$$x = -\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$$

A1

Question 6 (4 marks)

a. $f(g(x)) = \log_e(3x - 2 + 1)$
 $= \log_e(3x - 1)$ A1

domain $f(g(x)) = \text{domain } g(x)$
 $= \left(\frac{2}{3}, \infty\right)$ A1

b. Let $y = h(x)$.

$$\therefore y = \log_e(3x - 1)$$

To find rule for inverse, swap $x \leftrightarrow y$ and rearrange.

$$x = \log_e(3y - 1)$$
 M1

$$e^x = e^{\log_e(3y - 1)}$$

$$e^x = 3y - 1$$

$$e^x + 1 = 3y$$

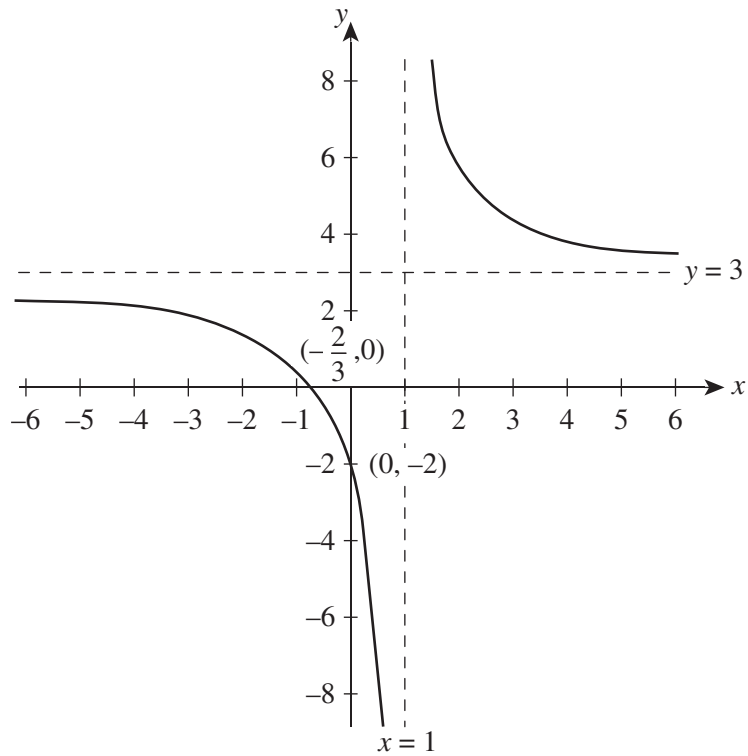
$$\frac{e^x + 1}{3} = y$$

So the rule for $h^{-1}(x)$ is: $h^{-1}(x) = \frac{e^x + 1}{3}$. A1

Question 7 (5 marks)

a. $3 + \frac{5}{x-1} = \frac{3(x-1) + 5}{x-1}$ M1
 $= \frac{3x - 3 + 5}{x-1}$
 $= \frac{3x + 2}{x-1}$

b.



correct shape A1
correct intercepts and asymptotes A1

c.
$$g(x) = 3 + \frac{5}{(x+2-1)} - 3$$

$$= \frac{5}{(x+1)}$$

$$= 5 \times h(x)$$

M1

Hence $g(x)$ is the graph of $h(x)$ after it has been dilated by a factor of 5 from the x -axis.

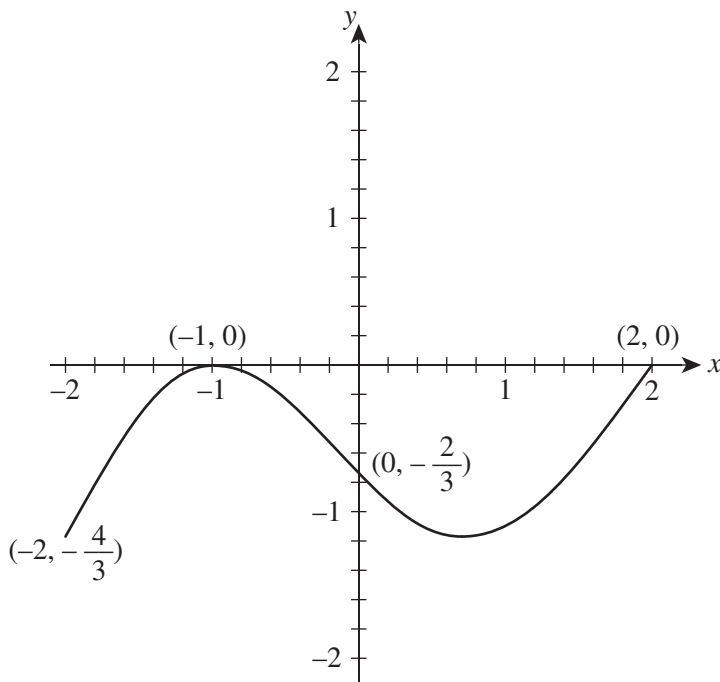
A1

Question 8 (4 marks)

