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MATHS METHODS 3 & 4
TRIAL EXAMINATION 2
SOLUTIONS
2016

SECTION A – Multiple-choice answers

1.	**C**	**9.**	**C**	**17.**	**E**
2.	**A**	**10.**	**D**	**18.**	**B**
3.	**D**	**11.**	**E**	**19.**	**D**
4.	**A**	**12.**	**C**	**20.**	**B**
5.	**A**	**13.**	**C**		
6.	**C**	**14.**	**B**		
7.	**E**	**15.**	**C**		
8.	**D**	**16.**	**D**		

SECTION A – Multiple-choice solutions

Question 1

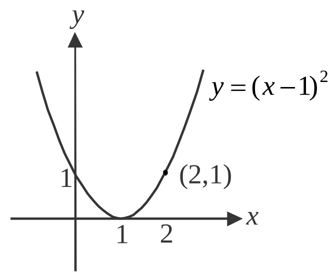
$$\text{period} = \frac{\pi}{n} \quad \text{where } n = \frac{3\pi}{4}$$
$$= \pi \div \frac{3\pi}{4}$$
$$= \frac{4}{3}$$

The answer is C.

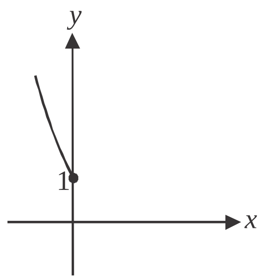
Question 2

Sketch the graph of $y = (x-1)^2$.

The range of $y = (x-1)^2$ is $y \in [0, \infty)$.



For the function *f*, the range is restricted to $[1, \infty)$.
The graph of *f* could therefore be



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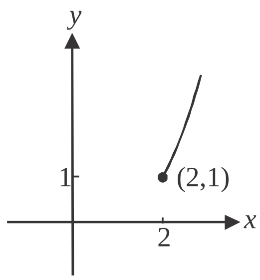
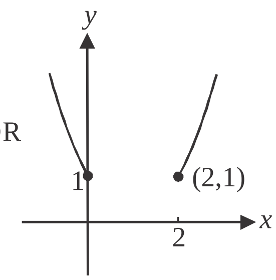


OR



So possible domains of *f* are $x \in (-\infty, 0]$ or $x \in [2, \infty)$ or $x \in (-\infty, 0] \cup [2, \infty)$.

Only the first domain is offered.

The answer is A.

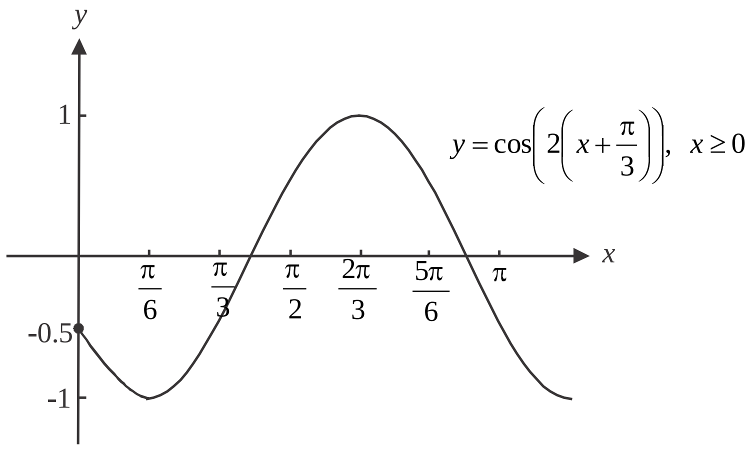
Question 3
The graph of g is an upright quartic which touches the origin so we need a positive x^2 term.
This eliminates options A, B and C.
Note that a must be a negative number since the point $(a,0)$ lies to the left of the origin.
Intuitively, we think that we need an $(x+a)$ factor but because a is a negative number, for example –1, this factor would become $(x-1)$ which is incorrect.
So we need an $(x-a)$ factor which eliminates option E.
The answer is D.

Question 4

We have $f:[0,a]\to R,\ \ f(x)=\cos\left(2\left(x+\frac{\pi}{3}\right)\right)$.

The inverse function f^{-1} exists if the function f is 1:1.

Sketch the graph of $y=\cos\left(2\left(x+\frac{\pi}{3}\right)\right),\ x\ge 0$.



Looking at this graph, we see that it is a 1:1 function for $x\in\left[0,\frac{\pi}{6}\right]$. (It's actually a 1:1 function for smaller intervals such as $x\in\left[0,\frac{\pi}{12}\right]$ etc but we want the largest interval possible ie. we want the maximum value of a.)
So if the function f with rule $f(x)=\cos\left(2\left(x+\frac{\pi}{3}\right)\right)$ is to have an inverse, then the largest possible value of a is $\frac{\pi}{6}$.
The answer is A.

