E

Free Exam
For 2016-2018 VCE Study Design

Engage
Education
Foundation

# Units 3 and 4 Maths Methods (CAS): Exam 2

**Practice Exam Solutions**

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

## Section A – Multiple-choice questions

Question 1
The correct answer is D.

Question 2
The correct answer is D.

Question 3
The correct answer is A.

Question 4
The correct answer is D.

Question 5
The correct answer is B.

Question 6
The correct answer is B.

Question 7
The correct answer is B.

Question 8
The correct answer is C.

Question 9
The correct answer is A.

Question 10
The correct answer is D.

Question 11
The correct answer is A.

Question 12
The correct answer is B.

Question 13
The correct answer is B.

Question 14
The correct answer is A.

Question 15
The correct answer is C.

Question 16
The correct answer is A.

Question 17
The correct answer is E.

Question 18
The correct answer is E.

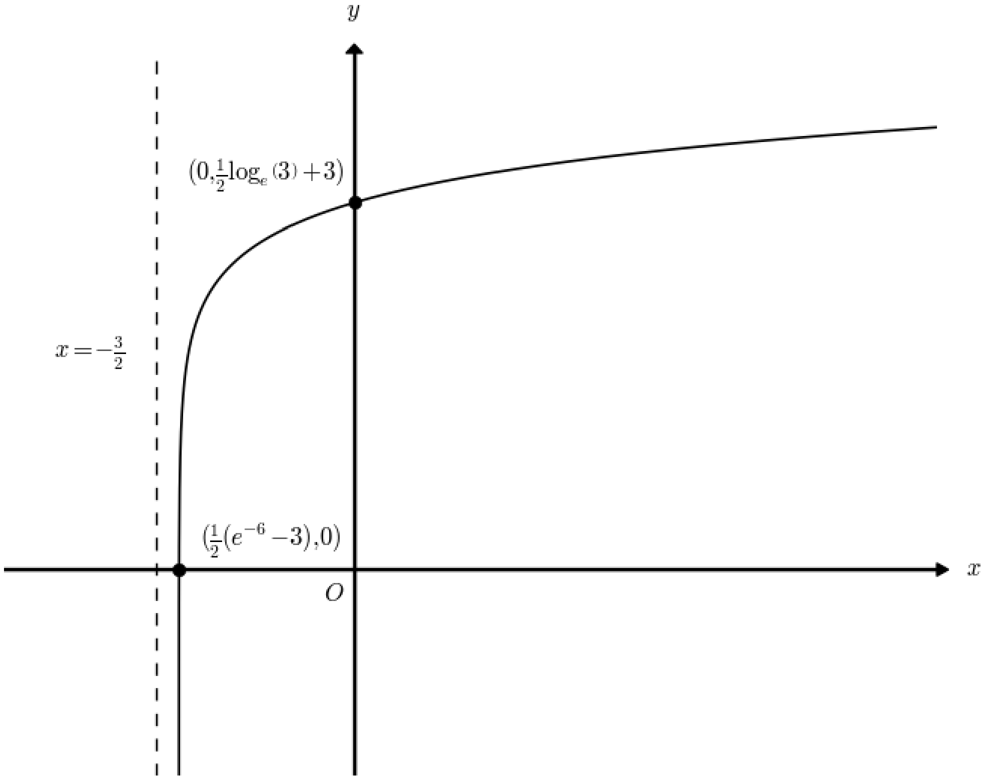
Question 19
The correct answer is B.

Question 20
The correct answer is A.

## Section B – Short-answer questions

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

### Question 1a




Requires a sketched graph with:

- $x$-intercept at $(\frac{1}{2}(e^{(-6)} - 3), 0)$ [1]
- $y$-intercept at $(0, \frac{1}{2}\ln(3) + 3)$ [1]
- Vertical asymptote at $x = -\frac{3}{2}$ [1]

### Question 1b i

Domain: $(\frac{-3}{2}, \infty)$, Range: $(-\infty, \infty)$ [1]

### Question 1b ii

$f'(x) = \frac{1}{2x+3}$ [1]

### Question 1b iii

f(x) has inverse by interchanging x and y such that we have

$$x = \frac{1}{2}\ln(2y+3)+3$$

$$2(x-3) = \ln(2y+3)$$

$$2y+3 = e^{2(x-3)}$$

$$y = \frac{e^{2(x-3)}-3}{2}$$

$$f^{-1}(x) = \frac{e^{2(x-3)}-3}{2}$$

### Question 1c

$a = 4.2184$ [1]

$b = 4.2184$ [1]

### Question 1d i

$q = \frac{1}{2}\ln(2p+3)+3$ [1]

### Question 1d ii

Gradient = $\frac{p-q}{q-p}$ [1]

Sub $(p,q)$ into $y = \frac{p-q}{q-p}x + c$

$$q = \frac{p-q}{q-p}p + c$$

So, $c = q - \frac{p-q}{q-p}p$

So, $y = \frac{p-q}{q-p}(x-p)+q$, as required [1]

### Question 1d iii

$$\begin{aligned}\text{length} &= \sqrt{(p-q)^2+(q-p)^2}\\ &= \sqrt{2(p-q)^2}\\ &= \sqrt{2\left(p-\left(\frac{1}{2}\ln(2p+3)+3\right)\right)^2}\end{aligned}$$

(as $q = \frac{1}{2}\ln(2p+3)+3$ from 1di) [2]

### Question 1e

Given that length = $\sqrt{2\left(p-\left(\frac{1}{2}\ln(2p+3)+3\right)\right)^2}$, this will be maximised when

$$\frac{d\,(\text{length})^2}{dp} = 4\left(1-\frac{1}{3+2p}\right)\left(-3+p-\frac{1}{2}\ln(2\mathrm{p}+3)\right) = 0$$

Solving for $p$, $p = -1.0000$ [2]

Question 2a i
$r = \frac{h}{2}$ [1]

Question 2a ii
$V = \frac{\pi h^3}{12}$ [1]

Question 2b
$\frac{dh}{dt} = -\frac{1}{40\pi}$ [1]

Depth is decreasing at $\frac{1}{40\pi}$ m/min

Question 2c
Full volume = $\frac{2000\pi}{3}$

Since $\frac{dV}{dt} = -0.1, V = \frac{2000\pi}{3} - 0.1t$ [1]

t = 20934.00 minutes [1]

Question 2d i
$t = 13.108$ hours [1]

Question 2d ii
$\frac{dv}{dt} = 3t^2 - 2t + 1$ [1]

$\frac{d^2v}{dt^2} = 6t - 2 = 0, \therefore t = \frac{1}{3}$ [1]

$t = \frac{1}{3}$ is a minimum by the second derivative test. (Just before $t = \frac{1}{3}, \frac{d^2v}{dt^2} < 0$, so the gradient is decreasing, and just after $t = \frac{1}{3}, \frac{d^2v}{dt^2} > 0$, so the gradient is increasing.) [1]

Substituting into $\frac{dv}{dt}$: $3(\frac{1}{3})^2 - 2\left(\frac{1}{3}\right) + 1 = \frac{2}{3}$ metres/hour [1]

Question 2e
The amount of uptime is 20934.00 minutes (from part 2d)

The amount of downtime is 13.108 * 60 minutes

$$\frac{13.108 * 60}{13.108 * 60 + 20934} \times 100 = 3.62\%$$

[1]

Question 3a i
$\Pr(S \geq 1.10) = 0.023$ [1]

Question 3a ii
Observe that $\frac{S-1.00}{0.05} \sim N(0,1)$ and $\frac{P-a}{b} \sim N(0,1)$.

Then:

$\Pr(P \geq 1.00) = \Pr(S \geq 1.10)$

$\Pr\left(\frac{P-a}{b} \geq \frac{1.00-a}{b}\right) = \Pr\left(\frac{S-1.00}{0.05} \geq \frac{1.10-1.00}{0.05}\right)$

By the observation above,

$\frac{1.00-a}{b} = \frac{1.10-1}{0.05}$ [1]

Similarly:

$\frac{1.10-a}{b} = \frac{1.30-1}{0.05}$ [1]

$1-a=2b$

$1.1-a=6b$

$a=0.95$ [1]

$b=0.025$ [1]

### Question 3b i

$0.0001$ [1]

### Question 3b ii

Where X is binomially distributed with p=0.4, n=10,

$\Pr(X \geq 2) = 1 - \Pr(X=0) - \Pr(X=1)$ [1]

$=0.9536$ [1]

### Question 3b iii

Mean $= n \times p = 4$ [1]

Variance $= n \times p \times (1-p) = 2.4$ [1]

### Question 3c i

$\hat{D} = \frac{X}{n}$ [1]

### Question 3c ii

$X \sim Bi\left(n, \frac{2s}{5}\right)$ [1]

The proportion of people who dislike their dish is 40% of the proportion of dishes which are salty.

