

Trial Examination 2016

MATHEMATICAL METHODS

Written Examination 2 - SOLUTIONS

SECTION A

1. A 2. C 3. E 4. C 5. A 6. D 7. E 8. B 9. B 10. A

11. E 12. D 13. A 14. E 15. B 16. D 17. D 18. C 19. C 20. B

SECTION A

Question 1

$$f: R \rightarrow R, f(x) = -2 - 3 \cos\left(\frac{x}{4} + 1\right)$$

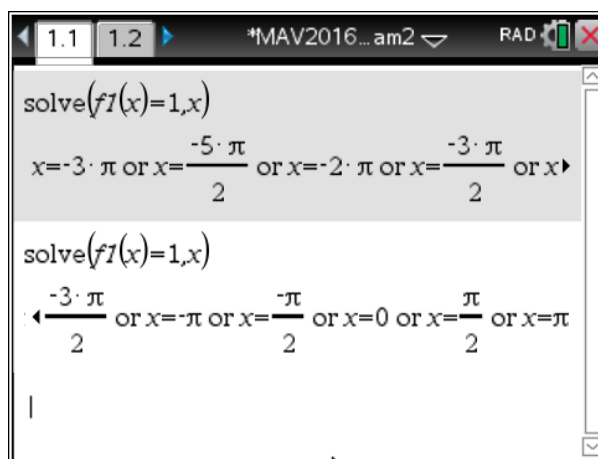
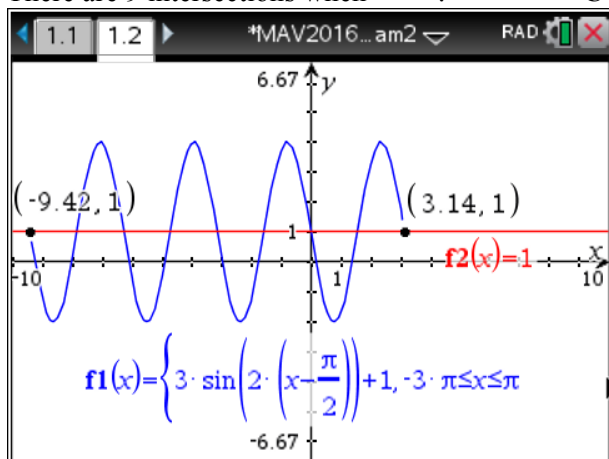
The range is $[-3 - 2, 3 - 2] = [-5, 1]$

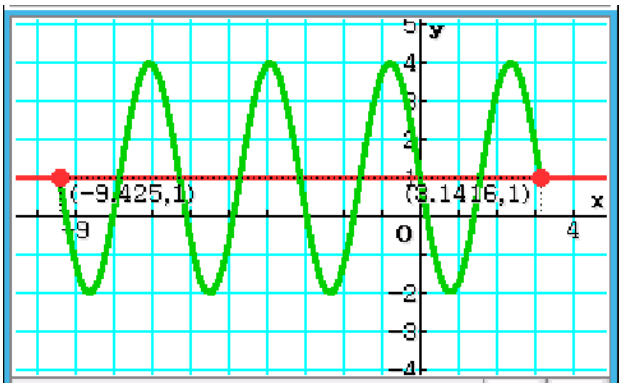
The Period is $\frac{2\pi}{\frac{1}{4}} = 8\pi$ A

Question 2

$$g: [-3\pi, \pi] \rightarrow R, g(x) = 3 \sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 1$$

There are 9 intersections when $k = 1$. C





$$\text{define } f(x) = 3\sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 1$$

$$\text{solve}(f(x) = 1 \mid -3\pi \leq x \leq \pi, x)$$

$$\left\{ x = 0, x = -3\pi, x = -2\pi, x = -\pi, x = \pi, x = \frac{-5\pi}{2}, x = \frac{-3\pi}{2}, x = \right\}$$

done

Question 3

The domain of $f(g(x))$ is the same as the domain of g where $g(x) = \tan(\pi x)$.

The asymptotes of g have equations $y = \frac{1}{2} + k, k \in \mathbb{Z}$.

Hence the domain of $f(g(x))$ is $\mathbb{R} \setminus \left\{ \frac{1}{2} + k \right\}, k \in \mathbb{Z}$ **E**

Question 4

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos(x)$$

$$\sqrt{15^2 - 2^2} = \sqrt{221}$$

$$\sin\left(\frac{3\pi}{2} - x\right) = -\frac{\sqrt{221}}{15} \quad \mathbf{C}$$

Question 5

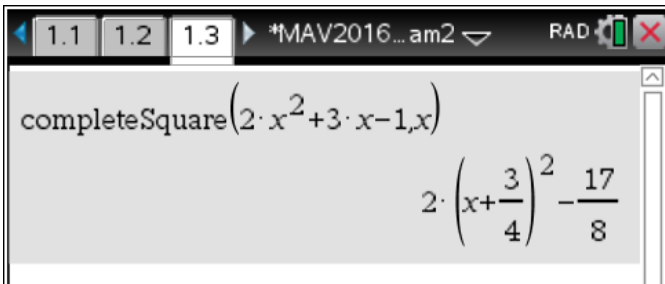
$$y = 2x^2 + 3x - 1 = 2\left(x + \frac{3}{4}\right)^2 - \frac{17}{8}$$

The graph of $y = x^2$ has been dilated by a factor of 2 from the x -axis and

Then translated $\frac{3}{4}$ units to the left and $\frac{17}{8}$ units down.

A possible rule for the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} \frac{3}{4} \\ \frac{17}{8} \end{bmatrix} \quad \mathbf{A}$$



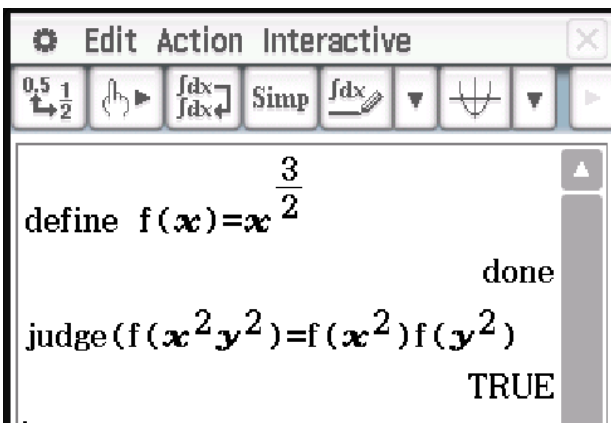
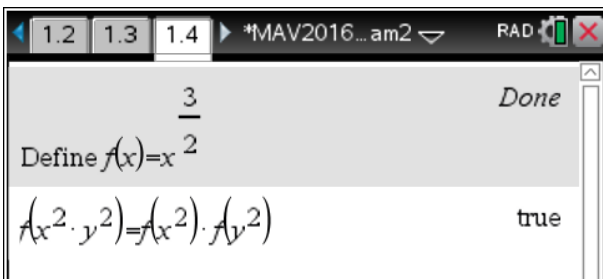
Question 6

$$f(x) = x^{\frac{3}{2}}$$

$$f(x^2y^2) = (x^2y^2)^{\frac{3}{2}} = x^3y^3$$

$$f(x^2)f(y^2) = (x^2)^{\frac{3}{2}}(y^2)^{\frac{3}{2}} = x^3y^3$$

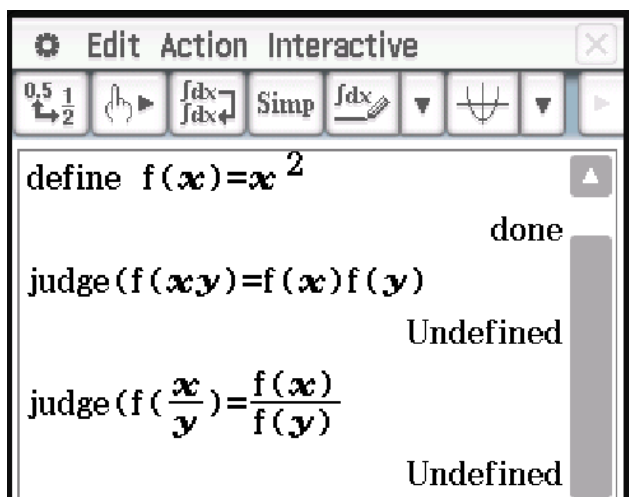
D



Using counter examples

$$f(xy) = f(x)f(y), f(-1 \times -1) = 1, f(-1) \times f(-1) \text{ has no real solution}$$

$$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}, f\left(\frac{-1}{-1}\right) = 1, \frac{f(-1)}{f(-1)} \text{ has no real solution}$$