Question 5

$$y=\log_e(ax),\quad a>0$$

$$\frac{dy}{dx}=\frac{a}{ax}=\frac{1}{x}$$

When $x=\frac{1}{a},\ y=\log_e\left(a\times\frac{1}{a}\right)=\log_e(1)=0$

At $\left(\frac{1}{a},0\right),\quad \frac{dy}{dx}=\frac{1}{1/a}=a$

Equation of tangent is $y-0=a\left(x-\frac{1}{a}\right)$

$$y=ax-1$$

y-intercept occurs when $x=0$ $\quad y=-1.$

The answer is A.

Question 6

A function is strictly decreasing over an interval if $f(x_2) < f(x_1)$ where $x_2 > x_1$.

The largest interval for which this is true is $\boldsymbol{x \in [-p, q]}$.

The answer is C.

Question 7

$g:[1,\infty) \to R,\ g(x) = \sqrt{x-1} + 2$

Let $\boldsymbol{y = \sqrt{x-1} + 2}$

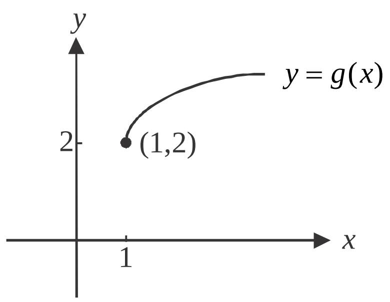
Swap x and y for inverse

$$x = \sqrt{y-1} + 2$$
$$x - 2 = \sqrt{y-1}$$
$$(x-2)^2 = y - 1$$
$$y = (x-2)^2 + 1$$
$$g^{-1}(x) = (x-2)^2 + 1$$

$d_g = [1,\infty)$

$r_g = [2,\infty)$ (from the graph)

So $d_{g^{-1}} = [2,\infty)$ because $d_{g^{-1}} = r_g$.



The inverse function is $g^{-1}:[2,\infty) \to R,\ g^{-1}(x) = (x-2)^2 + 1$.

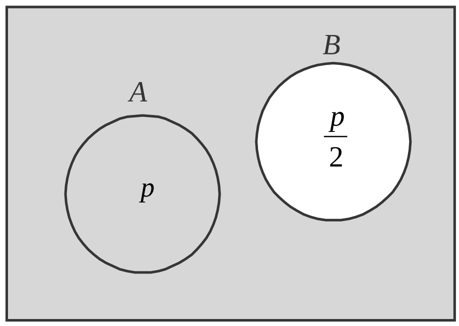
The answer is E.

Question 8

Draw a Venn diagram.

$\Pr(A \cup B')$ is shaded.

$$\Pr(A \cup B') = 1 - \Pr(B)$$
$$= 1 - \frac{p}{2}$$
$$= \frac{2-p}{2}$$



The answer is D.

Question 9

$\boldsymbol{f(x) = e^x - 2x}$

Method 1 – using CAS

$$\text{average rate of change} = \frac{f(1) - f(0)}{1-0} \quad \text{for } x \in [0,1]$$
$$= e - 3$$

Note that we are asked for the average rate of change and NOT the average value.

The answer is C.

Method 2 – by hand

$$\text{average rate of change} = \frac{f(1) - f(0)}{1-0} \quad \text{for } x \in [0,1]$$
$$= \frac{e-2-1}{1-0} \quad \text{since } f(1) = e - 2 \text{ and } f(0) = 1$$
$$= e - 3$$

Note that we are asked for the average rate of change and NOT the average value.

The answer is C.

Question 10

<u>Method 1</u>

$\mu = 14, \ \sigma = 2.5$ and $z = \dfrac{x-\mu}{\sigma}$.

When $x = 9$, $\quad z = \dfrac{9-14}{2.5} = -2$ and when $x = 16.5$, $\quad z = \dfrac{16.5-14}{2.5} = 1$.

So $\quad \Pr(9 < X < 16.5) = \Pr(-2 < Z < 1)$.

The answer is D.

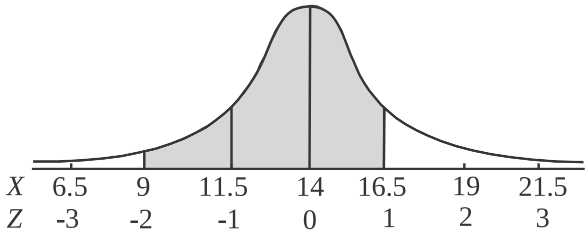
<u>Method 2</u>

Do a quick sketch.

$\Pr(9 < X < 16.5)$ is shaded.

