Neap:

Trial Examination 2015

VCE Mathematical Methods (CAS) Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

Question and answer booklet of 13 pages and a sheet of miscellaneous formulas.

Working space is provided throughout the booklet.

Instructions

Write your **name** and **teacher's name** in the space provided above on this page.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2015 VCE Mathematical Methods (CAS) Units 3&4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (3 marks)

If $h(x) = x\sin(x^2)$, find

a. $h'(x)$. 2 marks

b. $h'\left(\sqrt{\frac{\pi}{2}}\right)$. 1 mark

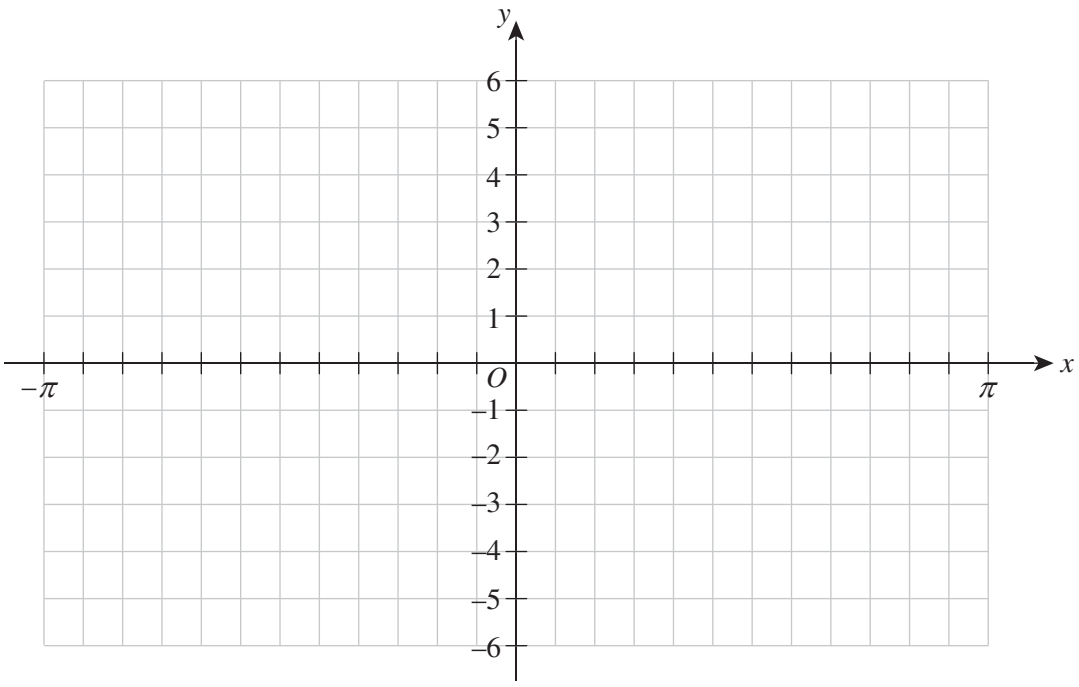
Question 2 (3 marks)

Find the antiderivative of the function $f(x) = e^{5x}(e^{x} + e^{-x})$, and hence find a function $g(x)$ such that $g'(x) = f(x)$ and $g(0) = 0$.

Question 3 (4 marks)

a. Solve $2\sin\left(2x-\frac{\pi}{6}\right)=\sqrt{3}$ for $-\pi<x<\pi$. 2 marks

b. Hence sketch the graph of $y=2\sin\left(2x-\frac{\pi}{6}\right)-\sqrt{3}$ for $-\pi<x<\pi$, showing all axial intercepts and endpoints. 2 marks



Question 4 (3 marks)

Solve the equation $3 \times 9^{x+1} + 1 = 5^2 \times 3^x + 3^{x+1}$.