$f(x + y) = f(x) + f(y)$, $f(1 + 1) = 2^{\frac{3}{2}}$, $f(1) + f(1) = 2$
 $f(x - y) = f(x) - f(y)$, $f(2 - 1) = 1$, $f(1) - f(1) = 0$

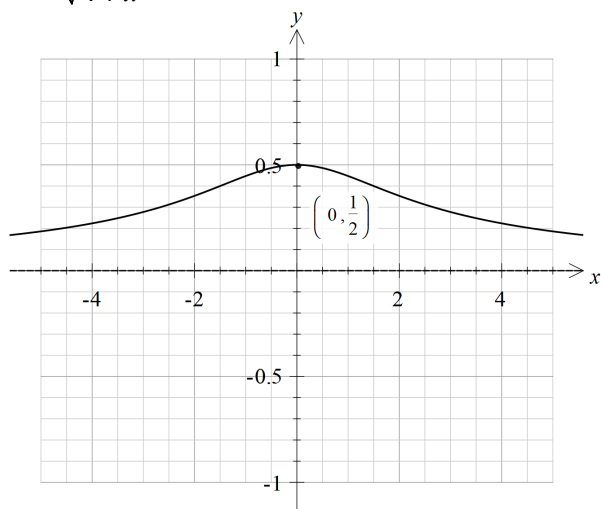
Question 7

When $x = 0$, $y = 0.5$.

Either **C** or **E**

When $x = 2$, $y \approx 0.35$

$$y = \frac{1}{\sqrt{4 + x^2}} \qquad \mathbf{E}$$



Question 8

$$f: (1, \infty) \rightarrow \mathbb{R}, f(x) = \frac{3}{(x-1)^2} + 2$$

Range is $(2, \infty)$ which is the domain of the inverse function.

$$\text{Let } y = \frac{3}{(x-1)^2} + 2$$

Inverse swap x and y .

$$x = \frac{3}{(y-1)^2} + 2$$

$$y = \pm \sqrt{\frac{3}{x-2}} + 1$$

$$y = \sqrt{\frac{3}{x-2}} + 1, \text{ as the domain of } f \text{ is } (1, \infty).$$

$$f^{-1}: (2, \infty) \rightarrow \mathbb{R}, f^{-1}(x) = \sqrt{\frac{3}{x-2}} + 1$$

B

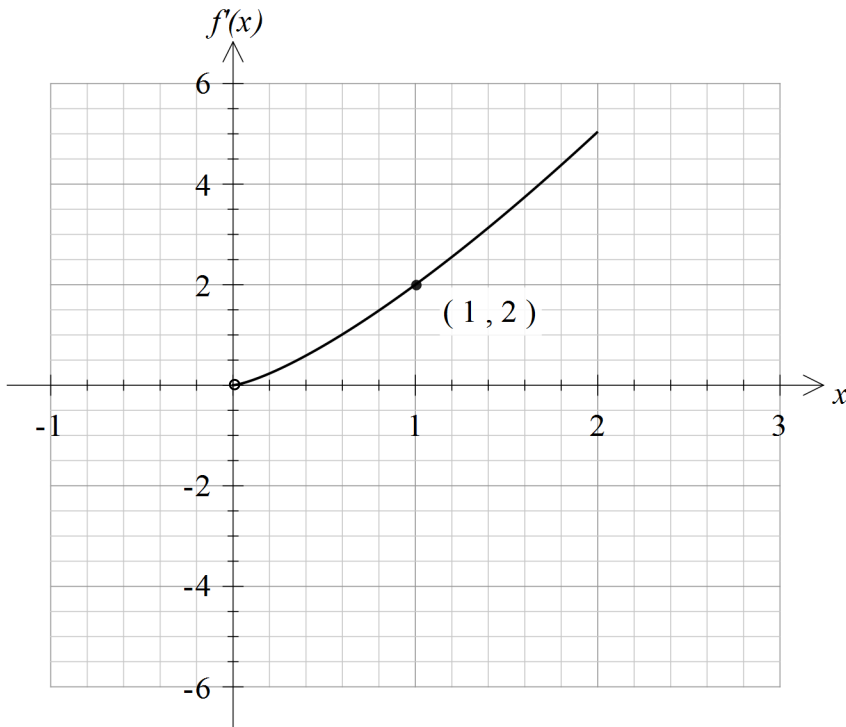
TI-84 Plus calculator interface showing the solution of the equation $x = \frac{3}{(y-1)^2} + 2$ for y . The screen displays the equation, the solutions $y = \frac{\sqrt{x-2} - \sqrt{3}}{\sqrt{x-2}}$ or $y = \frac{\sqrt{x-2} + \sqrt{3}}{\sqrt{x-2}}$, and the simplified form of the second solution: $\frac{\sqrt{3}}{\sqrt{x-2}} + 1$.

TI-84 Plus calculator interface showing the solution of the equation $x = \frac{3}{(y-1)^2} + 2$ for y . The screen displays the equation, the solutions $y = \frac{\sqrt{x-2} - \sqrt{3}}{\sqrt{x-2}}$ or $y = \frac{\sqrt{x-2} + \sqrt{3}}{\sqrt{x-2}}$, and the simplified form of the second solution: $\frac{\sqrt{3}}{\sqrt{x-2}} + 1$.

Question 9

The point $(1, 2)$ is on the curve of f' .

Thus the gradient of the graph of f at $x = 1$ is 2.

B**Question 10**

If $f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + h$ has a stationary point of inflection at $(-3, 0)$ and a turning point at $(2, 0)$ then $f(x) = a(x+3)^3(x-2)^2$.