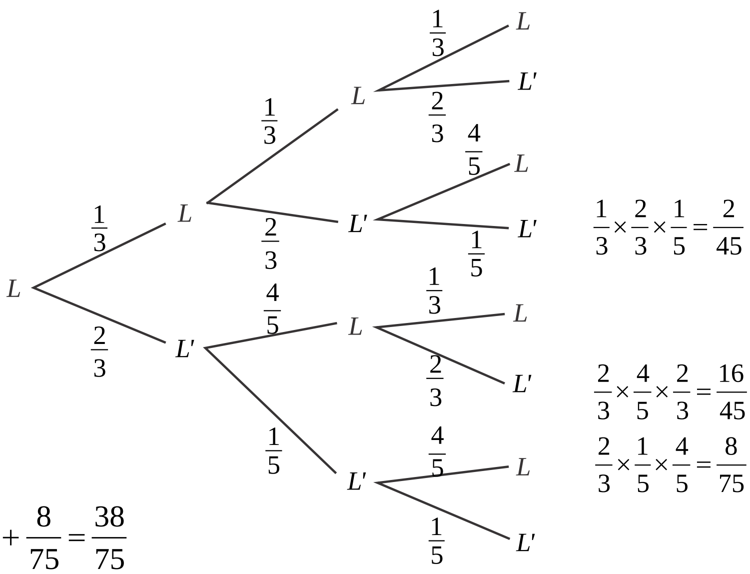
It is equal to $\Pr(-2 < Z < 1)$.

The answer is D.



Question 11

<u>Method 1</u> – using a tree diagram



Required probability $= \dfrac{2}{45} + \dfrac{16}{45} + \dfrac{8}{75} = \dfrac{38}{75}$

The answer is E.

<u>Method 2</u>

Pr(Leonie makes the call exactly once)

$= \Pr(L, L', L') + \Pr(L', L, L') + \Pr(L', L', L)$

$= \dfrac{1}{3}\times\dfrac{2}{3}\times\dfrac{1}{5} + \dfrac{2}{3}\times\dfrac{4}{5}\times\dfrac{2}{3} + \dfrac{2}{3}\times\dfrac{1}{5}\times\dfrac{4}{5}$

$= \dfrac{2}{45} + \dfrac{16}{45} + \dfrac{8}{75}$

$= \dfrac{38}{75}$

The answer is E.

Question 12

Let X represent the number of employees in the sample who are parents. X is a random variable with a binomial distribution where $n = 30$ **and** $p = 0.54$

$\Pr(X < 15) = 0.266093...$ $\qquad$ (i.e. binom Cdf (30,0.54,0,14))

Note that we want $X < 15$ so the upper bound is 14. The closest answer is 0.2661.

The answer is C.

Question 13

$$\int_0^3 (a-4f(x))\,dx = 7$$

$$\int_0^3 a\,dx - 4\int_0^3 f(x)\,dx = 7 \quad \text{(properties of antiderivatives)}$$

$$[ax]_0^3 - 4\times 2 = 7 \text{ because } \int_0^3 f(x)dx = 2 \quad \text{(given in the question)}$$

$$(3a-0)-8=7$$

$$3a=15$$

$$a=5$$

The answer is C.

Question 14

$$ax+3y=a-3$$

$$2x+(a+1)y=-1$$

Express this system as a matrix equation.

$$\begin{bmatrix} a & 3 \\ 2 & a+1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a-3 \\ -1 \end{bmatrix}$$

For no solutions or infinite solutions

$$a(a+1)-6=0$$

$$a^2+a-6=0$$

$$(a+3)(a-2)=0$$

$$a=-3 \text{ or } a=2$$

Test both these values of a.

If $a=-3$, we have $-3x+3y=-6 \quad -(1)$

$$2x-2y=-1 \quad -(2)$$

$(1)\div{}^-3 \qquad x-y=2$

$(2)\div 2 \qquad x-y=-\frac{1}{2}$

So for $a=-3$ there are no solutions.

If $a=2$, we have $2x+3y=-1$

$$2x+3y=-1$$

So for $a=2$ there are infinite solutions.

We require no solutions so $a=-3$.

The answer is B.

Question 15

$g(x)=ax^3+2bx^2+x+5.$ Stationary points occur when $g'(x)=0$

$g'(x)=3ax^2+4bx+1=0$

Note that the rule for $g'(x)$ is a quadratic function so we use our knowledge of the discriminant. For more then one solution, (i.e. 2 solutions and hence 2 stationary points)

$$(4b)^2-4\times 3a\times 1>0$$

$$16b^2-12a>0$$

$$a<\frac{4b^2}{3}$$

The answer is C.

Question 16

An approximate 95% confidence interval for p is given by

$\hat{p}-z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p}+z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ (formula sheet) where $\hat{p}=0.53,\ z=1.96$ and $n=820$.

We require $0.53-1.96\sqrt{\frac{0.53\times 0.47}{820}} < p < 0.53+1.96\sqrt{\frac{0.53\times 0.47}{820}}$

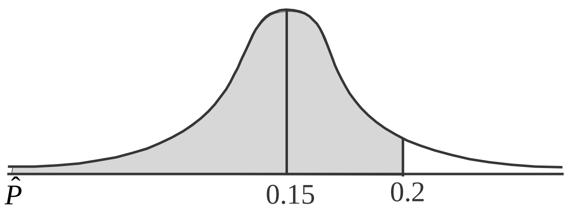
The answer is D.

