2015 VCE Mathematical Methods (CAS) 2 examination report

General comments

In the 2015 Mathematical Methods (CAS) examination 2, students achieved scores across the whole range of available marks. A small number of students sat the computer-based version of this examination. Responses showed that the examination was accessible and that it provided an opportunity for students to demonstrate their knowledge.

In Section 2, students need to take care with providing answers in the correct form. Exact answers are required unless otherwise stated. This caused problems in Questions 1 and 5. Some students were not working to the required number of decimal places. This was evident in Question 2. There were a number of rounding errors in Questions 2 and 3.

Brackets were often missing from equations, leading to incorrect answers. This occurred in Questions 1 and 2. Some students did not have their technology set for degrees or were unable to convert radians to degrees, as was needed for Question 2a. Students need to check that they have entered the correct expression in their technology.

Some students used inefficient methods when finding the area between two curves. This was evident in Question 1. Students took care when sketching the graphs in Questions 4 and 5. However, many were not able to describe the transformations correctly in Question 4.

Students need to check their responses to make sure they have answered all parts of a question. This was evident in Questions 2 and 5. They also need to re-read questions to make sure that they are answering the question asked. Some found the median instead of the mean, and some found the average value of the function when the average rate of change was required. Students also need to check if their answers are reasonable; for example, a probability cannot be more than 1 and the answers should not be outside the given domains. A good strategy in the graphing questions is to estimate the answers from the given values on the graphs.

Adequate working must be shown for questions worth more than one mark. This caused a problem in Questions 2 and 3. This often requires students to write down the equation they are going to solve on their technology, not to do the question by hand.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by the shading.

Question	% A	% B	% C	% D	% E	% No answer	Comments
1	95	4	1	0	0	0	
2	50	42	3	2	2	0	
3	61	14	20	2	4	0	The rule for the graph is in the form $f(x)=a(x-b)(x-c)^2(x-d)$, where a is negative and could be -2. $f(x)=-2(x-b)(x-c)^2(x-d)$ b is negative; for example if $b=-2$, the factor is $(x-(-2))=(x+2)$. Most students chose option A, $y=-2(x+b)(x-c)^2(x-d)$, but the factor $(x+b)$ is incorrect.
4	6	77	6	6	4	0	
5	4	5	3	16	71	0	
6	1	4	91	2	1	0	
7	2	31	56	4	6	0	
8	14	6	17	53	8	1	
9	16	37	21	15	9	1	Solve $\int_2^a \left(\frac{1}{6}\right)dx=1$, for a, $a=8$ $E(X)=\int_2^8 \left(\frac{x}{6}\right)dx=5$ or Area of the rectangle $=\frac{1}{6}(a-2)=1, a=8$. Since it is a uniform distribution, the expected value is halfway between 2 and 8, which is 5.
10	6	10	16	59	9	1	
11	24	7	7	32	29	0	$y_1=\sqrt{8x^3+1}$, $y_2=\sqrt{8\left(\frac{x}{2}\right)^3+1}=\sqrt{x^3+1}$ The graph of y_1 has been dilated by a factor of 2 from the y-axis to get the graph of y_2. This can be shown by sketching the graphs of both functions. For example, the point with coordinates (1, 3) is transformed to (2, 3).