$$h = a \times 3^3 \times (-2)^2 = 108a$$

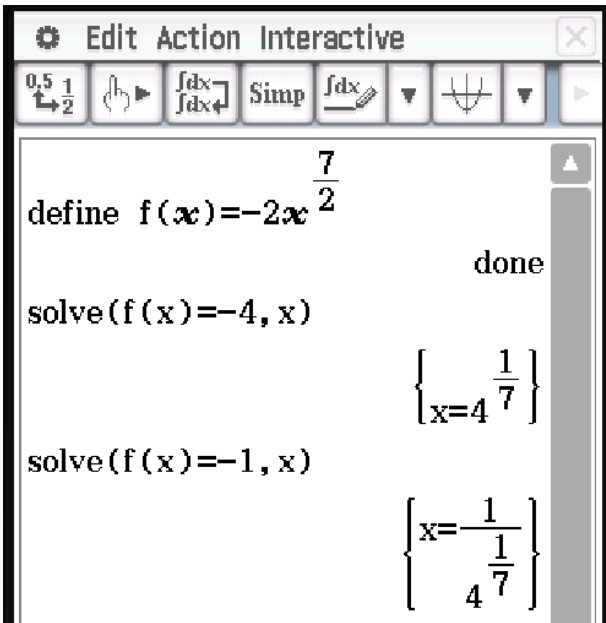
$$\frac{h}{a} = 108$$

A**Question 11**

$$f(x) = -2x^{\frac{7}{2}}, \quad y = -4 \text{ to } y = -1$$

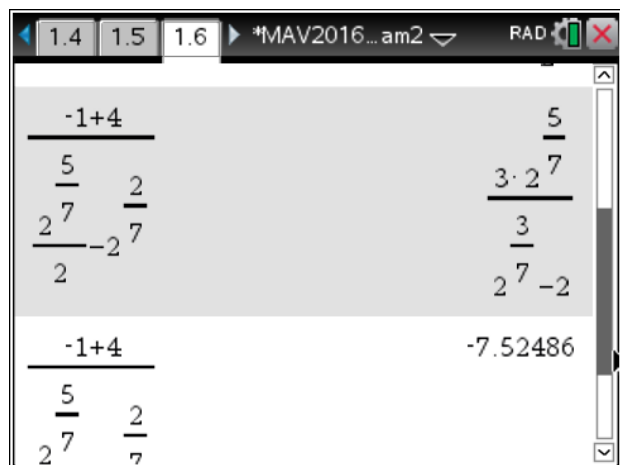
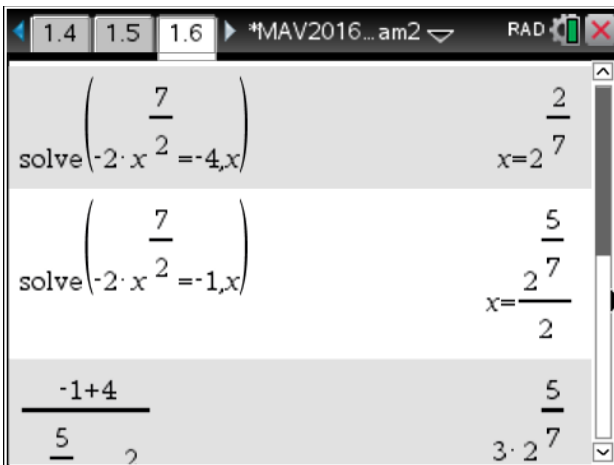
$$\text{Solve } f(x) = -4, x = 2^{\frac{2}{7}}$$

$$\text{and } f(x) = -1, x = 2^{-\frac{2}{7}}$$



$$\begin{aligned} \text{Average rate of change} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 + 4}{2^{\frac{2}{7}} - 2^{\frac{1}{7}}} \\ &\approx -7.5 \end{aligned}$$

E



$$\frac{f\left(4^{\frac{1}{7}}\right) - f\left(\frac{1}{4^{\frac{1}{7}}}\right)}{4^{\frac{1}{7}} - \frac{1}{4^{\frac{1}{7}}}} = -7.524864066$$

Question 12

$$\log_a(b) = c \text{ and } \log_c(a) = b$$

$$\log_a(b) + \log_c(a) = b + c$$

$$\log_a(b) + \frac{\log_a(a)}{\log_a(c)} = b + c$$

$$\log_a(b) + \frac{1}{\log_a(c)} = b + c$$

$$\log_a\left(\frac{b}{c}\right) \neq b + c \quad \mathbf{D}$$

Question 13

$$f'(x) = 2x^3 - 2x$$

$$f(x) = \int (2x^3 - 2x) dx$$

$$= \frac{x^4}{2} - x^2 + c$$

$$f(2) = \frac{9}{2}$$

$$\frac{9}{2} = \frac{2^4}{2} - 2^2 + c$$

$$c = \frac{1}{2}$$

$$\text{Solve } \frac{x^4}{2} - x^2 + \frac{1}{2} = 0 \text{ for } x.$$

$$x = \pm 1$$

A

The screenshot shows a TI-84 Plus calculator interface with the following content:

- Top bar: 1.5, 1.6, 1.7, *MAV2016...am2, RAD, and a close button.
- Input: $\int (2 \cdot x^3 - 2 \cdot x) dx$
- Output: $\frac{x^4}{2} - x^2$
- Input: $\text{solve}\left(\frac{2^4}{2} - 2^2 + c = \frac{9}{2}, c\right)$
- Output: $c = \frac{1}{2}$
- Input: $\text{solve}\left(\frac{x^4}{2} - x^2 + \frac{1}{2} = 0, x\right)$
- Output: $x = -1 \text{ or } x = 1$

Edit Action Interactive
 $\int 2 \cdot x^3 - 2 \cdot x dx$
 $\frac{x^4}{2} - x^2 + C$
 define $f(x) = \frac{x^4}{2} - x^2 + C$
 done
 solve $(f(2) = \frac{9}{2}, C)$
 $\{C = \frac{1}{2}\}$
 solve $(\frac{x^4}{2} - x^2 + \frac{1}{2} = 0, x)$
 $\{x = -1, x = 1\}$
 Alg Standard Real Rad

Question 14

$$\begin{aligned}
 \int_2^3 (g(x)) dx &= -3 \\
 &= 2 \int_3^2 (1 - 2g(x)) dx \\
 &= 2 \int_3^2 (1) dx - 4 \int_3^2 (g(x)) dx \\
 &= 2[x]_3^2 - 4 \times 3 \\
 &= -14
 \end{aligned}$$

E**Question 15**

$$\begin{aligned}
 \text{Area} &= \frac{1}{2}(e^0 + e + e^2 + e^3) \\
 &= \frac{1}{2}(1 + e + e^2 + e^3)
 \end{aligned}$$

The rectangles are below the curve.

Underestimate

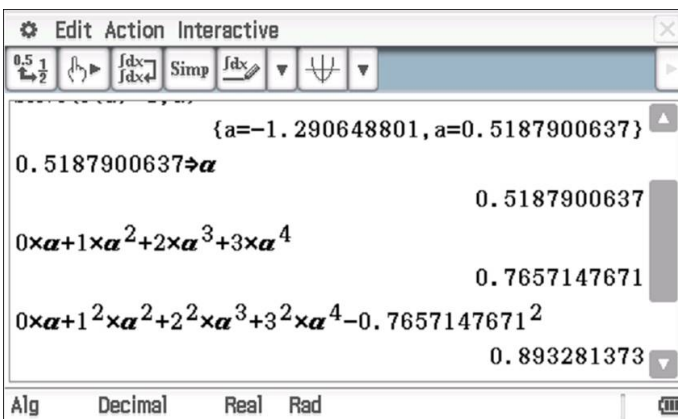
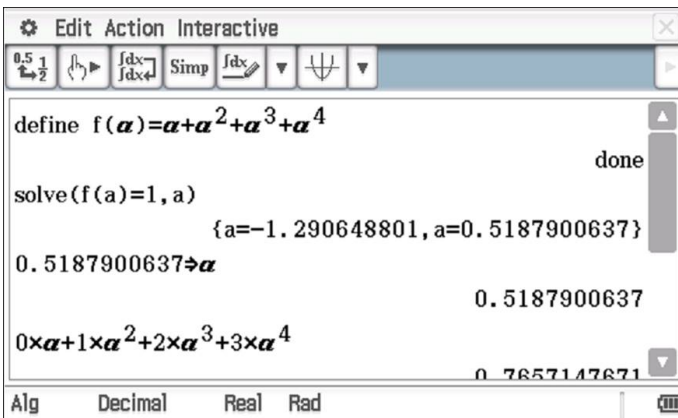
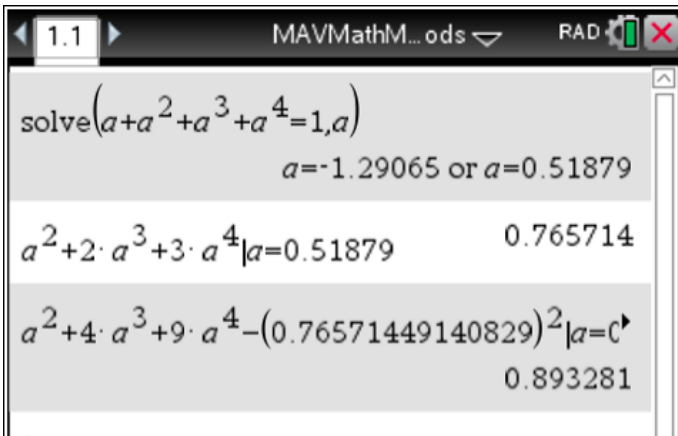
B**Question 16**Solve $a + a^2 + a^3 + a^4 = 1$ for a

