MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

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(*TSSM's *2015 trial exam updated for the current study design)

SOLUTIONS

Question 1

a. $3 - 5x \geq 0$

$x \leq \frac{3}{5}$

$Domain: \left(-\infty, \frac{3}{5}\right]$

A1

1 mark

b. $f'(x) = \frac{1}{2}(3-5x)^{-\frac{1}{2}} \times -5$

$f'(x) = -\frac{5}{2\sqrt{3-5x}}$

M1+A1

2 marks

c. $f'\left(\frac{1}{5}\right) = -\frac{5}{2\sqrt{3-1}} = -\frac{5}{2\sqrt{2}} = -\frac{5\sqrt{2}}{4}$

A1

1 mark

Question 2

a. $\int \sin(3x)\, dx = -\frac{\cos(3x)}{3} + c$

$0 = -\frac{1}{3} + c \ \ which\ gives\ \ c = \frac{1}{3}$

$F(x) = -\frac{\cos(3x)}{3} + \frac{1}{3}$

M2+A1

3 marks

b. $-\frac{\cos(3x)}{3}+\frac{1}{3}=\frac{1}{2}$

$\cos(3x)=-\frac{1}{2}$

$3x=\frac{2\pi}{3},\frac{4\pi}{3},\frac{8\pi}{3}$

$x=\frac{2\pi}{9},\frac{4\pi}{9},\frac{8\pi}{9}$

M2+A1

3 marks

Question 3

a. $y=\frac{x-2}{x+2}$

$x=\frac{y-2}{y+2}$

$\mathrm{yx}+2\mathrm{x}=\mathrm{y}-2$

$\mathrm{y}(\mathrm{x}-1)=-2-2\mathrm{x}$

$y=\frac{2+2x}{1-x}$

$f^{-1}(x)=\frac{2+2x}{1-x}$

M2+A1

3 marks

b. $Domain: R\backslash\{1\}$

$Range: R\backslash\{-2\}$

A2

2 marks

c. Using long division,

$f^{-1}(x)=-2+\frac{4}{1-x}$

$\int_0^{\frac{1}{2}}\left(-2+\frac{4}{1-x}\right)\,dx=[-2x-+4\log_e(1-x)]_0^{\frac{1}{2}}=-1-4\ln\left(\frac{1}{2}\right)=-1+4\ln 2$

M3+A1

4 marks

Question 4

a. $f'(x)=x^3-3x^2+2x$

$f'(x)=0\ \ gives\ \ x(x^2-3x+2)=0$

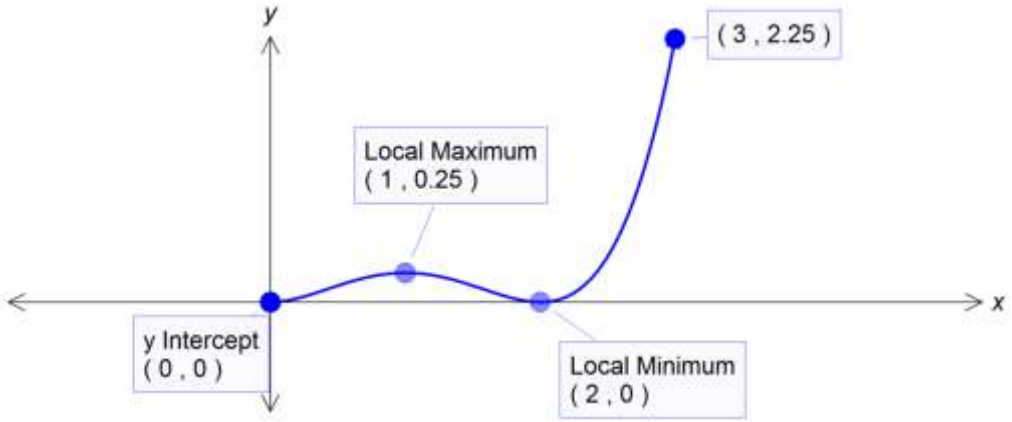
$x(x-2)(x-1)=0\ \ gives\ x=0,1,2$

$(0,0),\ \left(1,\frac{1}{4}\right),\ (2,0)$

M2+A1

3 marks

b.



1 for shape, 1 for stationary points, 1 for end points.

3 marks

c. $Area = \int_0^2 \left(\frac{1}{4}x^4 - x^3 + x^2\right)\, dx = \left(\frac{x^5}{20} - \frac{x^4}{4} + \frac{x^3}{3}\right)_0^2$

$Area = \frac{8}{5} - 4 + \frac{8}{3} = \frac{4}{15}\ \ square\ units$

M1+A1

2 marks

Question 5

a. $4000 = 5(2 + 7^{3x})$

$800 = 2 + 7^{3x}$

$798 = 7^{3x}$

$3x = log_7(798)$

$x = \frac{1}{3}log_7(798)$

M1+A1

2 marks

b. $2 \times 2^{2x} + 2^x - 1 = 0$

$2y^2 + y - 1 = 0,\ \ where\ \ y = 2^x$

$(2y - 1)(y + 1) = 0$

$y = \frac{1}{2}, -1$

$2^x = \frac{1}{2},\ 2^x = -1$

$x = -1\ \ (2^x = -1\ \ has\ no\ solution)$

M2+A1

3 marks

Question 6

a. $\frac{1}{5}+\frac{1}{10}+\frac{1}{3}+k=1$

$\mathrm{k}=\frac{11}{30}$

A1

1 mark

b. $\Pr(X<2)=\frac{1}{5}+\frac{1}{3}=\frac{8}{15}$

A1

1 mark

c. $\text{Mean}=\sum x\Pr(X=x)=0+\frac{1}{3}+\frac{1}{5}+\frac{11}{10}=\frac{49}{30}$

M1+A1

2 marks

Question 7

$\frac{dy}{dx}=-\frac{3}{x^2}$

$grad\ of\ tangent=-\frac{3}{a^2}$

$-\frac{3}{a^2}=-9$

$a=\pm\frac{1}{\sqrt{3}}$

$a=\frac{\sqrt{3}}{3}$

M1+A1

2 marks

Question 8

a. $\hat{p}=0.9$

A1

1 mark

b. $M=1.96\sqrt{\frac{0.9\times0.1}{r}}$

If you double r

$M=1.96\sqrt{\frac{0.9\times0.1}{2r}}$

Margin of error will decrease by a factor of $\sqrt{2}$

A1

1 mark