Question 17

A normal approximation is used because n is large.
For this normal distribution mean $= p = 0.15$ and

$$\begin{aligned}\text{standard deviation} &= \sqrt{\frac{p(1-p)}{n}}\\ &= \sqrt{\frac{0.15\times 0.85}{300}}\\ &= 0.0206155...\end{aligned}$$

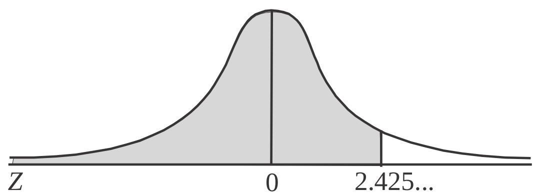
<u>Method 1</u>
Using normCdf$(-\infty, 0.2, 0.15, 0.0206155...)$
$\Pr(\hat{P} < 0.2) = 0.992353...$
The closest answer is 0.99235.
The answer is E.



<u>Method 2</u>

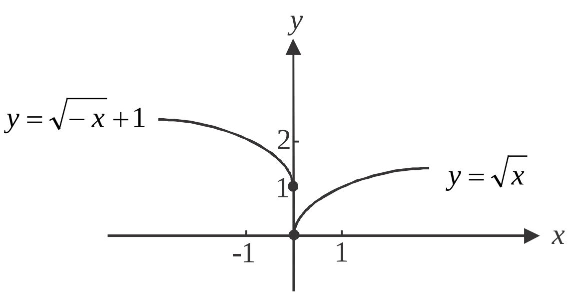
$$\begin{aligned}z &= \frac{0.2-0.15}{0.0206155...}\\ &= 2.42535...\end{aligned}$$

Using normCdf$(-\infty, 2.425..., 0, 1)$
$\Pr(Z < 2.42535...) = 0.992353...$
The closest answer is 0.99235.
The answer is E.



Question 18

<u>Method 1</u> – graphically
The transformation described by the matrix equation involves a reflection in the y-axis and a translation 1 unit down.
A function that could undergo this in order to become the graph $y=\sqrt{x}$ is shown below.



The graph has rule $y=\sqrt{-x}+1$.
The answer is B.

Method 2 – using matrices

Let the image point be (x', y').

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$x' = -x \qquad y' = y - 1$

The mapped function is $y' = \sqrt{x'}$

so $\qquad y - 1 = \sqrt{-x}$

$\qquad y = \sqrt{-x} + 1$

so $f(x) = \sqrt{-x} + 1$

The answer is B.

Question 19

Method 1

Using your CAS substitute $x+2$ into each of the expressions.

For option A, $\quad (x+2)^2 - 2(x+2) = x^2 + 2x$

For option B, $\quad (x+2)^2 + (x+2) = x^2 + 5x + 6$

For option C, $\quad (x+2)^2 - (x+2) - 3 = x^2 + 3x - 1$

For option D, $\quad (x+2)^2 - (x+2) + 5 = x^2 + 3x + 7$

The answer is D.

Method 2

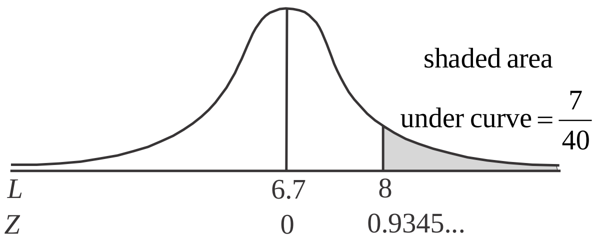
$$\begin{aligned} g(x+2) &= x^2 + 3x + 7 \\ &= (x^2 + 3x + \frac{9}{4}) - \frac{9}{4} + 7 \qquad \text{(completing the square)} \\ &= (x + \frac{3}{2})^2 + \frac{19}{4} \end{aligned}$$

$$\begin{aligned} \text{So } g(x) &= (x - 2 + \frac{3}{2})^2 + \frac{19}{4} \\ &= (x - \frac{1}{2})^2 + \frac{19}{4} \\ &= x^2 - x + 5 \end{aligned}$$

The answer is D.

Question 20

Draw a diagram.



$\Pr(L > 8) = \frac{1400}{8000}$

$= \frac{7}{40}$

So $\Pr(L < 8) = \frac{33}{40}$

Using $\text{invNorm}\left(\frac{33}{40}, 0, 1\right)$ we have $\Pr(L < 8) = \Pr(Z < 0.9345...)$

So, $0.9345... = \frac{8 - 6.7}{\sigma}$

$\sigma = \frac{8 - 6.7}{0.9345...} = 1.3909...$

The closest answer is 1.39

The answer is B.

SECTION B

Question 1 (8 marks)

a. a is the y-intercept of the graph of g.