$a \approx 0.51879$

$E(X) = a^2 + 2 \times a^3 + 3 \times a^4 \approx 0.766$

$Var(X) = a^2 + 4a^3 + 9a^4 - (0.765714 \dots)^2 \approx 0.893$

D



Question 17

	J	J'	
R	$0.3p$	$0.7p$	p
R'	$0.3 - 0.3p$	p^2	$1 - p$
	0.3	0.7	1

Independent events $\Pr(J \cap R) = \Pr(J) \times \Pr(R)$

Let $a = \Pr(J \cap R) = 0.3p$

$$0.3 + p - a + p^2 = 1$$

Solve $p^2 + 0.7p - 0.7 = 0$ for p .

$$p = \frac{\sqrt{329} - 7}{20}$$

$$a = 0.3p$$

$$a = \frac{3(\sqrt{329} - 7)}{200}$$

D

Calculator screen showing the solution to the equation $p^2 + \frac{7}{10}p - \frac{7}{10} = 0$. The solutions are $p = \frac{-(\sqrt{329} + 7)}{20}$ or $p = \frac{\sqrt{329} - 7}{20}$. The calculator also shows a decimal approximation: $p = -1.256918$ or $p = 0.5569179$.

Calculator screen showing the solution to the equation $0.7 \cdot p + p^2 = 0.7$. The solutions are $p = \frac{-\sqrt{329} - 7}{20}$ and $p = \frac{\sqrt{329} - 7}{20}$.

Question 18

Solve $\int_1^a (\log_e(x)) dx = 1$ for a .

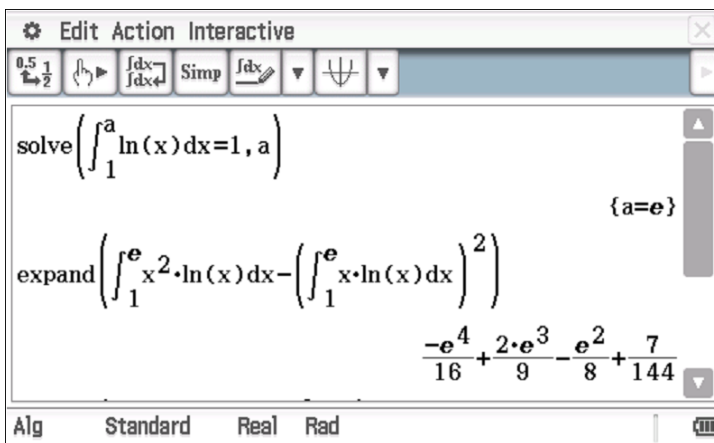
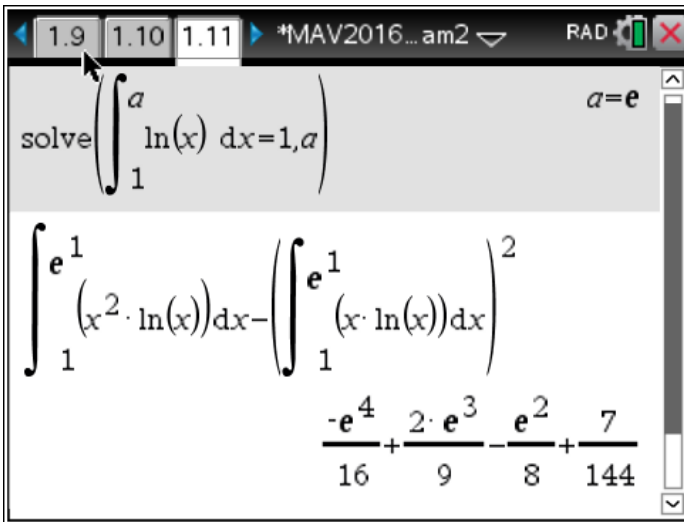
$$a = e$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$\text{Var}(X) = \int_1^e (x^2 \log_e(x)) dx - \left(\int_1^e (x \log_e(x)) dx \right)^2$$

$$= -\frac{e^4}{16} + \frac{2e^3}{9} - \frac{e^2}{8} + \frac{7}{144}$$

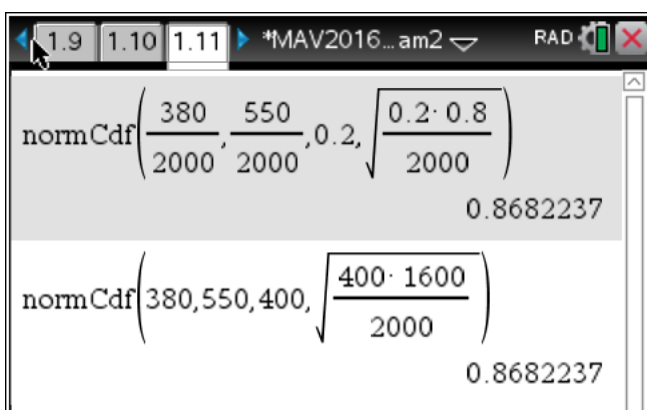
C



Question 19

$n = 2000$ and $p = 0.2$. Since n is large, the distribution of \hat{P} is approximately normal with $\mu = 0.2$ and $\sigma = \sqrt{\frac{0.2 \times 0.8}{2000}}$.

$\Pr\left(\frac{380}{2000} < X < \frac{550}{2000}\right) \approx 0.868$ **C**



The screenshot shows a TI-84 Plus calculator interface. The title bar reads "Edit Action Interactive". The top row of buttons includes a fraction template, a right arrow, a square root template, "Simp", a fraction template, and a square root template. The main display area shows two calculations of the normal cumulative distribution function (normCdf):

$$\text{normCdf}\left(\frac{380}{2000}, \frac{550}{2000}, \sqrt{\frac{0.2 \cdot 0.8}{2000}}, 0.2\right) = 0.8682237614$$
$$\text{normCdf}\left(380, 550, \sqrt{\frac{400 \cdot 1600}{2000}}, 400\right) = 0.8682237614$$

Below the calculations is a small square icon. At the bottom of the screen, the mode is set to "Alg", and the system is in "Standard" mode with "Real" and "Rad" options visible.

Question 20

If 200 such samples were taken, $0.9 \times 200 = 180$ of the confidence intervals would be expected to contain p . **B**

SECTION B EXTENDED RESPONSE QUESTIONS

Question 1 (14 marks)

a. $f(x) = ax^{\frac{2}{3}}$

Substitute the point $\left(-8, \frac{16}{3}\right)$

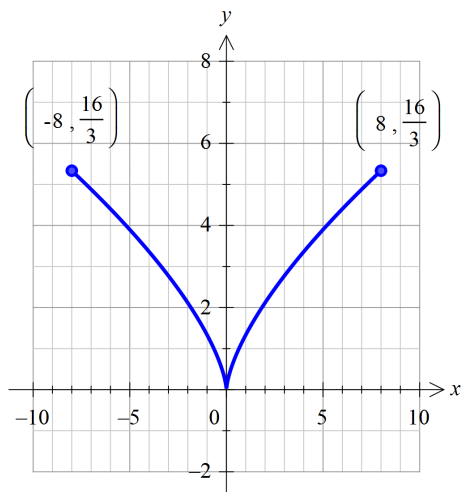
$$\frac{16}{3} = a(-8)^{\frac{2}{3}}$$

$$\frac{16}{3} = 4a$$

$$a = \frac{4}{3}$$

1M Show that

**b. Shape 1A
Endpoints 1A**



c. $f_1(x) = f(-x)$ reflects the graph over the y -axis.