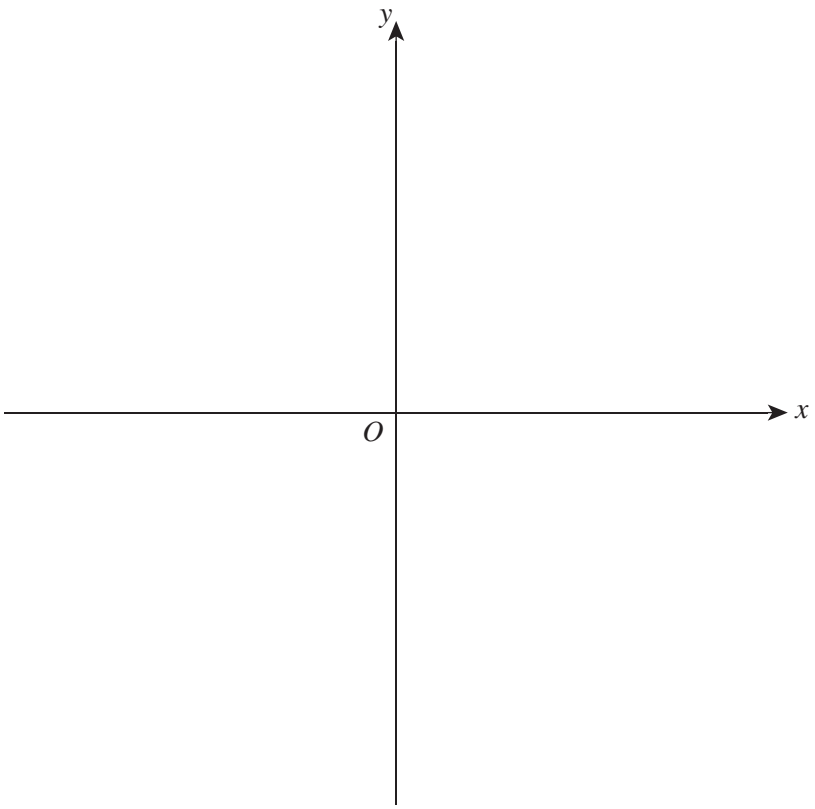
Question 5 (5 marks)

Consider $f\colon [-2, 3] \to R$, $f(x) = 6 + x - x^2$ and $g\colon (-3, 3) \to R$, $g(x) = |x|$.

a. Find the coordinates of the maximum point of f. 2 marks

b. **i.** Find the function $f(g(x))$. 1 mark

ii. Sketch the graph of $f(g(x))$, clearly labelling any axial intercepts and turning points with their coordinates. 2 marks



Question 6 (4 marks)

a. Given that $y = \log_e(x^3 + 1)$ for $x > -1$, find $\frac{dy}{dx}$. 2 marks

b. Hence find $\int (x + x^{-2})^{-1}\, dx$. 2 marks

Question 7 (4 marks)

A farmer is building a storage shed for wheat. He decides on a square-shaped floor of side length s metres, vertical walls of height h metres and a flat roof. The floor is to be built from concrete at a cost of $\frac{\$80}{9}$ per square metre. The roof is to be made of steel at a cost of $\frac{\$40}{9}$ per square metre. The walls will be built of reinforced concrete at a cost of \$10 per square metre and will require foundations around the perimeter of the square floor at a cost of \$10 per lineal metre.

a. Find an expression for C, the total cost of the project. 1 mark

b. Given that the total cost of the shed is to be \$4800, deduce an expression for h as a function of s. 1 mark

c. Hence, find an expression for the volume, V, of the shed that depends only on s. 1 mark

d. Find the height h and the side length s that will result in the maximum volume. 1 mark

Question 8 (3 marks)

A transformation $T: R^2 \to R^2$ that maps the graph of $g: R\backslash\left\{-\frac{1}{3}\right\} \to R,\ g(x) = \frac{1}{(3x+1)^2} + 2$ to the graph $h: R\backslash\{0\} \to R,\ h(x) = \frac{1}{x^2}$ has the rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b \\ c \end{bmatrix}\right)$, where a, b and c are non-zero real numbers.

Find the values of a, b and c.

Question 9 (5 marks)

The number of customers, X, waiting to be served in a bakery at 9:00 am has the probability distribution given in the table below.

X	0	1	2	3	4
$p(x)$	$\frac{3k^2-1}{7}$	$\frac{3k}{7}$	$\frac{4k}{7}$	$\frac{2k}{7}$	$\frac{k}{7}$

a. Find the value of k. 3 marks

b. Calculate the probability that there is at least one customer in the shop at 9:00 am. 2 marks

Question 10 (6 marks)

A continuous random variable has the following probability density function:

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 1, \text{ where } k \text{ is a constant} \\ \dfrac{7k - 3x}{4} & 1 < x \le \dfrac{7}{3} \\ 0 & \text{elsewhere} \end{cases}$$

a. Find the value of k. 2 marks

b. Find the exact value of the mode. 2 marks

c. Find the exact value of the mean. 2 marks

END OF QUESTION AND ANSWER BOOKLET