<u>y-intercept</u> occurs when $x = 0$

$$g(0) = \frac{1}{5}(-1)(-5)$$
$$= 1$$
$$\text{So } a = 1$$

(1 mark)

b. **i.** <u>Method 1</u> – using only CAS

The derivative of g at the point where $x = 4$ is given by $g'(4) = \frac{7}{5}$. **(1 mark)**

<u>Method 2</u> – by hand (and a little CAS)

$$g(x) = \frac{1}{5}(x^2 - 1)(x - 5)$$
$$= \frac{1}{5}(x^3 - 5x^2 - x + 5)$$
$$g'(x) = \frac{1}{5}(3x^2 - 10x - 1)$$
$$g'(4) = \frac{1}{5}(3 \times 16 - 40 - 1)$$
$$= \frac{7}{5}$$

(1 mark)

ii. Using the point $(4, -3)$

$$y - y_1 = m(x - x_1)$$
$$y - {}^-3 = \frac{7}{5}(x - 4)$$
$$y + 3 = \frac{7x}{5} - \frac{28}{5}$$
$$y = \frac{7x}{5} - \frac{43}{5}$$

(1 mark)

c. P is the point $(4, -3)$ and Q is the point $(0, 1)$.

distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (<u>not</u> on the formula sheet)

$= \sqrt{(4-0)^2 + (-3-1)^2}$ **(1 mark)**

$= \sqrt{32}$

$= 4\sqrt{2}$ **(1 mark)**

d. The graph of g and the tangent intersect when $g(x) = \frac{7x}{5} - \frac{43}{5}$.

Solve this equation for x using CAS. $x = -3$ or $x = 4$ **(1 mark)**

required area $= \int_{-3}^{4} \left(g(x) - \left(\frac{7x}{5} - \frac{43}{5} \right) \right) dx$ **(1 mark)**

$= \frac{2401}{60}$ square units

(1 mark)

Question 2 (12 marks)

a. Define f and g on your CAS.
Solve $f(t)=g(t)$ for t. **(1 mark)**
$t=6.54997...$ or $t=50.18649...$
So $t=6.550$ or $t=50.186$ (correct to three decimal places)
(1 mark)

b. Maximum value occurs when
$g'(t)=0$
Solve $g'(t)=0$ for t. **(1 mark)**
$t=20$
So $g(20)=13.3575...$
So the maximum value is \$13.36.
(1 mark)

c. $\text{average value}=\frac{1}{60-0}\int_0^{60} g(t)dt$ (<u>not</u> on the formula sheet) **(1 mark)**
$=11.33901\ldots$
Average value is \$11.34.
(1 mark)

d. Let $d(t)=g(t)-f(t)$, where d represents the difference in the value of the stocks.
(1 mark)
Solve $d'(t)=0$ for t **(1 mark)**
$t=21$ **(1 mark)**

Note that the period when the value of the Gold Inc. stock was greater than the value of the Foolsgold stock, is $t\in(6.550, 50.186)$ from part **a.** This answer satisfies that requirement.

e. **i.** <u>Method 1</u>
Solve $g'(t)=-e^{-2}$ for t. **(1 mark)**
$t=40$
So $p=40$. (Re-read the question!) **(1 mark)**

<u>Method 2</u>
Solve $g''(t)=0$ for t. **(1 mark)**
$t=40$
So $p=40$. (Re-read the question!) **(1 mark)**

ii. The rate at which the value of the Gold Inc. stock was decreasing was a maximum when $t=40$.
$g(40)=40e^{-2}+6$
$=11.4134...$
The value of Gold Inc. was \$11.41 at that time.
(1 mark)

Question 3 (17 marks)

a. X follows a binomial distribution with $n = 15$ and $p = \frac{2}{9}$. **(1 mark)**

Using $\text{binomCdf}(15, \frac{2}{9}, 0, 5)$,

$\Pr(X \le 5) = 0.90581...$

$= 0.906$ (correct to 3 decimal places).

(1 mark)

b. **i.** expected value of $\hat{P} = E(\hat{P})$

$= p$ (the population proportion)

$= \frac{2}{9}$

(1 mark)

ii. standard deviation of $\hat{P} = \sqrt{\frac{p(1-p)}{n}}$

$= \sqrt{\frac{\frac{2}{9} \times \frac{7}{9}}{15}}$ **(1 mark)**

$= \frac{\sqrt{210}}{135}$

(1 mark)

c. Solve $\sqrt{\frac{\frac{2}{9} \times \frac{7}{9}}{n}} < 0.1$ for n. **(1 mark)**

$n > 17.283….$

Since n must be an integer (i.e. we are dealing in whole numbers) the minimum number of customers needed in each sample is 18.

(1 mark)

d. The population proportion is $\frac{2}{9}$.