As the function $f(x) = \frac{4}{3}x^{\frac{2}{3}}$ is an even function the transformation described makes no difference. **1A**

d.

- a dilation of a factor of 2 units from the x -axis gives $y = \frac{8}{3}x^{\frac{2}{3}}$
then

- a translation of 3 units up gives $f_2(x) = \frac{8}{3}x^{\frac{2}{3}} + 3$ **1A**

e.

- a reflection in the x -axis gives $y_1 = -\frac{4}{3}x^{\frac{2}{3}}$
- a reflection in the y -axis gives $y_2 = -\frac{4}{3}(-x)^{\frac{2}{3}} = -\frac{4}{3}x^{\frac{2}{3}}$
- a translation of 1 unit in the positive direction of the y -axis gives $y_3 = -\frac{4}{3}x^{\frac{2}{3}} + 1$
- a translation of 3 units in the positive direction of the x -axis gives $y_4 = -\frac{4}{3}(x-3)^{\frac{2}{3}} + 1$

The image equation is $f_3(x) = -\frac{4}{3}(x-3)^{\frac{2}{3}} + 1$ **2A**

$$\text{f. } T: R^2 \rightarrow R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\text{gives } \begin{bmatrix} 2 & 0 \\ 0 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2x+1 \\ -\frac{1}{3}y+2 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Rearrange to get

$$x = \frac{x_1 - 1}{2}$$

$$y = \frac{y_1 - 2}{-\frac{1}{3}} = -3(y_1 - 2) \quad \text{1M}$$

Substitute to find the image equation for $f(x) = \frac{4}{3}x^{\frac{2}{3}}$.

$$-3(y_1 - 2) = \frac{4}{3}\left(\frac{x_1 - 1}{2}\right)^{\frac{2}{3}} \quad \text{1M}$$

Image equation is

$$y_1 = -\frac{4}{9}\left(\frac{x_1 - 1}{2}\right)^{\frac{2}{3}} + 2$$

$$\text{Giving } y = 2 - \frac{4}{9}\left(\frac{x-1}{2}\right)^{\frac{2}{3}} \quad \text{1A}$$

g. $y = 2 - \frac{4}{9}\left(\frac{x-1}{2}\right)^{\frac{2}{3}}$ gives a dilation of a factor of 2 from y -axis and a translation of 1 unit in the positive

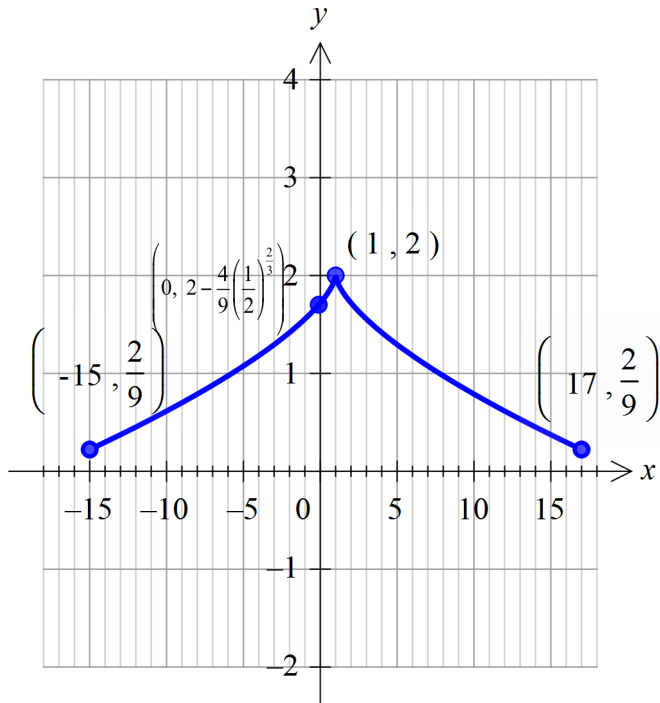
direction of the x -axis from the original function $f(x) = \frac{4}{3}x^{\frac{2}{3}}$ where the domain was $[-8, 8]$.

A dilation of a factor of 2 from y -axis gives the domain $[-16, 16]$.

Then a translation of 1 unit in the positive direction of the x -axis gives a domain $[-15, 17]$. **1A**

h. Shape 1A**Endpoints 1A****Coordinates of Cusp and**

$$\text{y-intercept } \left(0, 2 - \frac{4}{9} \left(\frac{1}{2} \right)^{\frac{2}{3}} \right) \quad \mathbf{1A}$$



Question 2 (15 marks)

$$\begin{aligned} \text{a. } P(0) &= 95 + 16 \sin\left(\frac{\pi}{15}(0 - 7.5)\right) \\ &= 95 - 16 \sin\left(\frac{\pi}{2}\right) \\ &= 95 - 16 \\ &= 79 \end{aligned}$$

79 beats per minute

1M Show that

The screenshot shows a TI-84 Plus calculator interface. At the top, there is a menu bar with 'Edit', 'Action', and 'Interactive' options. Below the menu bar is a toolbar with various calculator functions. The main display area shows the following text:

```
define f(t)=95+16sin(pi/15*(t-7.5))
f(0)
done
79
```

$$\begin{aligned} \text{b. Period} &= \frac{2\pi}{\pi/15} = 30 && \mathbf{1A} \\ \text{Amplitude} &= 16 && \mathbf{1A} \end{aligned}$$

$$\text{c. } P(t) = 95 + 16 \sin\left(\frac{\pi}{15}(t - 7.5)\right)$$

$$P'(t) = 16 \times \frac{\pi}{15} \cos\left(\frac{\pi}{15}(t - 7.5)\right) \quad \mathbf{1A}$$

$$\begin{aligned} P'(t) &= \frac{16\pi}{15} \cos\left(\frac{\pi t}{15} - \frac{\pi}{2}\right) \\ &= \frac{16\pi}{15} \cos\left(\frac{\pi}{2} - \frac{\pi t}{15}\right) \end{aligned}$$

$$\text{Giving } P'(t) = \frac{16\pi}{15} \sin\left(\frac{\pi t}{15}\right). \quad \mathbf{1M Show that}$$

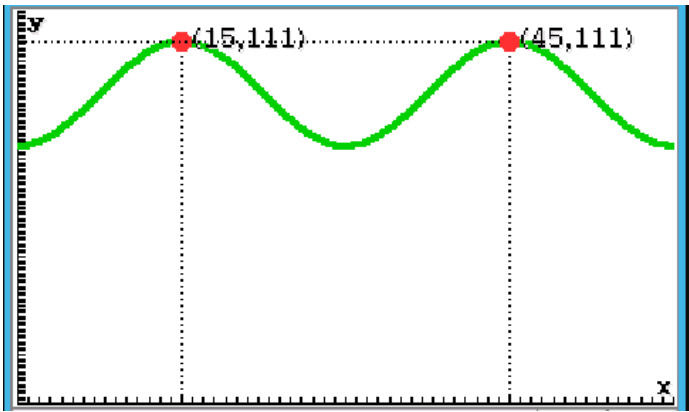
$$\text{d. Solve } \frac{16\pi}{15} \sin\left(\frac{\pi t}{15}\right) = 0 \text{ for } x \in [0, 60] \text{ 2 cycles} \quad \mathbf{1M}$$

$$\frac{16\pi}{15} \sin\left(\frac{\pi t}{15}\right) = 0$$

$$\sin\left(\frac{\pi t}{15}\right) = 0$$

$$\frac{\pi t}{15} = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$t = 0, 15, 30, 45, 60$$