$$\begin{aligned} \text{a. } f(-2) &= \frac{1}{3}(-2-2)(-2+1)^2 \\ &= \frac{1}{3}(-4)(-1)^2 \\ &= -\frac{4}{3} \quad \therefore \text{endpoint } \left(-2, -\frac{4}{3}\right) \end{aligned}$$

$$\begin{aligned} f(2) &= \frac{1}{3}(2-2)(2+1)^2 \\ &= 0 \quad \therefore \text{endpoint } (2, 0) \text{ and } x\text{-intercept} \\ &\quad \text{other } x\text{-intercept } (-1, 0) \end{aligned}$$

$$\begin{aligned} f(0) &= \frac{1}{3}(0-2)(0+1)^2 \\ &= \frac{1}{3}(-2)(1)^2 \\ &= -\frac{2}{3} \quad \therefore \text{y-intercept } \left(0, -\frac{2}{3}\right) \end{aligned}$$



correct shape A1
all coordinates correct A1

b. area of region = $-\frac{1}{3} \int_{-1}^2 \left(\frac{1}{3}(x-2)(x+1)^2\right) dx$

$$= -\frac{1}{3} \int_{-1}^2 (x^3 - 3x - 2) dx$$

$$= -\frac{1}{3} \left[\frac{x^4}{4} - \frac{3x^2}{2} - 2x \right]_{-1}^2$$

$$= -\frac{1}{3} \left[\left(\frac{16}{4} - \frac{12}{2} - 4 \right) - \left(\frac{1}{4} - \frac{3}{2} + 2 \right) \right]$$

$$= -\frac{1}{3} \left[(-6) - \left(\frac{3}{4} \right) \right]$$

$$= -\frac{1}{3} \times -\frac{27}{4}$$

$$= \frac{9}{4} \text{ units}^2$$

M1

A1

Question 9 (6 marks)

a. No, Jenny is not right. The sample is biased to the birds who prefer the type of seed Jenny offers and those types that are not naturally shy of human habitats. A1

b. $p = \frac{4}{10}$
 $= 0.4$ A1

c. 0, 1, 2 or 3 king parrots can come out of 3 birds.
 Therefore, \hat{p} can take values: 0, $\frac{1}{3}$, $\frac{2}{3}$ or 1. A1

d.

No. of king parrots	0	1	2	3
Proportion of king parrots (\hat{p})	0	$\frac{1}{3}$	$\frac{2}{3}$	1
$\Pr(\hat{P} = \hat{p})$	$\frac{1}{6}$	$\frac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = \frac{1}{2}$	$\frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}} = \frac{3}{10}$	$\frac{1}{30}$

The second of these values can be obtained by subtracting the first from 1.

correct third column of table A1
correct fourth column of table A1

$$\begin{aligned}
 \text{e. } \Pr(\hat{p} > 0.2) &= 1 - \Pr(\hat{p} < 0.2) \\
 &= 1 - \Pr(P = 0) \\
 &= 1 - \frac{1}{6} \\
 &= \frac{5}{6}
 \end{aligned}$$

A1

Question 10 (5 marks)

$$\text{a. } \int_0^{\frac{2\pi}{3}} \left(a \cos\left(x - \frac{\pi}{3}\right) \right) dx = 1$$

M1

$$a \left[\sin\left(x - \frac{\pi}{3}\right) \right]_0^{\frac{2\pi}{3}} = 1$$

$$a \left[\sin\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) - \sin\left(0 - \frac{\pi}{3}\right) \right] = 1$$

$$a \left(\frac{\sqrt{3}}{2} - -\frac{\sqrt{3}}{2} \right) = 1$$

$$a\sqrt{3} = 1$$

$$a = \frac{1}{\sqrt{3}}$$

A1

$$\text{b. } \int_0^m \left(\frac{1}{\sqrt{3}} \cos\left(x - \frac{\pi}{3}\right) \right) dx = \frac{1}{2}$$

M1

$$\frac{1}{\sqrt{3}} \left[\sin\left(m - \frac{\pi}{3}\right) - \sin\left(0 - \frac{\pi}{3}\right) \right] = \frac{1}{2}$$

$$\sin\left(m - \frac{\pi}{3}\right) - -\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\sin\left(m - \frac{\pi}{3}\right) = 0$$

$$m - \frac{\pi}{3} = \sin^{-1}(0)$$

$$m = \frac{\pi}{3}$$

A1

c. As the graph of $f(x)$ is a translation of $\cos(x)$ by $\frac{\pi}{3}$ units to the right, the highest point is at $x = \frac{\pi}{3}$.

Hence the mode is $\frac{\pi}{3}$.

A1