Student Name: _____

MATHEMATICAL METHODS (CAS)

Unit 2
Targeted Evaluation Task for School-assessed Coursework 1

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Creating VCE Success

2015 Multiple choice and extended response test on circular functions for Outcome 1

Recommended writing time*: 50 minutes
Total number of marks available: 40 marks

TASK BOOK

* The recommended writing time is a guide to the time students should take to complete this task. Teachers may wish to alter this time and can do so at their own discretion.

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Conditions and restrictions

- Students are permitted to bring into the room for this task: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the room for this task: blank sheets of paper and/or white out liquid/tape.
- A CAS calculator is permitted in this task.

Materials supplied

- Question and answer book of 15 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this task.

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SECTION A – Multiple-choice questions

Instructions for Section A

- Circle the correct response to each question.
- Each question is worth 1 mark.

Question 1

The radian measure of the angle $225°$ is:

A. $\frac{4\pi}{5}$

B. $\frac{5\pi}{4}$

C. $\frac{55}{14}$

D. $\frac{45\pi}{52}$

E. $\frac{3\pi}{4}$

Question 2

The maximum value of $f(x) = -2\sin\left(\frac{\pi}{2}x\right) + 3$ is:

A. -1

B. 5

C. 1

D. -6

E. 0

Question 3

The period of the graph $y = \tan\left(\frac{x-1}{4}\right)$ is:

A. 4π

B. 2π

C. π

D. $\frac{\pi}{2}$

E. $\frac{\pi}{4}$

Question 4

If $\sin(2\theta) = -\frac{1}{3}, \quad \frac{3\pi}{2} < \theta < 2\pi$, then $\tan(2\theta) - \cos(2\theta)$ is equal to:

A. $\frac{11\sqrt{2}}{12}$

B. $\frac{5\sqrt{2}}{12}$

C. $\frac{-5\sqrt{2}}{12}$

D. $-\frac{11\sqrt{2}}{12}$

E. $\frac{\sqrt{2}}{4}$

Question 5

The solutions of $\cos(2\theta - \pi) = \frac{1}{2}, \quad -\pi < \theta < \pi$ is/are:

A. $-\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

B. $-\frac{2\pi}{3}, -\frac{\pi}{3}$

C. $\frac{\pi}{3}, \frac{2\pi}{3}$

D. $\frac{\pi}{3}$

E. $-\frac{\pi}{3}, \frac{\pi}{3}$

Question 6

The period of $f(x) = -3\cos\left(\frac{2x-1}{\pi}\right)+1$ is:

A. 2π

B. π

C. 1

D. π^2

E. $4\pi^2$

Question 7

The domain and range of $f:(0\,,\,2\pi)\to R,\ \ f(x)=\frac{2}{3}\tan(2x)+1$ respectively are:

A. $(0\,,\,2\pi)$ *and* R

B. $(0\,,\,2\pi)$ *and* $\left(1\,,\,\frac{2\sqrt{3}}{3}+1\right)$

C. R *and* $\left(1\,,\,\frac{2\sqrt{3}}{3}+1\right)$

D. $(0\,,\,2\pi)$ *and* $(1\,,\,1)$

E. R *and* R

Question 8

The domain of one cycle of the function $f(x)=\frac{1}{3}\tan\left(\frac{x-3}{2}\right)+\pi$ could be:

A. $(\pi\,,\,3\pi)$

B. $(0\,,\,2\pi)$

C. $(\pi+3\,,\,3\pi+3)$

D. $(0,\,\pi)$

E. $(\pi-3\,,\,\pi+3)$

Question 9

The amplitude and range of the function $f(x) = -\frac{2}{3}\sin\left(\frac{x}{3} - 2\right) + 1$ is:

A. $\frac{2}{3}$ *and* $[1, 5]$

B. $-\frac{2}{3}$ *and* $(1, 5)$

C. $\frac{2}{3}$ *and* $[-1, 1]$

D. $\frac{2}{3}$ *and* $\left[-\frac{1}{3}, 1\right]$

E. $\frac{2}{3}$ *and* $\left[\frac{1}{3}, \frac{5}{3}\right]$

Question 10

The x-intercepts of the graph of the function $y = -2\cos\left(\frac{x}{3}\right)$ over the domain $-2\pi < x < \pi$ are:

A. $\left(-\frac{3\pi}{2}, 0\right)$

B. $\left(0, \frac{3\pi}{2}\right)$

C. $\left(\frac{3\pi}{2}, 0\right)$

D. $\left(-\frac{\pi}{2}, 0\right)$

E. $\left(-\frac{5\pi}{2}, 0\right)$

Question 11

Which of the following is the equation of an asymptote of the function $y = \tan\left(\frac{3x}{5}\right) + 4$?

A. $x = 0$

B. $x = \frac{\pi}{3}$

C. $x = \frac{-\pi}{6}$

D. $x = \frac{-5\pi}{6}$

E. $x = \frac{\pi}{2}$

Question 12

If $(2, 1)$ lies on the graph of $y = 2\sin\left(\frac{\pi ax}{2}\right)$, the value of *a* could be:

A. $\frac{\pi}{6}$

B. $\frac{-7}{6}$

C. $\frac{1}{3}$

D. $\frac{7}{6}$

E. $\frac{-5}{6}$

Question 13

The value of $\sin(420°) - \cos(360°)$ is:

A. $\frac{\sqrt{3}-1}{2}$

B. $\sqrt{3}-1$

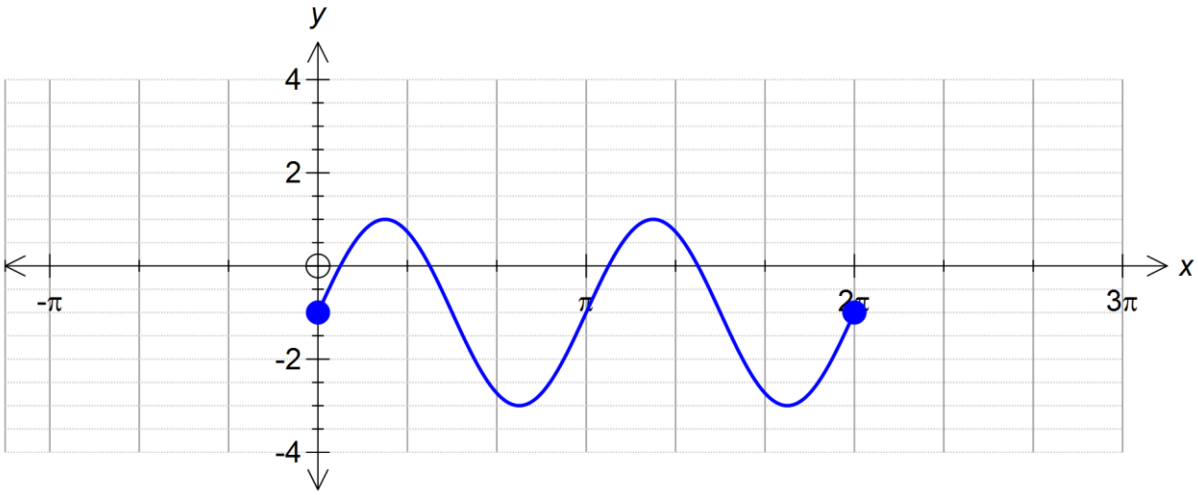
C. $\frac{\sqrt{3}-2}{2}$

D. - 0.13

E. $\frac{1-\sqrt{3}}{2}$

Question 14

The rule for the following graph could be:



A. $y = 2\sin\left(\frac{x}{2}\right) - 1$

B. $y = 2\sin(2x)$

C. $y = \sin(2x) - 1$

D. $y = 2\sin(2x) - 1$

E. $y = -2\sin(2x) - 1$

Question 15

If y-intercept of the function $f(x) = \sin(2x) + \cos(2x) - 1$ is:

A. -1

B. 0

C. 1

D. 2

E. 3

Question 16

The number of solutions of the equation $3\sin\left(\frac{x}{2}\right) = -1, \quad -\pi \le x \le \pi$, is/are:

A. 1

B. 2

C. 3

D. 4

E. 5

Question 17

If $\cos(90° + \theta) = 0.2851$, then the value of $\sin(\theta)$ is:

A. 0.2851

B. 0.7149

C. - 0.7149

D. - 0.2851

E. 0

Question 18

If the graph of the function $f(x) = -\sin(x)$ is reflected in the *y*-axis and translated +2 units parallel to the *y*-axis, the transformed function $g(x)$ has the rule:

A. $g(x) = -\sin(x) + 2$

B. $g(x) = \sin(x) - 2$

C. $g(x) = \sin(x) + 2$

D. $g(x) = -\sin(x) - 2$

E. $g(x) = \sin(x - 2)$

Question 19

If $f(x) = -\cos x \quad and \quad g(x) = 2\sin x$, then the value of $(fog)(0)$ is:

A. -1

B. 0

C. -2sin(1)

D. 1

E. 2

Question 20

The graph of the function $f(x) = \frac{1}{2}\sin(x)$ is transformed to the function $g(x) = -\frac{1}{2}\sin(2x - \pi) + 1$. The transformations required to transform the graph of $y = f(x)$ *to* $y = g(x)$ are:

A. Dilation of a factor of ½ unit from the *x*-axis, reflection in the *x*-axis, translation of $+\frac{\pi}{2}$ units parallel to the *x*-axis and translation of +1 unit parallel to the *y*-axis.

B. Dilation of a factor of 2 units from the *x*-axis, reflection in the *x*-axis, translation of $+\frac{\pi}{2}$ units parallel to the *x*-axis and translation of +1 unit parallel to the *y*-axis

C. Dilation of a factor of ½ unit from the *y*-axis, reflection in the *x*-axis, translation of $+\pi$ units parallel to the *x*-axis and translation of +1 unit parallel to the *y*-axis

D. Reflection in the *y*-axis, translation of $+\frac{\pi}{2}$ units parallel to the *x*-axis and translation of +1 unit parallel to the *y*-axis

E. Dilation of a factor of ½ unit from the *y*-axis, reflection in the *x*-axis, translation of $+\frac{\pi}{2}$ units parallel to the *x*-axis and translation of +1 unit parallel to the *y*-axis

SECTION B- Extended response questions

Instructions for Section B

- Answer each question in the space provided.
- Please provide appropriate workings and use exact answers when required.
- Unless otherwise stated, all decimals should be given correct to 2 decimal places.
- You are required to label axes appropriately.

Question 1

The height x metres, above the ground, of a person sitting on a Ferris wheel at time t seconds after the ride begins is modelled by the rule

$$x(t) = -6\cos\left(\frac{\pi}{24}t\right) + 8 \text{ , where } t \geq 0$$

a. How high is the person above ground before the ride begins?

_____

_____

1 mark

b. How long does the Ferris wheel take for one complete revolution?

_____

_____

2 marks

c. What is the height of the man above ground 4 seconds after the ride? Give your answer correct to three decimal places.

_____

_____

_____

2 marks

d. After how many seconds will the person be at a height of 9 m above the ground if the Ferris wheel completed 4 revolutions in one ride? Write to your answer correct to two decimal places.