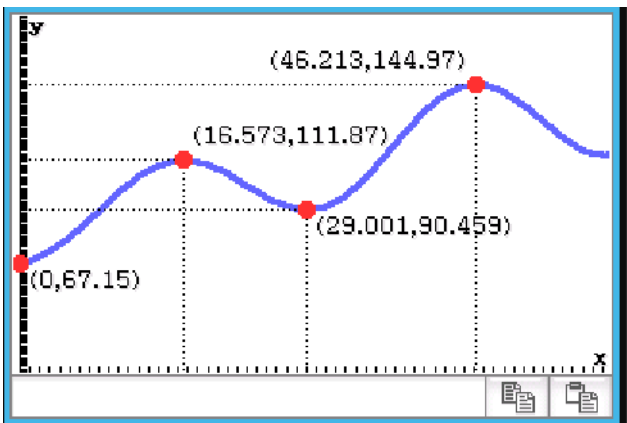
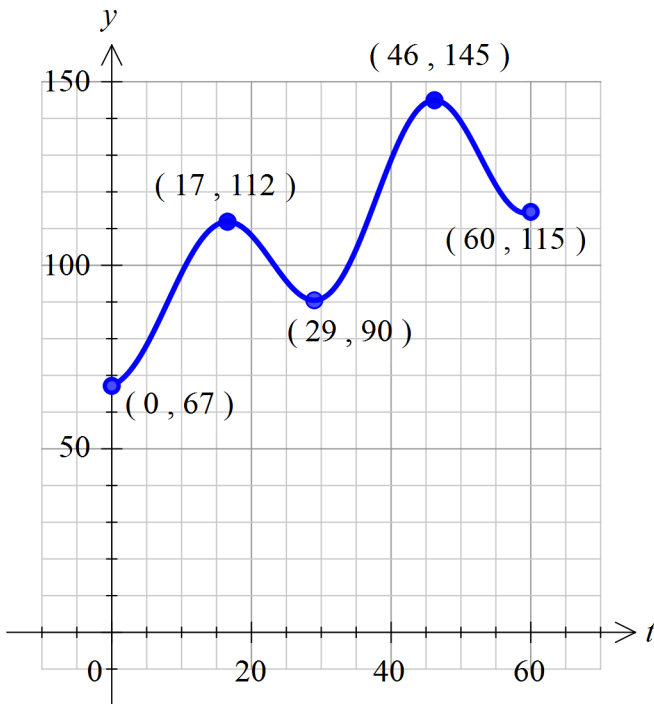


From the graph the local max occurs at $t = 15$ and $t = 45$

1A

e. $P_N : [0, 60] \rightarrow R, P_N(t) = 0.01(t + 85) \times P(t)$

- Shape 1A
- Turning points 1A
- Endpoints 1A



f. Maximum heart rate ≈ 144.97 beats /minute

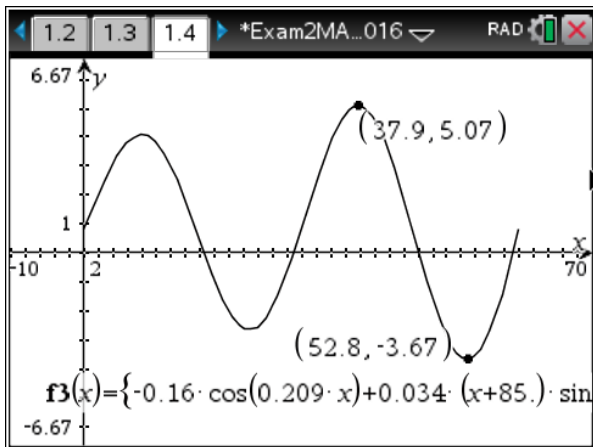
Maximum heart rate rounded to the nearest whole number = 145 beats /minute **1A**

g. Find when $P'_N(t)$ has its greatest magnitude

1M

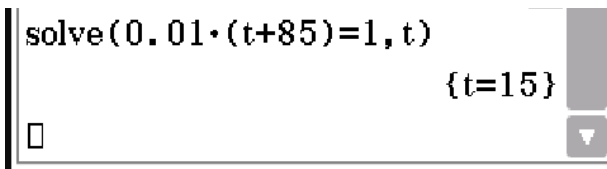
$t = 37.9$ minutes

1A



h. Solve the equation $0.01(t+85) = 1$ gives $t = 15$

1A

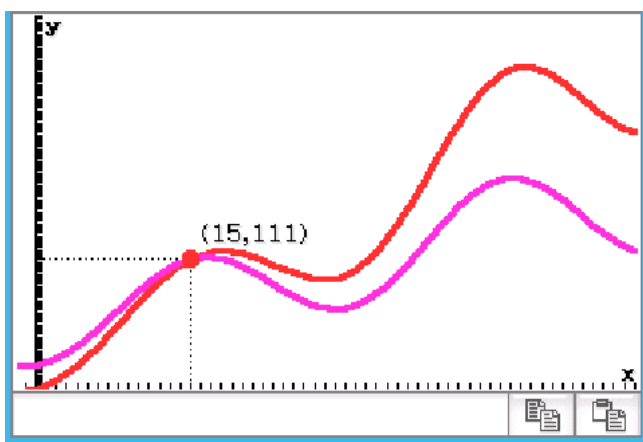


Solve $95 + 16 \sin\left(\frac{\pi}{15}(t-7.5)\right) = 0.01(t+85) \times P(t)$

$$95 + 16 \sin\left(\frac{\pi}{15}(t-7.5)\right) = 0.01(t+85) \times \left(95 + 16 \sin\left(\frac{\pi}{15}(t-7.5)\right)\right)$$

Gives the equation $1 = 0.01(t+85)$ where previously found solution $t = 15$

So Sarah's pulse equals Stephen's pulse at $t = 15$.



Looking at the graph

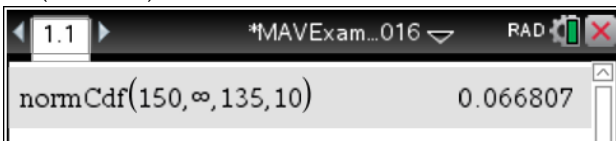
Sarah's pulse can be measured to be lower than her twin brother Stephen's

for $t \in (15, 60]$

1A

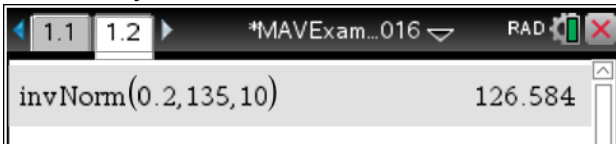
Question 3 (17 marks)a. $X : N(135, 100)$

$$\Pr(X > 150) = 0.0668\dots = 7\% \text{ to the nearest whole percent.} \quad \mathbf{1A}$$