Question	% A	% B	% C	% D	% E	% No answer	Comments
12	10	7	9	13	60	1	
13	7	4	14	11	63	1	
14	2	11	6	75	5	0	
15	3	69	20	4	3	1	
16	24	19	26	22	8	2	$f'(x)=\int\left(bx^n\right)dx=\frac{bx^{n+1}}{n+1}+c=\frac{bx^{n+1}}{n+1}$, $f'(x)=amx^{m-1}$ Hence, $n+1=m-1,\ n=m-2$, $\frac{b}{n+1}=am$, $\frac{b}{a}=m(n+1)=m(m-1)$ Hence, $\frac{b}{a}$ is an integer as *m* is an integer; for example, if *m* = 3, $\frac{b}{a}=3\times2=6$.
17	8	11	13	60	7	0	
18	48	20	15	10	6	1	If $f(x)=x^2$, then $\lvert f(x+y)-f(x-y\rvert=\lvert(x+y)^2-(x-y)^2$ $=\lvert 4xy\rvert$ $=4\sqrt{x^2y^2}$ $=4\sqrt{f(x)f(y)}$
19	5	7	68	14	5	1	
20	4	61	10	12	12	0	
21	7	20	30	37	5	1	$mx+c=ax^2$, $ax^2-mx-c=0$ The discriminant will be negative for no real solutions. Solve $m^2+4ac<0$ for c. $c<-\frac{m^2}{4a},\ a>0$ and $c>-\frac{m^2}{4a},\ a<0$
22	12	35	15	26	11	1	Let $g(x)=\tan(x)$ and $f(x)=-\lvert x\rvert$. $g\left(-f(x)\right)=\tan\left(\lvert x\rvert\right)$ and sketch the graph. The graph should be symmetrical about the *y*-axis.

Section 2

Question 1
1a.

Marks	0	1	Average
%	6	94	1

$f(x)=\frac{1}{5}(x-2)^2(5-x)$, $f'(x)=-\frac{3}{5}(x-4)(x-2)$ or an equivalent form

This question was answered well. However, there were some transcription errors.

1bi.

Marks	0	1	Average
%	20	80	0.8

The equation of the tangent at $x = 1$ is $y=-\frac{9}{5}x+\frac{13}{5}$.

This question was answered well. However, some students did not write an equation, leaving their answer as $-\frac{9}{5}x+\frac{13}{5}$.

1bii.

Marks	0	1	2	Average
%	20	9	70	1.5

$S\left(0,\frac{13}{5}\right)$, $Q\left(\frac{13}{9},0\right)$

This question was answered well. Some students did not write their answers as coordinates. Others labelled the coordinates incorrectly. Some wrote $\frac{13}{9}$ as 1.44, which was incorrect. An exact answer was required.

1c.

Marks	0	1	2	Average
%	37	10	52	1.2

$S\left(0,\frac{13}{5}\right)$ and $P\left(1,\frac{4}{5}\right)$, $PS=\sqrt{\left(\frac{13}{5}-\frac{4}{5}\right)^2+(0-1)^2}=\frac{\sqrt{106}}{5}$

Various incorrect formulas for the distance between two points were given. There was some poor substitution into the distance formula. Many students found the distance using *QS.*
$\sqrt{\left(\frac{13}{9}\right)^2+\left(\frac{13}{5}\right)^2}$ was a common incorrect response.

1d.

Marks	0	1	2	3	Average
%	25	14	7	54	1.9

Solve $\frac{1}{5}(x-2)^2(5-x)=\frac{1}{5}(13-9x)$ for x, $x=1$ or $x=7$, $\int_1^7\left(f(x)-\left(-\frac{9}{5}x+\frac{13}{5}\right)\right)dx=\frac{108}{5}$

The 'dx' was often missing and brackets were used poorly; for example, $\int_1^7\left(f(x)-\frac{-9}{5}x+\frac{13}{5}\right)dx$.

Many students split the areas up rather than just using $\int_a^b(upper\ function - lower\ function)dx$. Some students completed the solution by hand rather than using technology.

Question 2
2a.

Marks	0	1	2	Average
%	54	14	33	0.8

Let $g(x)=60-\frac{3}{80}x^2$, $g'(x)=-\frac{3}{40}x$, $g'(-40)=3$, $\tan(\theta)=3$, $\theta=71.56...=72^\circ$ to the nearest degree

This question was not answered well. A common incorrect response was 71°. Some students did not convert their answer to degrees. Others gave the answer as 56°, using $\tan(\theta)=\frac{ON}{OA}=\frac{60}{40}$.
Some found $m=3$ but were unable to find the angle.

2b.

Marks	0	1	2	Average
%	36	21	44	1.1

Let $h(x)=\frac{x^3}{25\ 600}-\frac{3x}{16}+35$, $h'(x)=\frac{3x^2}{25\ 600}-\frac{3}{16}$, $h'(0)=-\frac{3}{16}$

Some students did not interpret the question correctly and found the gradient of the straight line passing through X and Y. Some solved $h'(x)=0$ for x.

