VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2015

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter | |

MATHEMATICAL METHODS (CAS)

Written examination 1

Wednesday 4 November 2015

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, correction fluid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 16 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = (5x + 1)^7$.

Find $\dfrac{dy}{dx}$. 1 mark

b. Let $f(x) = \dfrac{\log_e(x)}{x^2}$.

i. Find $f'(x)$. 2 marks

ii. Evaluate $f'(1)$. 1 mark

Question 2 (3 marks)

Let $f'(x) = 1 - \frac{3}{x}$, where $x \neq 0$.

Given that $f(e) = -2$, find $f(x)$.

Question 3 (2 marks)

Evaluate $\int_1^4 \left(\frac{1}{\sqrt{x}}\right) dx$.

Question 4 (6 marks)

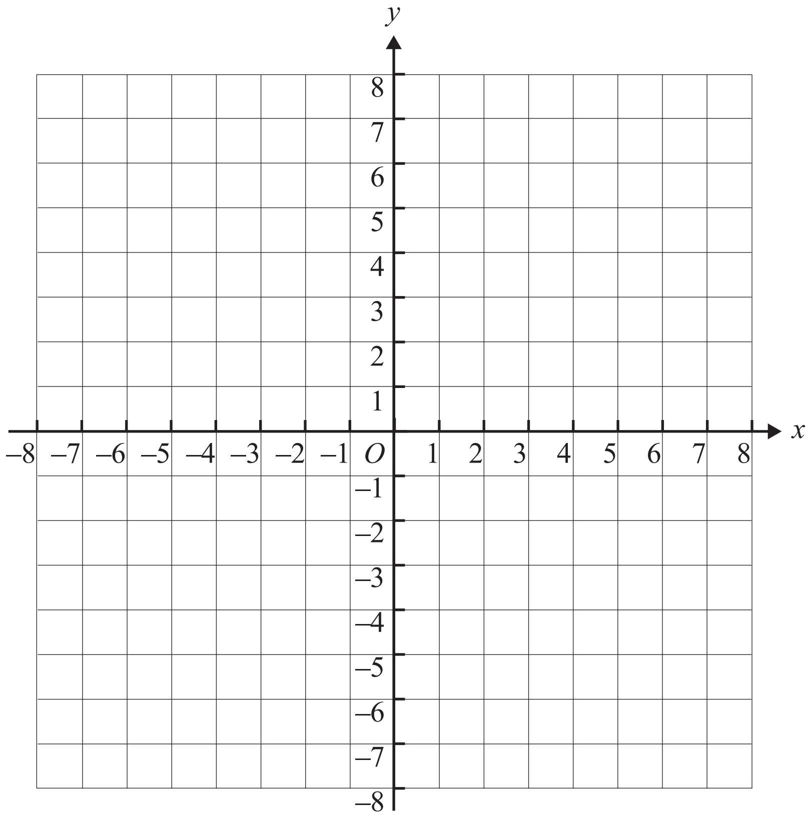
Consider the function $f:[-3,\ 2] \to R,\ f(x) = \frac{1}{2}\left(x^3 + 3x^2 - 4\right)$.

a. Find the coordinates of the stationary points of the function. 2 marks

Question 4 – continued

The rule for f can also be expressed as $f(x) = \frac{1}{2}(x-1)(x+2)^2$.

b. On the axes below, sketch the graph of f, clearly indicating axis intercepts and turning points. Label the end points with their coordinates. 2 marks



c. Find the average value of f over the interval $0 \le x \le 2$. 2 marks

Question 5 (3 marks)

On any given day, the depth of water in a river is modelled by the function

$$h(t) = 14 + 8\sin\left(\frac{\pi t}{12}\right), \ 0 \le t \le 24$$

where h is the depth of water, in metres, and t is the time, in hours, after 6 am.

a. Find the minimum depth of the water in the river. 1 mark

b. Find the values of t for which $h(t) = 10$. 2 marks

Question 6 (3 marks)

Let the random variable X be normally distributed with mean 2.5 and standard deviation 0.3

Let Z be the standard normal random variable, such that $Z \sim N(0, 1)$.

a. Find b such that $\Pr(X > 3.1) = \Pr(Z < b)$. 1 mark

b. Using the fact that, correct to two decimal places, $\Pr(Z < -1) = 0.16$, find $\Pr(X < 2.8 | X > 2.5)$.
Write the answer correct to two decimal places. 2 marks

Question 7 (5 marks)

a. Solve $\log_2(6 - x) - \log_2(4 - x) = 2$ for x, where $x < 4$. 2 marks

b. Solve $3e^t = 5 + 8e^{-t}$ for t. 3 marks

Question 8 (3 marks)

For events A and B from a sample space, $\Pr(A|B) = \frac{3}{4}$ and $\Pr(B) = \frac{1}{3}$.

a. Calculate $\Pr(A \cap B)$. 1 mark

b. Calculate $\Pr(A' \cap B)$, where A' denotes the complement of A. 1 mark

c. If events A and B are independent, calculate $\Pr(A \cup B)$. 1 mark

Question 9 (4 marks)

An egg marketing company buys its eggs from farm A and farm B. Let p be the proportion of eggs that the company buys from farm A. The rest of the company's eggs come from farm B. Each day, the eggs from both farms are taken to the company's warehouse.