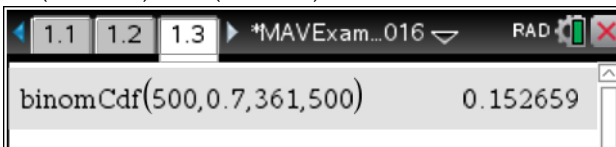
b. $\Pr(X > x) = 0.8$

$$\Pr(X < x) = 0.2$$

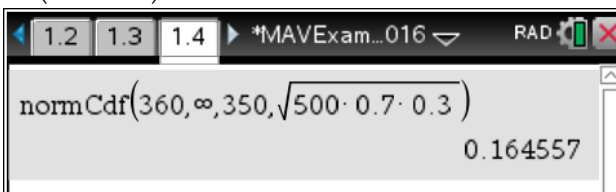
$$x = 127 \text{ days to the nearest whole number} \quad \mathbf{1A}$$

c. $Y : \text{Bi}(500, 0.7)$

$$\Pr(Y > 360) = \Pr(Y \geq 361) = 0.1527 \text{ correct to 4 decimal places.} \quad \mathbf{1A}$$

d. $W : N(350, 500 \times 0.7 \times 0.3)$

$$\Pr(W > 360) = 0.1646 \text{ correct to 4 decimal places.} \quad \mathbf{1M}$$

e. $\mu = 0.7$

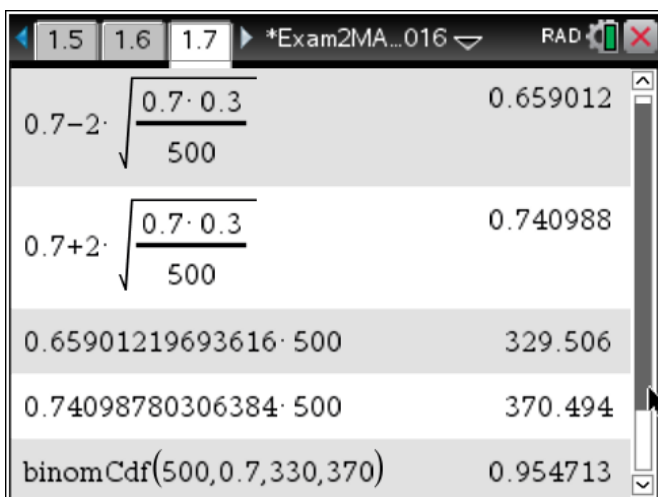
$$\sigma = \sqrt{\frac{0.7 \times 0.3}{500}} = \frac{\sqrt{105}}{500}$$

$$\left(0.7 - 2 \times \frac{\sqrt{105}}{500}, 0.7 + 2 \times \frac{\sqrt{105}}{500} \right) \quad \mathbf{1M}$$

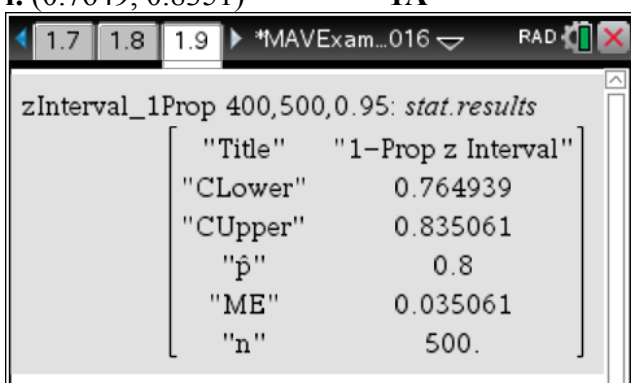
$$\Pr(0.6590\dots \leq \hat{P} \leq 0.7409\dots) = \Pr(329.5\dots \leq X \leq 370.4\dots) \quad \mathbf{1M}$$

$$= \Pr(330 \leq X \leq 370)$$

$$= 0.9547 \text{ correct to four decimal places.} \quad \mathbf{1A}$$



f. (0.7649, 0.8351) **1A**



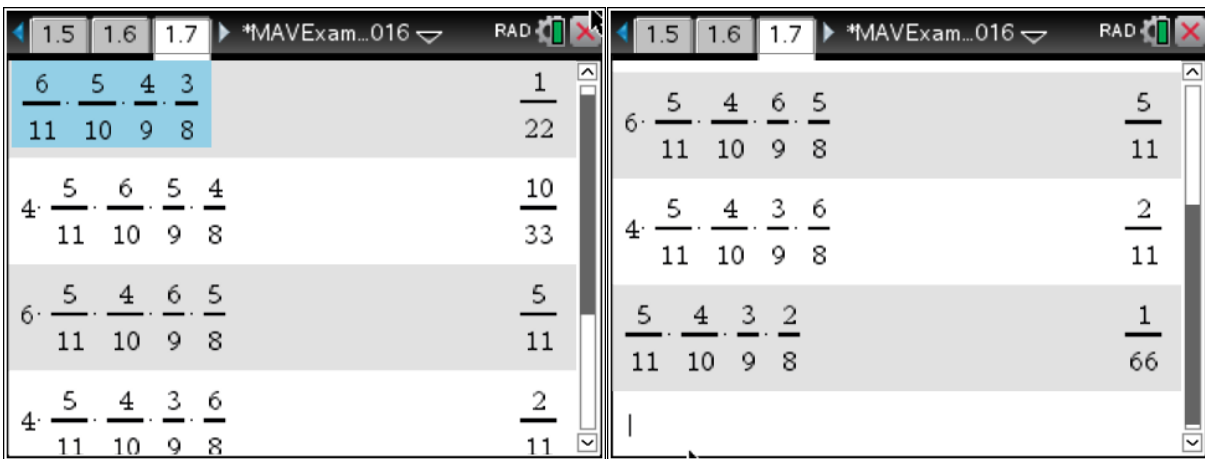
g.

Proportion of yellow balls \hat{p}	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$\Pr(\hat{P} = \hat{p})$	$\frac{1}{22}$	$\frac{10}{33}$	$\frac{5}{11}$	$\frac{2}{11}$	$\frac{1}{66}$

Without replacement

$$\frac{6}{11} \times \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{22} \quad \mathbf{1A}$$

$$4 \times \frac{5}{11} \times \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{10}{33} \quad \mathbf{1A}$$



h. $\Pr(\hat{p} = 1 | \hat{p} > \frac{1}{2}) = \frac{\Pr(\hat{p} = 1 \cap \hat{p} > \frac{1}{2})}{\Pr(\hat{p} > \frac{1}{2})}$ 1M

$$= \frac{1}{\frac{66}{13}} = \frac{1}{13}$$

1A

i. $\Pr(X > 200) = 0.1, \Pr(X < 185) = 0.05$ 1M

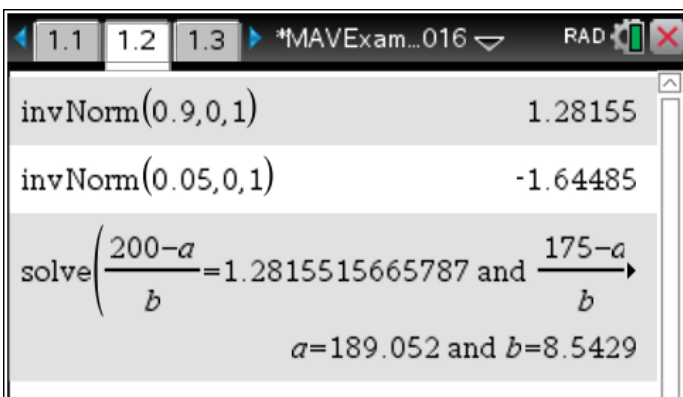
$$\Pr\left(Z > \frac{200 - \mu}{\sigma}\right) = 0.1, \Pr\left(Z < \frac{175 - \mu}{\sigma}\right) = 0.05$$

$$\frac{200 - \mu}{\sigma} = 1.2815\dots, \frac{175 - \mu}{\sigma} = -1.6448\dots$$