We want $\Pr\left(\frac{2}{9} - \frac{\sqrt{210}}{135} \le \hat{P} \le \frac{2}{9} + \frac{\sqrt{210}}{135}\right)$ **(1 mark)**

$= \Pr(0.11487... \le \hat{P} \le 0.32956...)$

$= \Pr(0.11487... \times 15 \le X \le 0.32956... \times 15)$ **(1 mark)**

$= \Pr(1.72318... \le X \le 4.94348...)$

$= \Pr(2 \le X \le 4)$ since X must take integer values

$= 0.65209...$ (using $\text{binomCdf}(15, \frac{2}{9}, 2, 4)$)

$= 0.652$ (correct to 3 decimal places)

(1 mark)

e. **i.** $\text{E}(T)=\int_2^{15} t\,f(t)\,dt$ **(1 mark)**

$=9.00498...$

$=9.005$ minutes (correct to 3 decimal places)

(1 mark)

ii. $\Pr(T>10)=\int_{10}^{15} f(t)\,dt$ **(1 mark)**

$=0.421391...$

$=0.421$ (correct to 3 decimal places)

(1 mark)

iii. $\Pr(T>12\,|\,T>10)=\dfrac{\Pr(T>12\cap T>10)}{\Pr(T>10)}$ (conditional probability formula)

$=\dfrac{\Pr(T>12)}{\Pr(T>10)}$ **(1 mark)**

$=\dfrac{\int_{12}^{15} f(t)\,dt}{\Pr(T>10)}$

$=\dfrac{0.239963...}{0.421391...}$ (from part ii)

$=0.5694545...$

$=0.569$ (correct to 3 decimal places)

(1 mark)

f. <u>Method 1</u> – using CAS

Using 1–Prop *z* interval, with $x=1860$, $n=2000$ and $C\text{ Level}=0.90$, the 90% confidence interval for *p* is (0.920615…, 0.939384…) or (0.921, 0.939) where values are given correct to 3 decimal places.

(1 mark)

<u>Method 2</u> – by hand (and a little CAS)

An approximate 90% confidence interval for *p* is given by

$$\left(\hat{p}-1.64\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+1.64\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

where $\hat{p}=\dfrac{1860}{2000}$ (the sample proportion) and $n=2000$

$=0.93$

The approximate 90% confidence interval for *p* is therefore

$$\left(0.93-1.64\sqrt{\frac{0.93\times0.07}{2000}},\ 0.93+1.64\sqrt{\frac{0.93\times0.07}{2000}}\right)$$

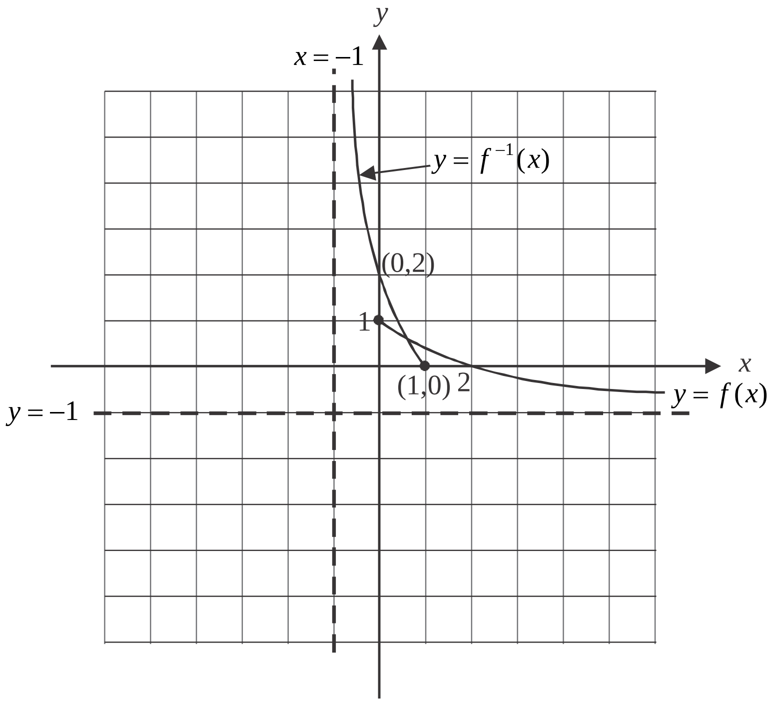
$=(0.920643...,\ 0.939356...)$

$=(0.921,\ 0.939)$ (values correct to 3 decimal places)

(1 mark)

Question 4 (8 marks)

a.



(1 mark) – correct shape including endpoint at (1,0) and y-intercept at (0,2)
(1 mark) – correct asymptote and equation

b. Method 1 - algebraically

$$f(x)=\frac{4}{x+2}-1$$

Let $y=\frac{4}{x+2}-1$

- dilation by a factor of 2 units from the y-axis means replace x with $\frac{x}{2}$

So we have $y=\frac{4}{x/2+2}-1$

so $y=\frac{8}{x+4}-1$ **(1 mark)**

- reflection in the x-axis means replace y with $-y$

So we have $-y=\frac{8}{x+4}-1$

so $y=1-\frac{8}{x+4}$

- translation of 3 units vertically upwards means add 3

So we have $y=1-\frac{8}{x+4}+3$

so $y=4-\frac{8}{x+4}$

So $h(x)=4-\frac{8}{x+4}$.

