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Free Exam

For 2016-2018 VCE Study Design

Engage Education Foundation

Units 3 and 4 Maths Methods (CAS): Exam 2

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 2 hours writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of marks
A	20	20	20
B	4	4	60
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers, calculators and a bound reference.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied:

- This question and answer booklet of 233 pages including a formula sheet.

Instructions:

- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Section A – Multiple-choice questions

Instructions

Answer all questions by circling your choice.
Choose the response that is correct or that best answers the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Questions

Question 1

The graph of a cubic function has a local maximum at $(m, 2)$ and a local minimum at $(n, -2)$. For what values of c does $f(x) + c = 0$ have 3 solutions?

A. $c > -2$
B. $c < -2$ or $c > 2$
C. $c < -2$
D. $-2 < c < 2$
E. $c < 2$

Question 2

$f(x) = \ln(x) + 2$. The average rate of change over the interval [1, 4] is:

A. $\frac{1}{4}$
B. $2\ln(2)$
C. 1
D. $\frac{2\ln(2)}{3}$
E. $\frac{3}{4}$

Question 3

$(-2k + 9)x - 6y = -2k - 4$
$6x + (-2k - 3)y = -7$

The simultaneous linear equations above will have an infinite number of solutions when k equals:

A. $\frac{3}{2}$
B. $\frac{2}{3}$
C. 1
D. -2
E. $\frac{-2}{3}$

Question 4

f is a function. The derivative of $\sin(f(x))$ is:

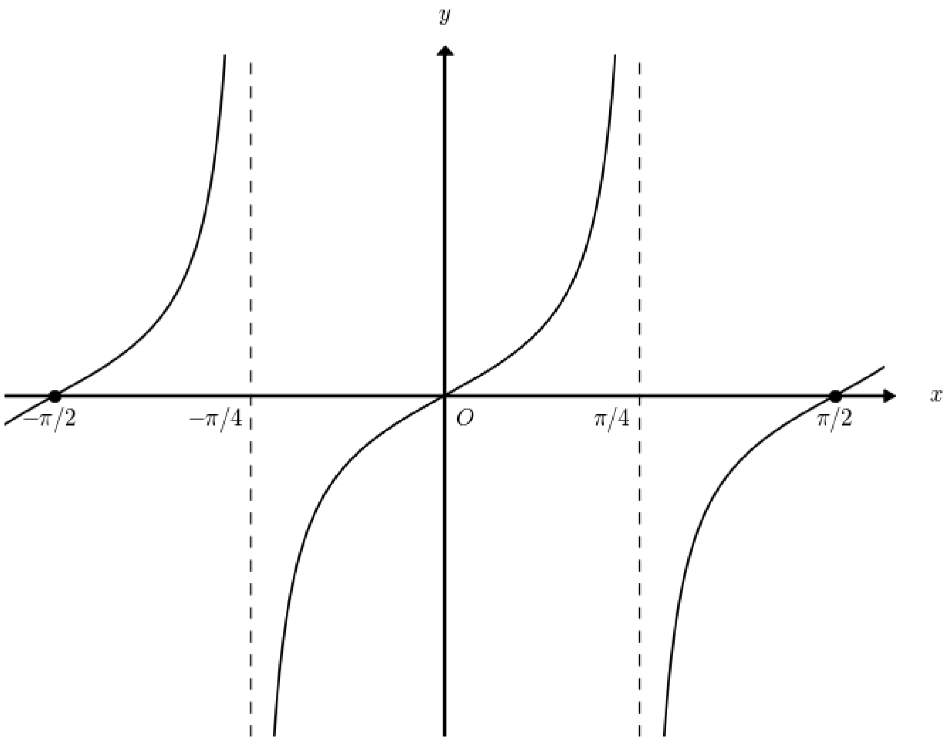
A. $f'(x)\sin(x)$
B. $f'(x)\sin(f(x))$
C. $f(x)\cos(f(x))$
D. $f'(x)\cos(f(x))$
E. $f'(x)\cos(x)$

Question 5

$f(x) = \ln\left(\sqrt{(x-3)^2} - 2\right) + 3$. The domain of f is:

A. (1, 5)
B. $(-\infty, 1) \cup (5, \infty)$
C. $(-\infty, 1] \cup [5, \infty)$
D. $(-\infty, 1) \cup (3, \infty)$
E. (−3, 3)

Question 6



Which of the following is the equation of the graph above?