2c.

Marks	0	1	2	3	Average
%	57	5	6	33	1.2

$d(x)=g(x)-h(x)$, solve $d'(x)=0$ for x, $x=2.4903...$, $h(2.49...)=34.5337...$, so the coordinates of M are (2.49, 34.53) correct to two decimal places

Some students did not give their answers correct to two decimal places. Some worked to one decimal place and others rounded their answers incorrectly. Others did not set up the distance formula correctly or did not use brackets correctly in the distance formula. Some substituted *u* into *g* instead of *h*. A common incorrect response was *v* = 24.53.

2d.

Marks	0	1	2	3	Average
%	64	6	9	21	0.9

$h(-2.4903...) = 35.4663...$, $w = 35.47$, $PQ = g(-2.4903...) - h(-2.4903...) = 24.3011\ldots = 24.30$ correct to two decimal places, $MN = g(2.4903...) - h(2.4903...) = 25.2338\ldots = 25.23$ correct to two decimal places

Some students did not work to the required number of decimal places or rounded incorrectly. Others had $PQ = 25.23$ and $MN = 24.30$.

2e.

Marks	0	1	2	3	Average
%	26	9	15	49	1.9

Solve $f(x) = g(x)$ for x. at E, $x = -23.7068... = -23.71$ correct to two decimal places, at F, $x = 27.9963... = 28.00$ correct to two decimal places

Some students did not work to the required number of decimal places. Others rounded to 27.00 instead of 28.00. Some students gave answers without showing any working.

2f.

Marks	0	1	Average
%	46	54	0.6

$$\text{Area} = \int_{-23.706...}^{27.996...} (f(x) - g(x))dx = 869.619\ldots = 870 \text{ m}^2 \text{ to the nearest m}^2$$

Some students rounded incorrectly to 869 or did not work to the required number of decimal places. Some gave their answers in exact form.

Question 3
3ai.

Marks	0	1	2	Average
%	17	3	80	1.7

$$\Pr(X > 7) = \int_7^8 f(x)dx = \frac{11}{16} \text{ or } 0.6875$$

Some students omitted the dx. Some had incorrect terminals such as $\int_6^7 f(x)dx$, $\int_{7.0001}^8 f(x)dx$ or $\int_{6.9999}^8 f(x)dx$. Others gave the answer without showing any working.

3aii.

Marks	0	1	2	Average
%	40	14	47	1.1

$Y \sim \mathrm{Bi}\left(3, \frac{11}{16}\right)$, $\Pr(Y=1) = 3 \times \frac{11}{16} \times \left(\frac{5}{16}\right)^2 = \frac{825}{4096}$

Many students were able to identify the binomial distribution with the correct n and p values. A common incorrect answer was $\frac{11}{16} \times \left(\frac{5}{16}\right)^2 = \frac{275}{4096}$.

3b.

Marks	0	1	Average
%	27	73	0.8

$E(X) = \int_6^8 (x \times f(x))\,dx = \frac{36}{5} = 7.2$

Some students worked out the median, solving $\int_6^8 f(x)dx = 0.5$ for x, instead of the mean. Others evaluated $\int_6^8 (f(x))\,dx$, leaving out x.

3c.

Marks	0	1	2	Average
%	37	13	50	1.1

$Oranges \sim \mathrm{N}(74, 9^2)$, $\Pr(O < 85 \mid O > 74) = \frac{\Pr(O < 85 \cap O > 74)}{\Pr(O > 74)} = \frac{\Pr(74 < O < 85)}{\Pr(O > 74)} = \frac{0.38918...}{0.5} =$
$0.7783\ldots = 0.778$, correct to three decimal places

Many students were able to recognise that the problem involved conditional probability. Some students evaluated $\frac{0.38918...}{0.49999}$ or $\frac{0.889188...}{0.5}$.

3di.

Marks	0	1	2	Average
%	39	11	50	1.1

$Lemons \sim \mathrm{Bi}(4, 0.03)$, $\Pr(lemons \geq 1) = 1 - \Pr(lemons = 0) = 1 - (0.97)^4 = 0.114707\ldots = 0.1147$ correct to four decimal places

Some students wrote 3% as 0.3. Others had the incorrect value for n, using $Lemons \sim \mathrm{Bi}(3, 0.03)$. Some gave the answer without showing any working, while others attempted to use the normal distribution.

3dii.

Marks	0	1	2	Average
%	52	13	35	0.9

$\Pr(X \geq 1) > 0.5$, $1-(0.97)^n > 0.5$, $n > 22.7566...$, $n = 23$

Some students rounded their answer to 22. Others did not state the minimum value, leaving their answer as $n > 22.7566$. Some students used the trial and error methods and this was acceptable. Some students did not show any working.

Question 4
4a.

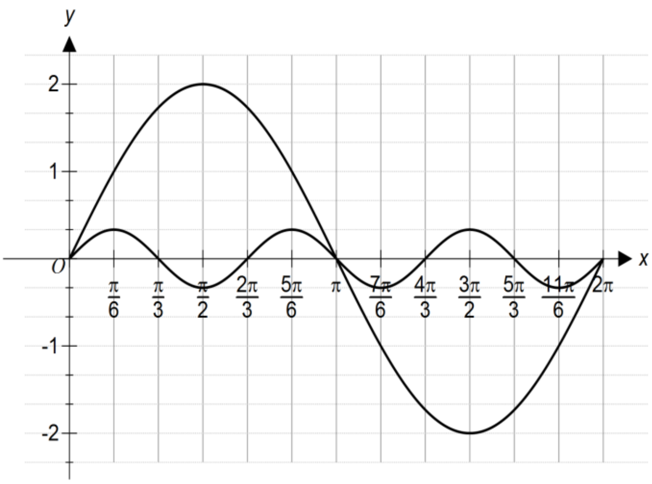
Marks	0	1	Average
%	64	36	0.4

$\text{Total area} = 2\int_0^{\pi}(2\sin(x))dx = 4\int_0^{\pi}(\sin(x))dx$, $a = 4$

This question was not answered well. Many students wrote $a = 2$.

4b.

Marks	0	1	2	Average
%	19	14	67	1.5



Many students drew accurate graphs. Some students did not have the *x*-intercepts and turning points in the right positions. Most students drew one continuous curve, without any shading.

4c.

Marks	0	1	2	Average
%	35	24	41	1.1

A dilation by a factor of $\frac{1}{6}$ from the *x*-axis and a dilation of a factor of $\frac{1}{3}$ from the *y*-axis

Many students did not give the correct wording when describing the dilations.

4di.

Marks	0	1	2	Average
%	73	16	12	0.4

$\text{Area} = 2\int_0^{\pi}\left(k(x)-q(x)\right)dx = 4m + \dfrac{2\cos(n\pi)-2}{n^2} = 4m + \dfrac{0}{n^2}$, if n is even $\cos(n\pi)=1$

This question was not answered well. Many students evaluated $\int_0^{2\pi}\left(k(x)-q(x)\right)dx$. Many did not realise that when n is even, $\cos(n\pi)=1$.

4dii.

Marks	0	1	2	Average
%	81	10	9	0.3

$\text{Area} = 2\int_0^{\pi}\left(k(x)-q(x)\right)dx = 4m + \dfrac{2\cos(n\pi)-2}{n^2} = 4m + \dfrac{{}^{-}4}{n^2}$, if n is odd $\cos(n\pi)=-1$

This question was not answered well. Many students did not realise that when n is odd, $\cos(n\pi)=-1$.

Question 5
5ai.

Marks	0	1	Average
%	32	68	0.7

$S(t)=2e^{\frac{t}{3}}+8e^{\frac{-2t}{3}}$, $S(0)=10$, $S(5)=2e^{\frac{5}{3}}+8e^{-\frac{10}{3}}$

Some students gave an approximate answer for $S(5)$ when an exact answer was required.

5aii.

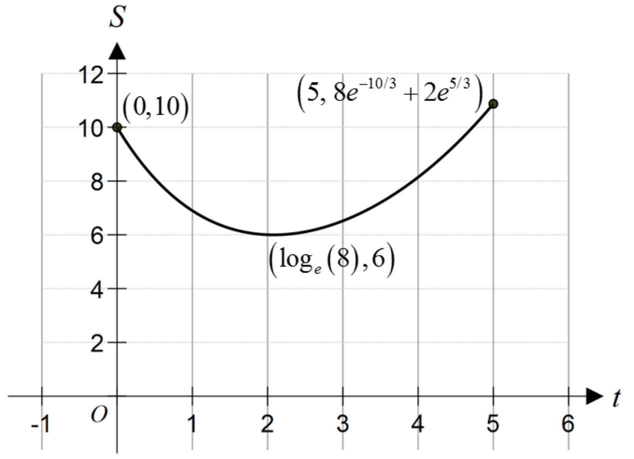
Marks	0	1	2	Average
%	22	27	51	1.3

Solve $S'(t)=0$ for t, $t=3\log_e(2)=\log_e(8)$, c = 8, $S\left(\log_e(8)\right)=6$

Some students did not find the minimum value.

5aiii.

Marks	0	1	2	Average
%	14	23	63	1.5



Many students drew accurate graphs. Some had the coordinates the wrong way around and others did not put the coordinates on the graph. Some did not sketch the graph over the required domain.

5aiv.

Marks	0	1	2	Average
%	45	12	43	1

$$\text{Average rate of change} = \frac{S(\log_e(8)) - S(0)}{\log_e(8) - 0} = \frac{6-10}{\log_e(8)} = -\frac{4}{\log_e(8)}$$

Some students worked out the average value of the function and not the average rate of change. Others left the negative sign off their answer, writing $\frac{4}{\ln 8}$. Some students gave $\frac{\log_e(8)-0}{6-10}$.

5b.

Marks	0	1	2	Average
%	35	7	58	1.3

$V(t) = de^{\frac{t}{3}} + (10-d)e^{\frac{-2t}{3}}$, solve $V'(\log_e(9)) = 0$ for d, $d = \frac{20}{11}$

This question was answered reasonably well. Some students tried to solve the equation by hand but were unsuccessful.

5ci.

Marks	0	1	2	Average
%	47	48	4	0.6

Solve $V'(0)=0$ for *d*, $d=\frac{20}{3}$, $\frac{20}{3}\le d<10$

Many students found $d=\frac{20}{3}$ but did not consider the set of values for *d*. Some had the inequality as $\frac{20}{3}\le d\le 10$ or $\frac{20}{3}<d<10$.

5cii.

Marks	0	1	2	Average
%	51	46	4	0.6

Solve $V'(5)=0$ for *d*, $d=\frac{20}{2+e^5}$, $0<d\le\frac{20}{2+e^5}$

Many students found $d=\frac{20}{2+e^5}$, but did not consider the set of values for *d*. Some had the inequality written incorrectly.

5d.

Marks	0	1	2	Average
%	82	7	11	0.3

Solve $V'(a)=0$ for *a*, $a=\log_e\left(\frac{20}{d}-2\right)$, solve $V\left(\log_e\left(\frac{20}{d}-2\right)\right)=\frac{k}{2}d^{\frac{2}{3}}(10-d)^{\frac{1}{3}}$ for *k*, $k=3\times 2^{\frac{1}{3}}$

Alternatively, choose a value of a where $0\le a\le 5$, let a = 0, $V(0)=10$, $d=\frac{20}{3}$ from 5ci., solve

$10=\frac{k}{2}\left(\frac{20}{3}\right)^{\frac{2}{3}}\left(10-\frac{20}{3}\right)^{\frac{1}{3}}$ for *k*, $k=3\times 2^{\frac{1}{3}}$

This question was not answered well. Some students were able to find $a=\log_e\left(\frac{20}{d}-2\right)$ but did not substitute it into *V*. A common incorrect response was $k=-3\times 2^{\frac{1}{3}}$.