(1 mark)

Method 2 – using matrices
Let (x', y') be the image point.

So, $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ **(1 mark)**

$$= \begin{bmatrix} 2x \\ -y+3 \end{bmatrix}$$

$x' = 2x \qquad y' = -y+3$

$x = \frac{x'}{2} \qquad y = 3 - y'$

$$y = \frac{4}{x+2} - 1$$

becomes $3 - y' = \dfrac{4}{\frac{x'}{2}+2} - 1$

$$-y' = \frac{8}{x'+4} - 1 - 3$$

$$y' = 4 - \frac{8}{x'+4}$$

So $\qquad h(x) = 4 - \dfrac{8}{x+4}$.

(1 mark)

c. $\boldsymbol{q(x) = \dfrac{a}{x+2} - 1}$

Let $\quad y = \dfrac{a}{x+2} - 1$

<u>x-intercept</u> occurs when $\boldsymbol{y = 0}$

$$\boldsymbol{0 = \frac{a}{x+2} - 1}$$

$$\boldsymbol{x + 2 = a}$$

$$\boldsymbol{x = a - 2}$$

x-intercept is $(a-2, 0)$

(1 mark)

d. A quick sketch shows the area required which has been shaded.

area = $\displaystyle\int_0^{a-2} q(x)\,dx$ **(1 mark)**

Using CAS, we have

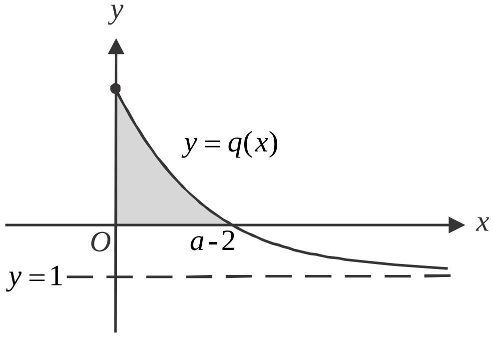
area $= a\log_e(a) + a(-\log_e(2) - 1) + 2, \quad a \ge 2$ **(1 mark)**

$= \log_e(a^a) - a\log_e(2) - a + 2$

$= \log_e(a^a) - \log_e(2^a) - a + 2$

$= \log_e\left(\dfrac{a^a}{2^a}\right) - a + 2$ **(1 mark)**

as required



Question 5 (15 marks)

a. Since $v(x)=3\sin(\pi ax)$, the amplitude of the function is 3 so the maximum distance below the surface of the water that a dolphin missile can reach is 3 metres. **(1 mark)**

b. Victoria is at the point (2,0) so the missile entered the water at the point (1.8, 0). This represents one and a half periods of the graph. So one period is 1.2.

For $v(x)=3\sin(\pi ax)$, the period is given by $\frac{2\pi}{n}=\frac{2\pi}{\pi a}=\frac{2}{a}$.

So $\frac{2}{a}=1.2$

$a=\frac{2}{1.2}$

$=\frac{5}{3}$ as required

(1 mark)

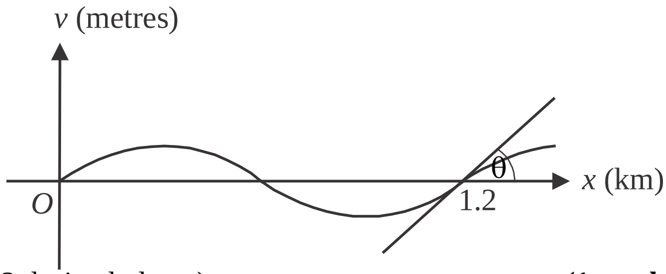
c. The acute angle between the horizontal and the tangent to the path of the first missile as it emerges from the water is the same each time it emerges from the water. Let's consider the second time it emerges from the water.

For the first missile $v(x)=3\sin\left(\frac{5\pi x}{3}\right)$ so $v'(x)=5\pi\cos\left(\frac{5\pi x}{3}\right)$.

The period is 1.2 so the missile emerges from the water at the point (1.2, 0).