A. $\tan(x)$
B. $\tan\left(2\left(x - \frac{\pi}{2}\right)\right)$
C. $\tan\left(x - \frac{\pi}{2}\right)$
D. $\tan\left(\frac{1}{2}x\right)$
E. $\tan\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$

Question 7

The voltage in a wire at hour x after midnight is given by $f(x) = 13 - 7\sin\frac{\pi}{12(x-3)}$. Which is closest to the average voltage between 9am and 9pm?

A. 7
B. 13
C. 6
D. 20
E. 15

Question 8

A function is monotonically increasing if its gradient is always greater than or equal to 0. Which of the following polynomials is monotonically increasing?

A. $\int(x+2)(x-2)dx$
B. $\int(x-1)(x-2)\,dx$
C. $\int(3(x+1)^2)\,dx$
D. $\int(-3(x+1)^2)\,dx$
E. $\int(-(x+1)(x+3))\,dx$

Question 9

The normal of the graph $y = e^{kx}$ has gradient of 2 when $x = 0$. Find k.

A. $k = -\frac{1}{2}$
B. $k = 2$
C. $k = \frac{1}{2}$
D. $k = -2$
E. $k = -1$

Question 10

The average value of $f(x) = \frac{3}{x}$ over [1, 10] is:

A. $\frac{1}{3}(\ln(10) - 1)$
B. $\frac{27}{10}$
C. $3\ln(10)$
D. $\frac{1}{3}\ln(10)$
E. $\frac{3}{10}$

Question 11

The weight of baskets is normally distributed with mean 900g and standard deviation 17g. The manufacturer says 10% are less than x grams. x is equal to:

A. 878.21
B. 860.45
C. 832
D. 872.04
E. 939.55

Question 12

A newspaper is surveying individuals to estimate the proportion of the population who support a particular politician. The minimum number of individuals the newspaper must survey to guarantee their answer is correct to within 1%, with 95% confidence is:

A. 6764
B. 9604
C. 27056
D. 38415
E. Cannot be determined as the width of the confidence interval depends on the proportion.

Question 13

The table below gives incomplete probabilities for the events A and B.

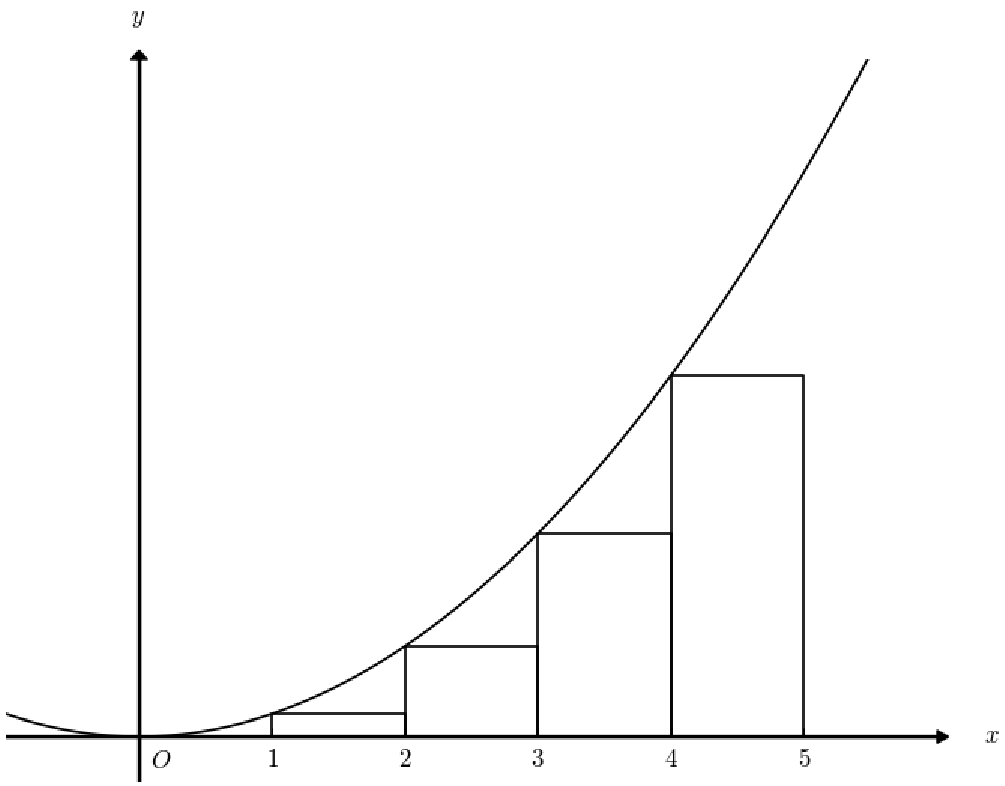
	A	A'	
B	1/7		1/6
B'			
	2/3		1

Hence, $Pr(A\cup B')$ is:

A. $\frac{6}{7}$
B. $\frac{41}{42}$
C. $\frac{11}{21}$
D. $\frac{10}{21}$
E. $\frac{13}{42}$

Question 14

The graph of $f(x) = x^2$ is shown below. In order to find an approximation to the area of the region bounded by the graph, the y-axis and the line $x = 5$, there have been 4 rectangles drawn as shown. Calculate the value of this approximation.

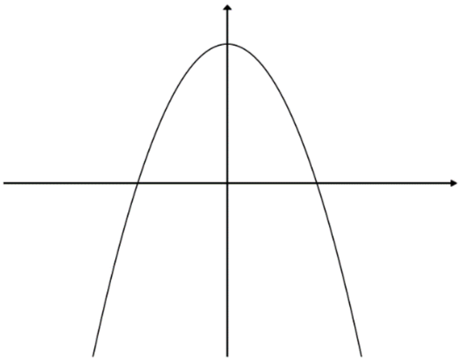


A. 30
B. 55
C. 29
D. $\frac{125}{3}$
E. 25

Question 15

$f'(x) = -x^2 + 4$. $f(x)$ could be:

A.



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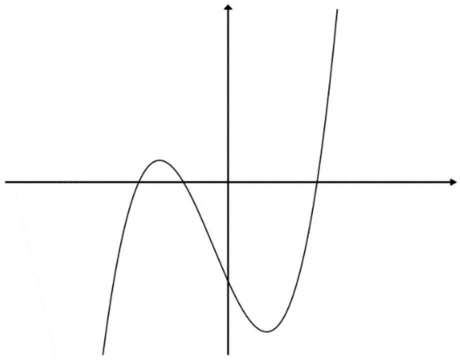

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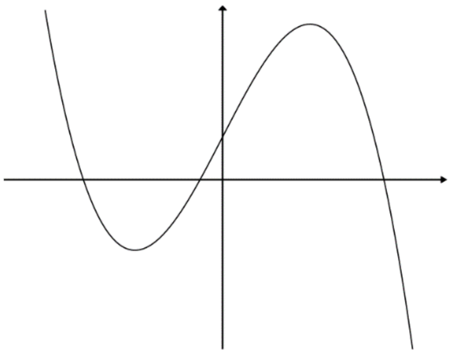

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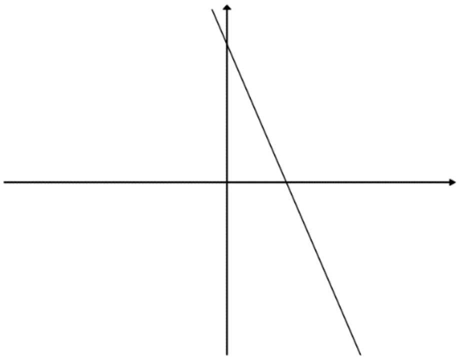