### Question 3c iii

$E[\hat{D}] = E\left[\frac{X}{n}\right]$

$= \frac{n \cdot \frac{2s}{5}}{n}$

$= \frac{2s}{5}$ [1]

$Var(\hat{D}) = Var\left(\frac{X}{n}\right)$

$= \frac{1}{n^2} \cdot n \cdot \frac{2s}{5} \cdot \left(1 - \frac{2s}{5}\right)$

$= \dfrac{2s(5-2s)}{25n}$ [1]

### Question 3c iv

Let $d$ be the proportion of people who dislike their dish. While we know that $d = \frac{2s}{5}$, i.e. 40% of people who receive a salty dish dislike it, the critic believes $d = \frac{s}{2}$, i.e. that 50% will dislike it.

So, $\hat{D} = \frac{X}{n}$, and $X \sim Bi(n, d)$.

Thus $\frac{\hat{D}-d}{\sqrt{\frac{d(1-d)}{n}}} \sim N(0,1)$ [0.5] approximately, for large $n$, and an approximate 95% confidence interval for $d$ is given by $d \in \left(\hat{d} - z\sqrt{\frac{\hat{d}(1-\hat{d})}{n}}, \hat{d} + z\sqrt{\frac{\hat{d}(1-\hat{d})}{n}}\right)$, where $z \approx 1.96$ satisfies $\Pr(Z < z) = 0.975, Z \sim N(0,1)$.

Substituting $\hat{d} = 0.0096, z = 1.959964$, obtain

$d \in \left(0.0096 - 0.1911\sqrt{\frac{1}{n}}, 0.0096 + 0.1911\sqrt{\frac{1}{n}}\right)$ [0.5]

Now, the critic believes $d = \frac{s}{2} \Rightarrow s = 2d$. Thus

$s \in \left(0.0192 - 0.3822\sqrt{\frac{1}{n}}, 0.0192 + 0.3822\sqrt{\frac{1}{n}}\right)$ [1]

### Question 3c v

$\hat{d} + z\sqrt{\frac{\hat{d}(1-\hat{d})}{n}} = 0.0192 + 0.3822\sqrt{\frac{1}{n}} < 0.03 \Rightarrow n > 88$ [1]

### Question 4a i

Gradient = $\frac{-5}{k}$ [1]

### Question 4a ii

$y' = \frac{-5}{x^2}$ [1]

$x = \sqrt{k}$ [1]

### Question 4b i

$10$ [1]

### Question 4b ii

$\int_a^1 f(x) = 10$ [1]

$a = e^{-2}$ [1]

### Question 4c i

$A(k) = \frac{1}{2} \times (a+b) \times h, where\ a = 5, b = \frac{5}{k}, h = k-1$ [1]

$A(k) = \dfrac{1}{2} \times \left(5 + \dfrac{5}{k}\right) \times (k-1)$

$A(k) = \dfrac{1}{2}\left(\dfrac{5k+5}{k}\right)(k-1)$

$A(k) = \frac{5}{2}(\frac{k^2-1}{k})$ [1]

Question 4c ii

$A(k) = \frac{5(k+1)(k-1)}{2k} = 10$ [1]

$k = 2 \pm \sqrt{5}$, but $k > 0$ so $k = 2 + \sqrt{5}$ [1]

Question 4c iii

At all points between 1 and $k$, the curve $\frac{5}{x}$ is lower than the line $AK$, therefore the integral (i.e. the area under the curve) of $\frac{5}{x}$ will be less than the area of the parallelogram formed by the line $AK$.

Since $\int_1^k f(x) < 10, k < e^2$ because $\int_1^{e^2} f(x) = 10$ [1]

Question 4d

$$\int_1^{pq} f(x) = 5\ln(pq) = 6$$

$$\int_1^{\frac{p}{q}} f(x) = 5\ln(\frac{p}{q}) = 4$$

$pq = e^{\frac{6}{5}}$

$\frac{p}{q} = e^{\frac{4}{5}}$ [1]

$p = qe^{\frac{4}{5}}$

$q^2 \times e^{\frac{4}{5}} = e^{\frac{6}{5}}$

$q = e^{\frac{1}{5}}$

$p = e$ [1]

Question 4e i

The point of intersection is $(\frac{5}{n}, n)$ [1]

$f'\left(\frac{5}{n}\right) = -\frac{n^2}{5}$

Tangent: $y = -\frac{n^2}{5}x + 2n$ [1]

Question 4e ii

$x - int = \frac{10}{n}$

$y - int = 2n$

$n = \sqrt{5}$ [1]