At $x=1.2$,
$v'(x)=5\pi\cos(2\pi)=5\pi$
(1 mark)

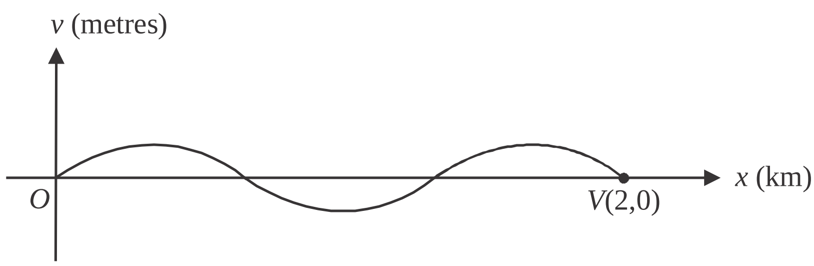
So $\tan(\theta)=5\pi$
$=\tan^{-1}(5\pi)$
$=86.35735...$



Required angle is 86.36° (to 2 decimal places). **(1 mark)**

If we had considered the first time it emerged from the water, we would have had the missile emerging from the water at (0, 0).
At $x=0$, $v'(x)=5\pi\cos(0)=5\pi$ and so on.

d. A missile would enter the water for the second time after completing 1.5 periods.



Since Victoria is at (2, 0), the period of the path of this missile would be $\frac{2}{3}\times 2=\frac{4}{3}$.

So $\frac{2\pi}{\pi a}=\frac{4}{3}$

$a=\frac{3}{2}$ **(1 mark)**

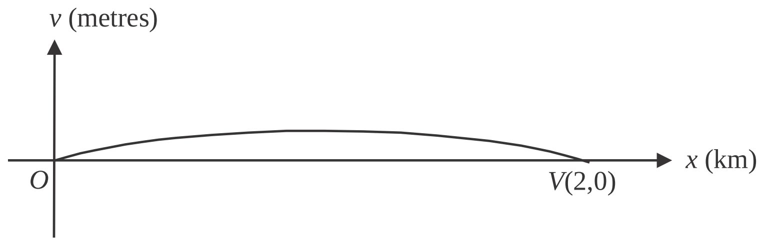
e. A missile which passes over Victoria at $V(2,0)$ before hitting the water for the first time will have a path with a period that is greater than 4 (so half a period will be greater than 2).

So we require $\text{period} = \frac{2}{a} > 4$

so $a < \frac{1}{2}$

Also, from the question, a is a positive constant so $a > 0$.

So we require $0 < a < \frac{1}{2}$.



(1 mark) – correct left endpoint **(1 mark)** – correct right endpoint

f. Since Victoria's speed is 2km/h when she is 2km from the ship, then $s(2) = 2$.
Solve this equation for x.

$$(2+k)^2 - 2^{2+1} + 1 = 2$$
$$4 + 4k + k^2 - 2^3 + 1 = 2$$
$$k^2 + 4k - 5 = 0$$
$$(k+5)(k-1) = 0$$
$$k = -5 \text{ or } k = 1$$

but $k > 0$, so $k = 1$.

OR
$$(2+k)^2 - 2^{2+1} + 1 = 2$$
$$(2+k)^2 = 9$$
$$2 + k = \pm 3$$
$$k = -2 \pm 3$$
$$k = -5 \text{ or } k = 1$$

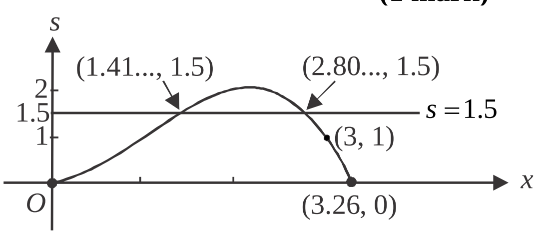
but $k > 0$ so $k = 1$ **(1 mark)**

g. d is Victoria's distance from the ship when she stops swimming.
Solve $s(x) = 0$ for x. **(1 mark)**
$x = -1$ or $x = 0$ or $x = 3.25746...$
The first solution is outside the domain. The second is when she is at the ship.
So $d = 3.26$ (correct to 2 decimal places). **(1 mark)**

h. **i.** Solve $s'(x) = 0$ for 0. **(1 mark)**
$x = -0.5149...$ or $x = 2.212432...$
The first solution is outside the domain.
Victoria is 2.2124km (correct to 4 decimal places) from the ship when her speed is a maximum. **(1 mark)**

ii. $s(2.212432...) = 2.050601...$
Her maximum speed is 2.051km/h (correct to 3 decimal places).
(1 mark)

i. Sketch the function s.
Solve $s(x) = 1.5$ for x.
$x = -2$ or $x = 1.4158...$ or $x = 2.8054...$
The first solution is outside the domain and the next two are when Victoria is too close to the ship (ie within 3 km).



When Victoria is 3km from the ship her speed is 1km/h which is an acceptable speed to detonate the bomb, ie she needs to be swimming at 1.5km/h or slower.
The left endpoint of the interval is therefore 3. This endpoint **is** included because Victoria can be travelling at 1km/h. The right endpoint is 3.26 (from part **g.**) but it **is not** included since at $s = 3.26$, Victoria has stopped swimming and she must detonate the bomb whilst she is still swimming.
The required interval is therefore $x \in [3, 3.26)$ or $3 \le x < 3.26$.

(1 mark) – correct left endpoint **(1 mark)** – correct right endpoint