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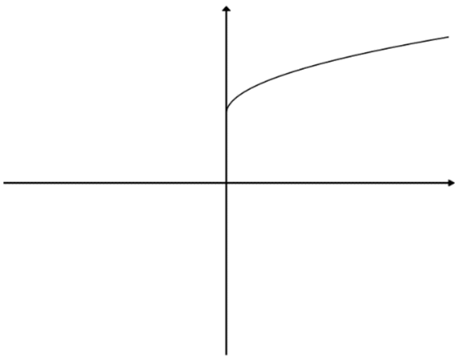


Question 16

The function $\frac{3\tan 2\pi x}{2} + 5$ has period:

A. $\frac{1}{2}$
B. 1
C. 2π
D. $\frac{\pi}{2}$
E. 3

Question 17

The range of $f(x) = x^2 - 8x + 12$ over [2, 8) is:

A. [-4,0]
B. [-4,-12]
C. [0,12)
D. [0,12]
E. [-4,12)

Question 18

Given that $f(x) = \sin(x)$ and $g(x) = \frac{1}{2}\sin(2x) - 3$ which of the following describes the true set of transformations from $f(x)$ to $g(x)$?

A. Dilation from the y-axis by factor 2, dilation from the x-axis by factor 2, translation in the negative y-direction by 3
B. Dilation from the y-axis by factor 1/2, dilation from the x-axis by factor 2, translation in the negative y-direction by 3
C. Dilation from the y-axis by factor 1/2, dilation from the x-axis by factor 1/2, translation in the positive y-direction by 3
D. Dilation from the y-axis by factor 1/2, dilation from the x-axis by factor 2, translation in the positive y-direction by 3
E. Dilation from the y-axis by factor 1/2, dilation from the x-axis by factor 1/2, translation in the negative y-direction by 3

Question 19

A function f has the property that for all real values of x: $f(x) = f(-x)$. The function could be:

A. $\sin(x)$
B. $\cos(x)$
C. $\tan(x)$
D. $\frac{x^3}{4}$
E. $\frac{x}{2}$

Question 20

A discrete random variable X has the probability function $\Pr(X = k) = \frac{2^k e^{-2}}{k!}, k = 0,1,2 \ldots$
What is $\Pr(X \geq 1)$?

A. $1 - e^{-2}$
B. e^{-2}
C. $2e^{-2}$
D. $3e^{-2}$
E. $1 - 2e^{-2}$

Section B – Short-answer questions

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

$f(x) = \frac{1}{2}\ln(2x + 3) + 3$

a. Sketch the graph of $f(x)$.

3 marks

b. Find:

i. the range and domain of f

1 mark

ii. $f'(x)$

1 mark

iii. Show that $f^{-1}(x) = \frac{e^{2(x-3)}-3}{2}$

2 marks

c. (a, b) is a point of intersection between $f(x)$ and $f^{-1}(x)$, where $a > 0$. Find a and b correct to four decimal places.

2 marks

d. Every point (p, q) on $f(x)$ has corresponding point is (q, p) on $f^{-1}(x)$.

i. Find q in terms of p.

1 mark

ii. Show that the equation of the line passing through (p, q) and (q, p) is $y = \frac{(p-q)}{(q-p)}(x-p)+q$.

2 marks

iii. Find the length of the line segment between (p, q) and (q, p) in terms of p.

2 marks

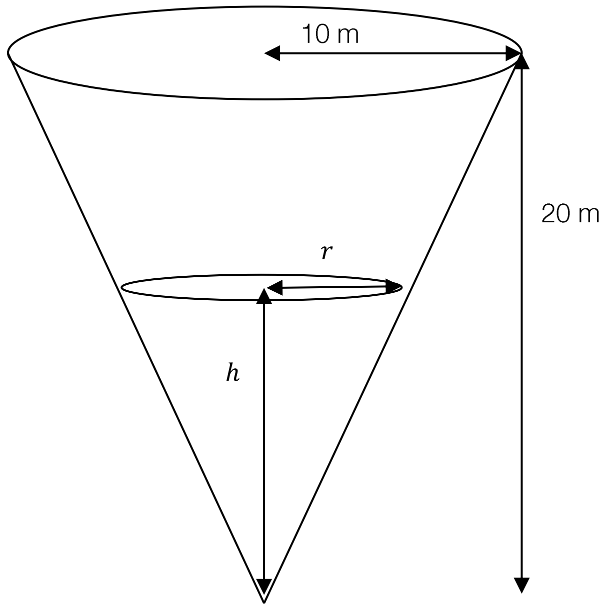
e. Given that $-1.4999 < \mathrm{p} < 4.2184$ find the value of p that maximises this length, correct to four decimal places (there is no need to find the length itself).

