Engage
Education
Foundation

Units 3 and 4 Maths Methods (CAS): Exam 1

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$\frac{dy}{dx} = 2x\sin(3x) + 3x^2\cos(3x)$ [2]

[1] for an application of chain rule

Question 1b

$f(x) = xe^{-x}$

$f'(x) = e^{-x} - xe^{-x}$

$= (1-x)e^{-x}$ [1]

$f'(2) = (1-2)e^{-2}$

$= -e^{-2}$ [1]

Question 2

$\int_1^2 \frac{1}{\sqrt{2x-1}}dx = \int_1^2 (2x-1)^{-1/2}dx$

$= [(2x-1)^{1/2}]_1^2$ [1]

$= (2(2)-1)^{1/2} - (2(1)-1)^{1/2}$

$= \sqrt{3} - 1$ [1]

Question 3

Let a $= \frac{\theta}{2} + \frac{\pi}{4}$

a $\in [\frac{\pi}{4}, \frac{11\pi}{4}]$

[1] for acknowledging the domain

Substitute a for $\frac{\theta}{2} + \frac{\pi}{4}$:

$\tan(a) = \sqrt{3}$

$a = \tan^{-1}\sqrt{3}$

$= \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}$ [1]

Hence, $\frac{\theta}{2} + \frac{\pi}{4} = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}$

$\therefore \theta = \frac{\pi}{6}, \frac{13\pi}{6}, \frac{25\pi}{6}$ [1]

Question 4

$e^{2x} - 6e^x + 8 = 0$

$(e^x - 2)(e^x - 4) = 0$ [1]

$e^x = 2 \text{ or } 4$

$\therefore x = \ln(2) \text{ or } \ln(4)$ [1]

Question 5a
$\text{ran}_f = R$

$\text{dom}_g \in [\frac{1}{2}, \infty)$

$\therefore \text{ran}_f \not\subseteq \text{dom}_g$

Hence, $g(f(x))$ does not exist. [1]

Question 5b
$f(g(x)) = -4(2x-1)^{3/2}$ [1]

$\therefore \text{dom}_{fog} \in [\frac{1}{2}, \infty)$ [1]

$\text{ran}_{fog} \in (-\infty, 0]$ [1]

Question 6a
Find the y- intercept:

$y = \ln(3-0) = \ln(3)$ [1]

Rearrange the equation in terms of x:

$y = \ln(3-x)$

$e^y = 3 - x$

$x = 3 - e^y$ [1]

Find the area enclosed:

$\int_0^{\ln(3)} 3 - e^y \, dy$

$= [3y - e^y]_0^{\ln(3)}$ [1]

$=\left(3\ln(3) - e^{\ln(3)}\right) - (0 - e^0)$

$= 3\ln(3) - 3 + 1$

$=3\ln(3) - 2$ [1]

Question 6b
$(x, y) \to (x, -y) \to (x, -2y) \to (x+1, -2y) \to (x', y')$ [1]

$x' = x + 1$

$x = x' - 1$

$y' = -2y$

$y = -\frac{y'}{2}$

$-\frac{y'}{2} = \log_e(3 - (x' - 1))$

$\therefore y' = -2\log_e(4 - x')$ [1]

Question 7a i
E(X) = 0.2*4 + 0.4*5 + 0.3*6 + 0.1*7

=5.3 [1]

Question 7a ii
Pr(X>5.3) = Pr (X=6) + Pr(X=7)

= 0.3 + 0.1

= 0.4 [1]

Question 7b
Var(X) = 0.81

Sd(X) = $\sqrt{\text{Var}(X)}$ = 0.9 [1]

sd(2X-1) = 4sd(X)

= 4*0.9

=3.6 [1]

Question 8
$f'(x) = 6x - 4$

$f'(2) = 12 - 4 = 8$ [1]

Gradient of the tangent at x=2 is 8.

Hence the gradient of the normal is $\frac{-1}{8}$ [1]

$f(2) = 4$

$\therefore$the equation of normal:

$y - 4 = \frac{-1}{8}(x - 2)$

$y = \frac{-1}{8}x + \frac{17}{4}$ [1]

Question 9
$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

$0.2^2 = \frac{0.1*0.9}{n}$ [1]

$n = 2.25$ [1]

Question 10a

$= \sqrt{9} = 3$

$\Pr(33<X<42) = \Pr(36 - \ <X<36 + 2\ )$ [1]

$= 0.68 * 0.5 + 0.95 * 0.5$

$= 0.815$ [1]

Question 10b

$\Pr(X<39|X>36) = \frac{0.68*0.5}{0.5} = 0.68$ [1]

Question 11a

Let radius of the oil in container = r

h:r = 20:5 = 4:1

Hence, $r = \frac{h}{4}$ [1]

$V = \frac{1}{3}\ \text{base} * \text{height}$

$= \frac{1}{3} * \pi r^2 * h$

$= \frac{1}{3} * \pi(\frac{h}{4})^2 * h$

$V = \frac{h^3\pi}{48}$ [1]

Question 11b

$$V(15) = \frac{15^3\pi}{48}$$

$V(15) = \frac{1125\pi}{16}\ \text{cm}^3$ [1]

Question 11c

$$V = \frac{h^3\pi}{48}$$

$\frac{dV}{dh} = \frac{h^2\pi}{16}$ [1]

$$\pi = \frac{h^2\pi}{16}$$

$h^2 = 16$

$h = \pm 4$

$h = 4$ since $h > 0$ [1]