1M

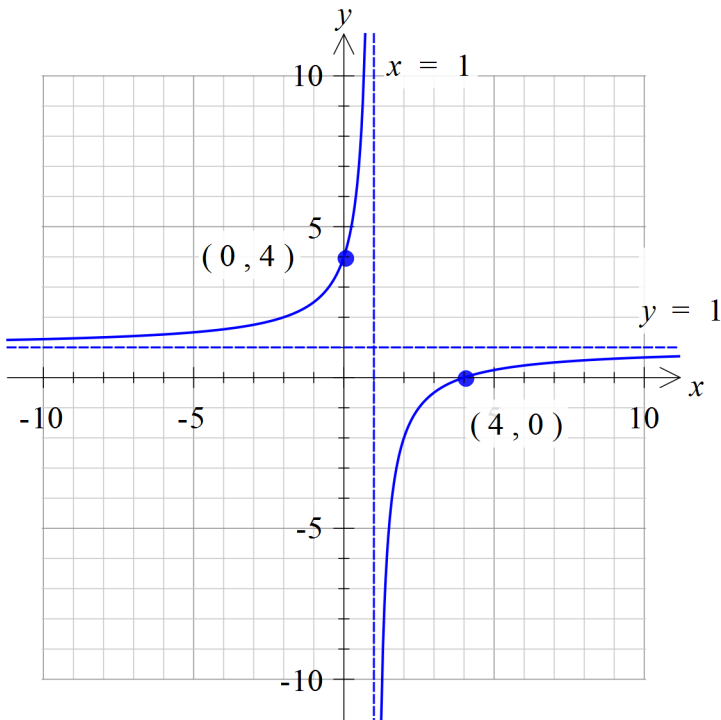
$$\mu = 189 \text{ days}, \sigma = 9 \text{ days}$$

1A



Question 4 (14 marks)

- a. shape **1A**
- asymptotes with equations **1A**
- coordinates of axial-intercepts **1A**



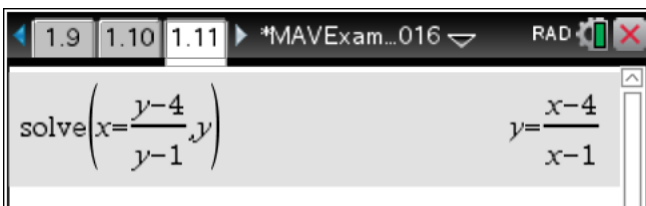
b. Let $y = \frac{x-4}{x-1}$

Inverse swap x and y .

Solve $x = \frac{y-4}{y-1}$ for y .

$$f^{-1}(x) = \frac{x-4}{x-1}$$

Hence $f(x) = f^{-1}(x)$ **1A**



c. $g : R \setminus \{-n\} \rightarrow R, g(x) = \frac{x+m}{x+n}$

$$g^{-1}(x) = \frac{-nx+m}{x-1}$$

$n = -1$ **1A**

$m \in R \setminus \{-1, 0\}$ **1A**

OR

$$g(x) = 1 + \frac{m+n}{x+n}$$

Horizontal asymptote at $y = 1$

Vertical asymptote at $x = 1$

$$n = -1 \quad \mathbf{1A}$$

$$m \in \mathbb{R} \setminus \{-1, 0\} \quad \mathbf{1A}$$

$$\mathbf{d.} \quad h: \mathbb{R} \setminus \left\{ -\frac{b}{n} \right\} \rightarrow \mathbb{R}, h(x) = \frac{ax + m}{bx + n}$$

$$h^{-1}(x) = \frac{-nx + m}{bx - a}$$

$$ax + m = -nx + m, \quad bx - a = bx + n$$

$$a = -n$$

1M Show that

$$\mathbf{e.} \quad h_1: \mathbb{R} \setminus \left\{ -\frac{n}{4} \right\} \rightarrow \mathbb{R}, h_1(x) = \frac{3x + 2}{4x + n}$$

$$n = -3$$

1A

$$\mathbf{f.} \quad w: \mathbb{R} \rightarrow \mathbb{R}, w(x) = -(2x - 1)^3 + \frac{1}{2}$$

$$w^{-1}(x) = \frac{1}{2} \left(\sqrt[3]{\left(\frac{1}{2} - x \right)} + 1 \right)$$

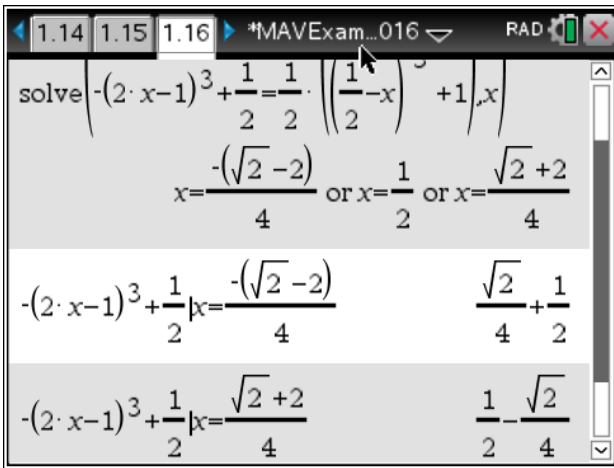
Solve $w(x) = w^{-1}(x)$ for x .

1M

$$x = \frac{2 - \sqrt{2}}{4}, x = \frac{1}{2} \text{ or } x = \frac{2 + \sqrt{2}}{4}$$

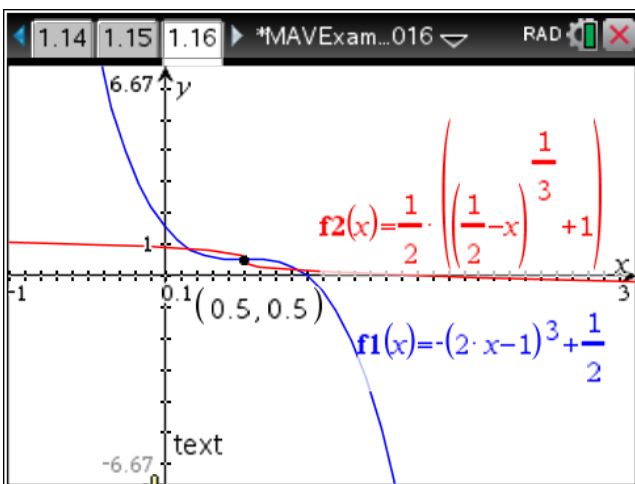
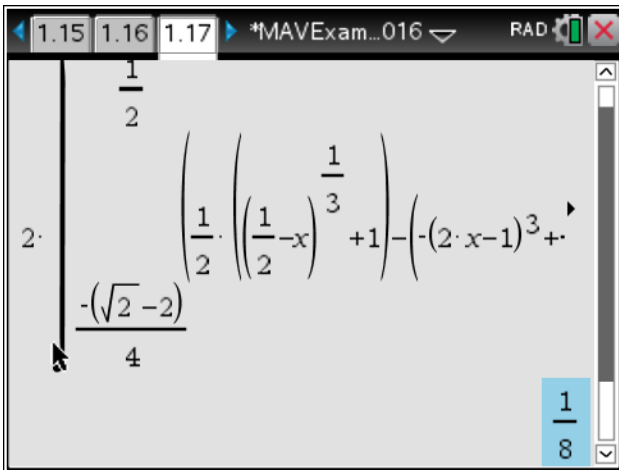
1A

$$\left(\frac{2 - \sqrt{2}}{4}, \frac{2 + \sqrt{2}}{4} \right), \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{2 + \sqrt{2}}{4}, \frac{2 - \sqrt{2}}{4} \right) \quad \mathbf{1H}$$



g. Area = $2 \int_{\frac{2-\sqrt{2}}{4}}^{\frac{1}{2}} \left(\frac{1}{2} \left(\sqrt[3]{\frac{1-x}{2}} + 1 \right) - \left(-(2x-1)^3 + \frac{1}{2} \right) \right) dx$ **1M**

$= \frac{1}{8}$ **1A**



h. $w_k : R \rightarrow R, w_k(x) = -(rx-1)^3 + s$

$s = \frac{1}{r}$ **1A**