2 marks

Total: 16 marks

Question 2

There is an upside-down cone shape with dimensions (radius 10 m, height 20 m) housing a certain level of water inside. Let h be the height of the water level inside the cone, and r be the radius of the surface of the water.



a.

i. Find r in terms of h.

1 mark

ii. The volume of water in the cone is defined by a function of h. Find this function.

1 mark

b. If $h = 4$, find the rate at which the depth is decreasing.

1 mark

c. When the volume of water falls below 1 m^3, the tank must be refilled to the top. Given that the cone starts full - from the moment water starts flowing, how long will it take until the tank needs to be refilled? Give your answer correct to 2 decimal places.

2 marks

d. The tap at the bottom is closed so that no more water flows out until the tank is filled. Water is added to the tank such that the volume at time t hours is equal to $t^3 - t^2 + t + 1$ metres cubed.

i. Find the time t, correct to three decimal places, at which the tank will be full again.

1 mark

ii. Find the rate of volume increase, when the volume is increasing the slowest.

4 marks

e. ‘Down-time’ is when the tank is being refilled. What percentage of time is being spent as ‘down-time’? Give your answer correct to 2 decimal places.

1 mark

Total: 11 marks

Question 3

The amount of salt, S (in grams), on a dish in a restaurant is distributed normally with a mean of one gram and standard deviation of 0.05 grams.

a.

i. Find $\Pr(S \geq 1.10)$ correct to three decimal places.

1 mark

ii. The amount of pepper P (in grams) is distributed normally with mean a and standard deviation b. In this restaurant, $\Pr(P \geq 1.00) = \Pr(S \geq 1.10)$ and $\Pr(P \geq 1.10) = \Pr(S \geq 1.30)$. Find the exact values of a and b.

4 marks

b. When the amount of salt in a dish is below 1.10g, the dish is liked by all customers. However, when the amount of salt is 1.10g or above 40% of customers do not like the dish. We refer to dishes with 1.10g or more salt as being ‘salty dishes’. Let X be the random variable representing the number of people who do not like the dish.

Given on one night there have been 10 'salty dishes' served:

i. What is the probability, correct to four decimal places, that all 10 customers do not like the dish?

1 mark

ii. What is the probability, correct to four decimal places, that at least 2 customers do not like the dish?

2 marks

iii. What is the mean and variance of X?

2 marks

c. A food critic is attempting to estimate the proportion of dishes which are salty, by counting the number of customers that dislike the dish. If less than 3% of the dishes are salty, the restaurant will be added to the critic's 'SaltSafe' list.

Let s be the proportion of dishes which are salty, i.e. the proportion of dishes which have greater than 1.10g of salt. Let $\hat{D}$ be the proportion of people measured by the critic who dislike the dish.

i. Express $\hat{D}$ in terms of X, the number of people who dislike the dish, and n, the total number of people counted by the critic.

1 mark

ii. Write down the distribution of X, with parameters in terms of s and n.

1 mark

iii. Calculate the expected value and variance of $\hat{D}$ in terms of s and n.

2 marks

iv. The critic measured $\hat{d} = 0.0096$. Given that the critic believed that 50% of individuals receiving a salty dish disliked the dish, calculate the approximate 95% confidence interval for s obtained by the critic, in terms of n, with numbers correct to four decimal places.

2 marks

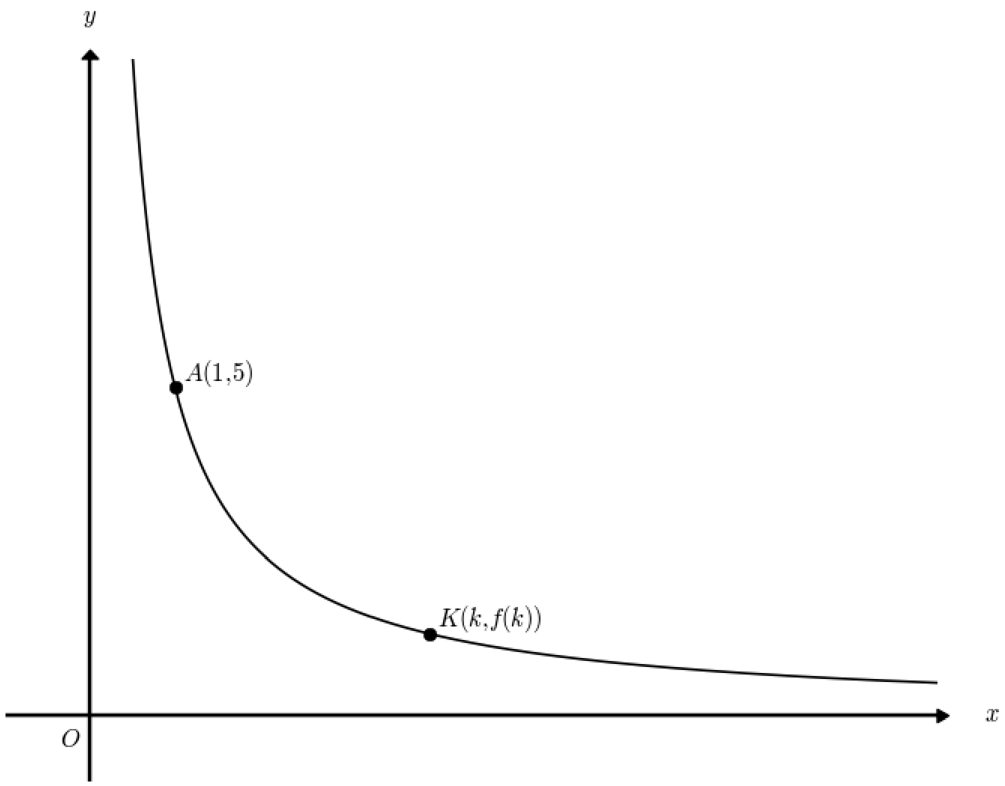
v. The critic only adds restaurants to their 'SaltSafe' list if the entire approximate 95% confidence interval for s is less than 3%. Given that the critic adds this restaurant to the list, calculate the minimum number of people counted.

1 mark

Total: 17 marks

Question 4

The following is a graph of $y = f(x) = \frac{5}{x}$. Consider the first quadrant ($x > 0, y > 0$) only.



a.

i. Calculate the gradient between the two points in terms of k.

1 mark

ii. At what value of x between 1 and k does the tangent to the graph of f have the same gradient as AK?

2 marks

b.

i. Evaluate the definite integral of f, between 1 and e^2.

1 mark

ii. Let a be a positive real number less than 1. Find the value of a such that the value of the definite integral of f between a and 1 is 10.

2 marks

c.

i. Find the area $A(k)$ bounded by the line segment AK, the lines $x = 1$, $x = k$, and the x-axis, in terms of k.

2 marks

ii. For what value of k does this area equal 10?

2 marks

iii. Using the value for k determined in part ii, explain in words, without evaluating the integral, why the definite integral of $f(x)$ between 1 and k is less than 10. Hence, use this result to explain why $k < e^2$.

1 mark

d.

i. Find the exact values of p and q such that $\int_1^{pq} f(x) = 6$ and $\int_1^{\frac{p}{q}} f(x) = 4$, given $p, q > 0$.

2 marks

e.

i. The line $y = n$ intersects $f(x)$. What is the equation of the tangent at this point?

2 marks

ii. For what value of n are the x and y-intercepts of the tangent the same value?

1 marks

Total: 16 marks

Formula Sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq-1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an\left(ax+b\right)^{n-1}$		$\int(ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq-1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}(\log_e(x))=\frac{1}{x}$		$\int\frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}(\sin(ax))=a\ \cos(ax)$		$\int\sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}(\cos(ax))=-a\ \sin(ax)$		$\int\cos(ax)dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}(\tan(ax))=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

End of Booklet

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