3 marks

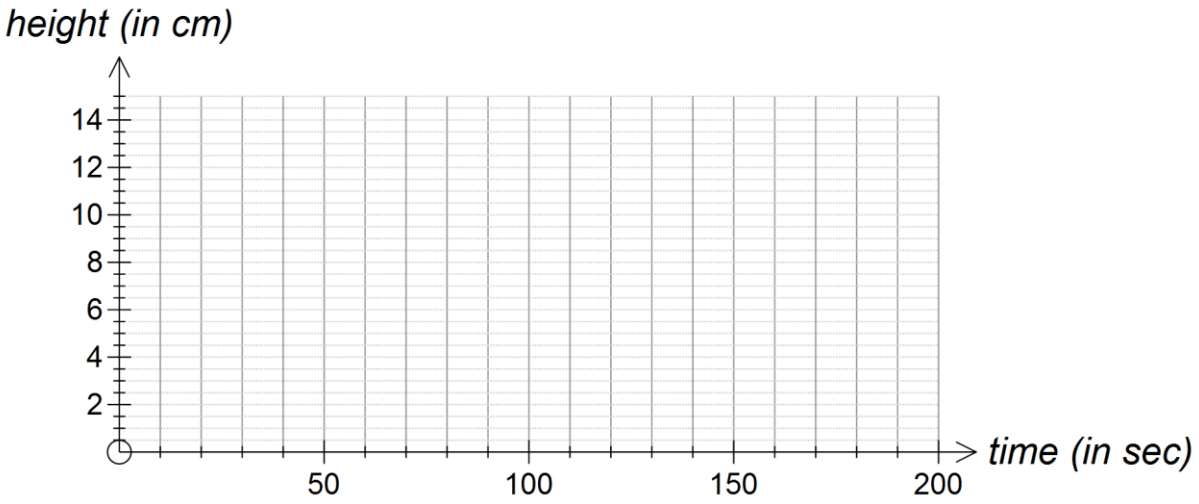
e. Find the average rate of change in height above the ground correct to two decimal places, in the first 5 seconds.

2 marks

f. What is the maximum height of the person above the ground?

1 mark

g. Sketch the graph of $x(t)$ for one ride, clearly indicating any axes intercepts and coordinates of turning points, with coordinates to the nearest whole number.



4 marks

Total 15 marks

Question 2

The velocity of a child sliding down a slide can be modelled by the following rule

$$v(t) = a\tan\left(\frac{\pi t}{n}\right) \quad for \quad 0 \le t \le 5$$

where v is the velocity in m/sec, t seconds after he starts to slide.

a. If the child takes 12 seconds to come down the slide, find the value of n.

2 marks

b. If the velocity of the child is 5m/sec after 3 seconds of the slide, find the value of a.

_____

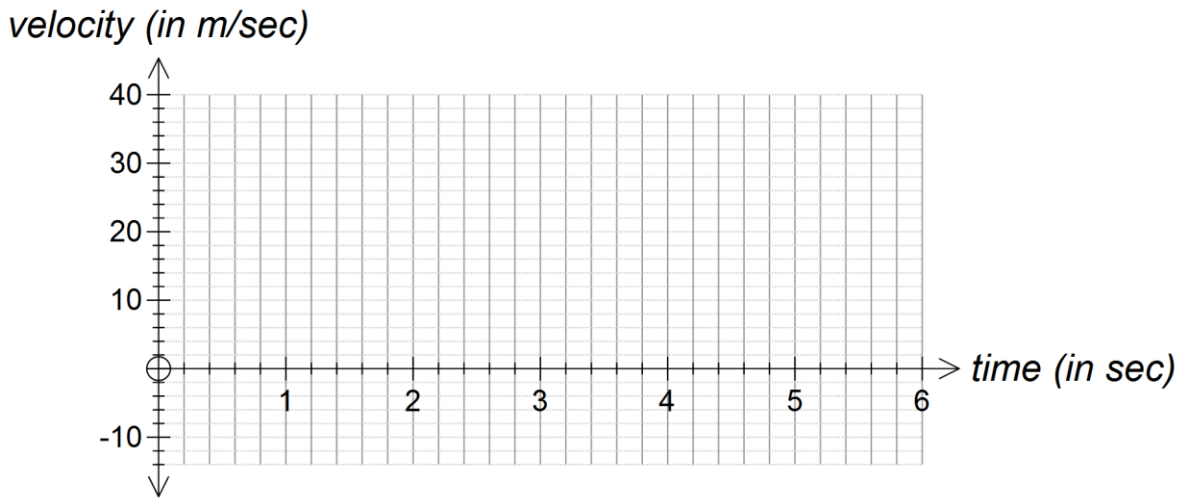
_____

_____

_____

1 mark

c. Hence, sketch the graph of $v(t)$ for the first five seconds of the slide, labelling all axes intercepts and end-points to the nearest whole number.



2 marks

Total 5 marks

END OF TASK BOOK