Assume that $\frac{3}{5}$ of all eggs from farm A have white eggshells and $\frac{1}{5}$ of all eggs from farm B have white eggshells.

a. An egg is selected at random from the set of all eggs at the warehouse.

Find, in terms of p, the probability that the egg has a white eggshell. 1 mark

Question 9 – continued

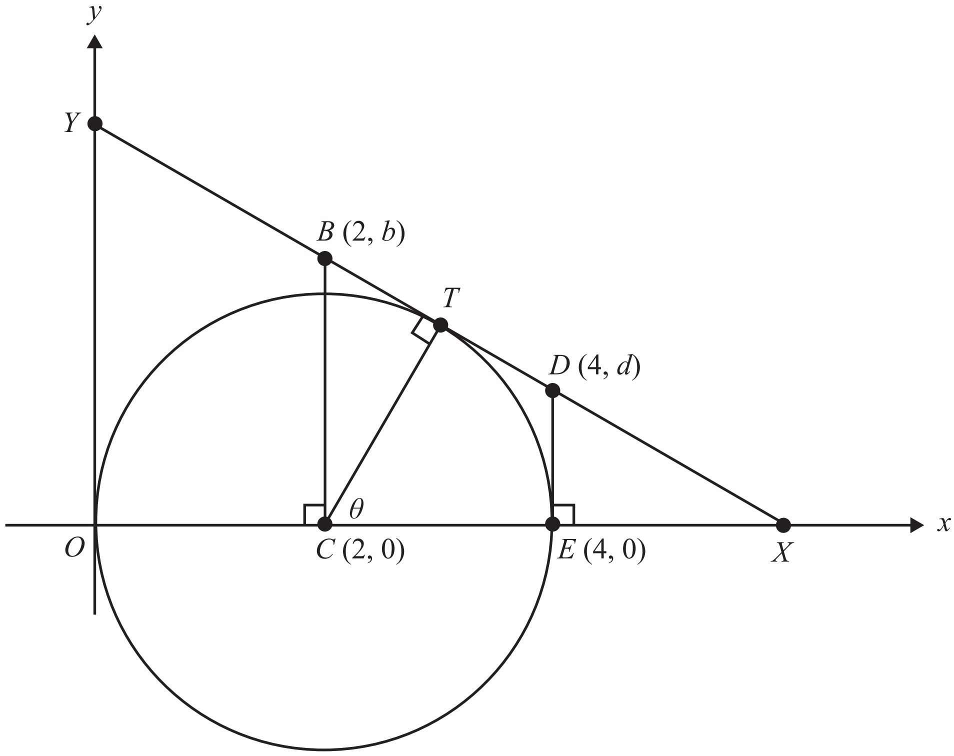
b. Another egg is selected at random from the set of all eggs at the warehouse.

i. Given that the egg has a white eggshell, find, in terms of p, the probability that it came from farm B. 2 marks

ii. If the probability that this egg came from farm B is 0.3, find the value of p. 1 mark

Question 10 (7 marks)

The diagram below shows a point, T, on a circle. The circle has radius 2 and centre at the point C with coordinates (2, 0). The angle ECT is θ, where $0 < \theta \le \frac{\pi}{2}$.



The diagram also shows the tangent to the circle at T. This tangent is perpendicular to CT and intersects the x-axis at point X and the y-axis at point Y.

a. Find the coordinates of T in terms of θ. 1 mark

b. Find the gradient of the tangent to the circle at T in terms of θ. 1 mark

Question 10 – continued

c. The equation of the tangent to the circle at T can be expressed as

$$\cos(\theta)x + \sin(\theta)y = 2 + 2\cos(\theta)$$

i. Point B, with coordinates $(2, b)$, is on the line segment XY.

Find b in terms of θ. 1 mark

ii. Point D, with coordinates $(4, d)$, is on the line segment XY.

Find d in terms of θ. 1 mark

d. Consider the trapezium *CEDB* with parallel sides of length b and d.

Find the value of θ for which the area of the trapezium *CEDB* is a minimum. Also find the minimum value of the area. 3 marks

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Instructions

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

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Mathematical Methods (CAS)
Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$

$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\int \frac{1}{x} dx = \log_e |x| + c$

$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$

$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET