The Mathematical Association of Victoria

Trial Exam 2015

MATHEMATICAL METHODS (CAS)

Written Examination 2

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of examination

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 18 pages with a detachable sheet of miscellaneous formulas in the centrefold
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple – choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

SECTION A

Question 1

If $\cos(x) = \frac{2}{5}$ and $\frac{3\pi}{2} < x < 2\pi$ then $\tan(x)$ equals

A. $-\frac{\sqrt{21}}{2}$

B. $-\frac{\sqrt{21}}{5}$

C. $\frac{\sqrt{21}}{2}$

D. $\frac{2}{5}$

E. $-\frac{2}{5}$

Question 2

The amplitude and period of the graph of $f : R \to R, f(x) = -3\sin\left(\frac{x}{2}+1\right)+1$ are respectively

A. 3 and 4
B. 3 and 4π
C. -3 and 2π
D. -3 and 4
E. 4π and 6

Question 3

The graph of g where $g(x) = \tan(3x-1)+\dfrac{\pi}{2}$ has asymptotes with equations

A. $y = \dfrac{(2k+1)\pi+2}{6}, k \in Z$

B. $x = \dfrac{(2k+3)\pi}{6}, k \in Z$

C. $x = \dfrac{(4k+1)\pi+2}{6}, k \in Z$

D. $x = \dfrac{(2k+1)\pi+2}{6}, k \in Z$

E. $y = \dfrac{(2k+1)\pi}{6}, k \in Z$

Question 4

$A\cos^2(x) - B\cos(x) = 0$ where $x \in [0, 2\pi]$ and A and B are non zero real constants will have two solutions only if

A. $A = B$

B. $A = -B$

C. $|B| > |A|$

D. $A = \dfrac{3B}{2}$

E. $A = 2B$

Question 5

For the function $g(x) = \dfrac{1}{2}\sqrt{x^2+2x+1}$, then $g(a+1)$, where $a > -2$ equals

A. $a+2$

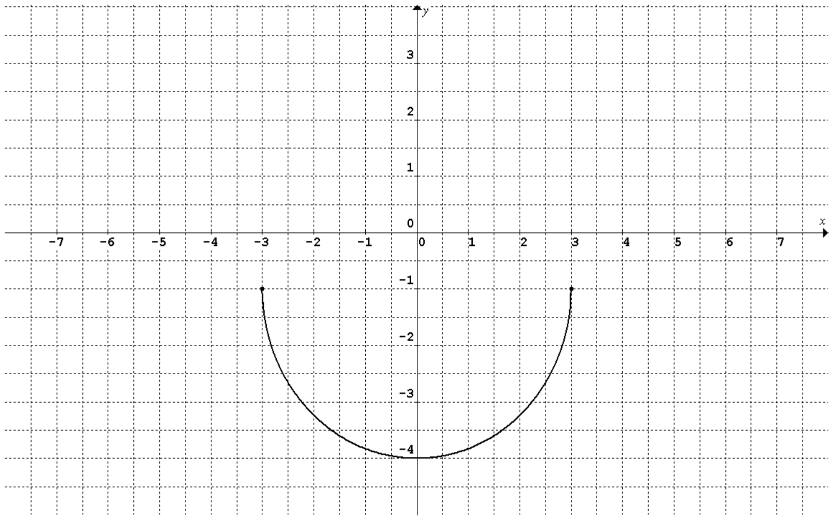
B. $\dfrac{1}{2}\sqrt{a(a+1)}$

C. $\dfrac{1}{2}\sqrt{a^2+1}$

D. $\dfrac{1}{2}(a+2)$

E. $-\dfrac{1}{2}|a+2|$

Question 6



A possible equation for the graph shown above is

A. $y = \dfrac{1}{4 - x^2}$

B. $y = -\sqrt{9 - x^2} - 1$

C. $y = -\sqrt{9 - x^2} + 1$

D. $y = \dfrac{1}{3}x^2 - 4$

E. $y = x^2 - 4$

Question 7

Consider the functions $g: R \setminus \{1\} \to R, g(x) = \dfrac{2}{x-1}$ and $f: [0, \infty) \to R, f(x) = (x+2)^2 + 1$.

The range of $f \times g$ is

A. $[0,1) \cup (1, \infty)$

B. $(-\infty, -10] \cup [12 + 4\sqrt{10}, \infty)$

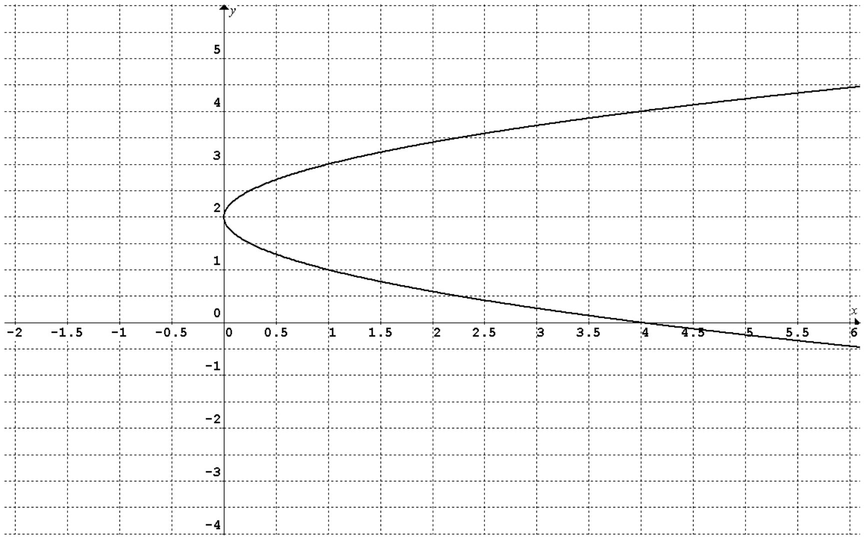
C. $[0, \infty)$

D. $(-\infty, 12 - 4\sqrt{10}] \cup [12 + 4\sqrt{10}, \infty)$

E. $(-\infty, -10] \cup [\sqrt{10} + 1, \infty)$

Question 8

The graph of a relation is shown below.



Which one of the following statements is **incorrect**?

A. The y-intercept is 2.

B. The range of the graph is R.

C. The domain of the graph is $[0, \infty)$.

D. The graph could have an equation in the form $y = \pm\sqrt{x} + 2$.

E. The graph is a many to one function.

Question 9

The image of the function $g(x) = 2x^4$ is $y = -\frac{2}{3}\left(\frac{x}{3}+1\right)^4$. The transformations that could have been applied are

A. reflection in the x-axis, then translation in the positive direction of the x axis by 1 unit, followed by a dilation from the y-axis by a factor of $-\frac{1}{3}$.

B. reflection in the x-axis, then translation in the negative direction of the x axis by $\frac{1}{3}$, followed by a dilation from the x-axis by a factor of $\frac{1}{3}$.

C. reflection in the x-axis, then a dilation from the y-axis by a factor of 3, then a translation in the negative direction of the x-axis by $\frac{1}{3}$, followed by a dilation from the x-axis by a factor of $\frac{1}{3}$.

D. reflection in the x-axis, then a dilation from the y-axis by a factor of 3, then a translation in the negative direction of the x-axis by 3 units, followed by a dilation from the x-axis by a factor of $\frac{1}{3}$.

E. reflection in the y-axis, then translation in the positive direction of the x axis by $\frac{1}{3}$, followed by a dilation from the y-axis by a factor of 3.

Question 10

The inverse f^{-1} of the function $f:(-\infty,-2)\to R, f(x)=\dfrac{-3}{x+2}$ is

A. $f^{-1}:(0,\infty)\to R, f^{-1}(x)=-\dfrac{3}{x}-2$

B. $f^{-1}:(-\infty,0)\to R, f^{-1}(x)=\dfrac{3}{x}-2$

C. $f^{-1}:(-\infty,-2)\to R, f^{-1}(x)=\dfrac{3}{x}-2$

D. $f^{-1}:(0,\infty)\to R, f^{-1}(x)=\dfrac{-3}{x+2}$

E. $f^{-1}:(0,\infty)\to R, f^{-1}(x)=\dfrac{x+2}{-3}$

Question 11

The graph with equation $y=(x+1)^2$ is transformed to its image equation $y=5(x-3)^2+2$.

A possible rule for the transformation $T:R^2\to R^2$ is

A. $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}1&0\\0&5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}0\\2\end{bmatrix}$

B. $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}5&0\\0&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}4\\2\end{bmatrix}$

C. $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}-1&0\\0&5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}2\\2\end{bmatrix}$

D. $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}-1&0\\0&5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}4\\2\end{bmatrix}$

E. $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}-1&0\\0&5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}-4\\2\end{bmatrix}$

Question 12

The instantaneous rate of change for the function with rule $f(x)=-\dfrac{3}{x}$ at $x = 3$ is

A. 1
B. -1
C. 3
D. $-\dfrac{1}{3}$
E. $\dfrac{1}{3}$

Question 13

Consider the function $h(x)=x^2+2x+1$.

The value(s) of x for $\{x : h'(x)=1\}$ is/are

A. 4
B. 0
C. $-\frac{1}{2}$
D. 0 and -2
E. 1 and -1

Question 14

A hybrid function is described by $f(x)=\begin{cases} x+1, x\in[2,\infty) \\ \sqrt{2x+1}, x\in(0,2) \\ x^{\frac{2}{3}}, x\in(-\infty,0] \end{cases}$

$f'\left(\frac{1}{2}\right)$ is equal to

A. $\sqrt{\frac{1}{2}}$
B. 1 and $\frac{\sqrt{2}}{2}$ and $\frac{2}{3}(2)^{\frac{1}{3}}$
C. $\frac{3}{2}$ and $\sqrt{2}$ and $\left(\frac{1}{2}\right)^{\frac{2}{3}}$
D. $\frac{2^{\frac{4}{3}}}{3}$
E. 1

Question 15

Right endpoint rectangles with widths of one unit are used to find the approximate area bounded by the graph with equation $f(x)=\frac{2}{(x-3)^2}+1$, the x-axis and the lines with equations $x=-1$ and $x=2$.

This area can be found by evaluating

A. $f(-1)+f(0)+f(1)$
B. $f(-1)+f(0)+f(1)+f(2)$
C. $f(0)+f(1)$
D. $f(5)+f(6)+f(7)$
E. $f(4)+f(5)+f(6)$

Question 16

The average value of the function *f*, where $f(x) = -e^{2\cos(x+\pi)}$ over the interval $[-\pi, 0]$ is closest to

A. -7.2
B. -2.3
C. -1.1
D. 1.1
E. 2.3

Question 17

If $\int_{-1}^{3} h(x)dx = 5$ then $\int_{3}^{-1} (1+5h(x))dx$ equals

A. 5
B. -25
C. -29
D. -21
E. 21

Question 18

If X is the random variable with probability density function f and rule

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

then the mean, standard deviation and median are respectively

A. 1, 1 and 1
B. 1, $\sqrt{2}$ and $\log_e(2)$
C. $\log_e(2)$, $\sqrt{2}$ and 1
D. $\log_e(2)$, 1 and 1
E. 1, 1 and $\log_e(2)$

Question 19
Consider the following probability distribution where *a*, *b* and *c* are real constants.

x	1	3	5	7
$\Pr(X = x)$	0.1	a	b	c

If the mean is 4 and the variance 2 then which one of the following is **false**?

A. $a = 0.325$, $b = 0.55$ and $c = 0.025$
B. $a + b + c = 0.9$
C. $3a + 5b + 7c = 3.9$
D. $9a + 25b + 49c = 1.9$
E. The median is 5.

Question 20

Olga sat five examinations. The mean, standard deviation and Olga's score for each examination are shown in the following table.n The scores for each examination are normally distributed.

Examination	**Mean μ (%)**	**Standard Deviation σ (%)**	**Olga's Score (%)**
Mathematics	55	7	60
English	70	10	75
Chemistry	60	8	65
Physics	50	6	55
History	80	7	70

Relatiive to other students in each subject, Olga's best examination result was in

A. Mathematics
B. English
C. Chemistry
D. Physics
E. History

Question 21

The heights of a certain population of children are normally distributed with mean 130 cm and variance 36 cm. The $\Pr\left(X < 135 \mid X > 130\right)$ equals

A. $\dfrac{\Pr(130 < X < 135)}{2}$

B. $2\Pr\left(125 < X < 130\right)$

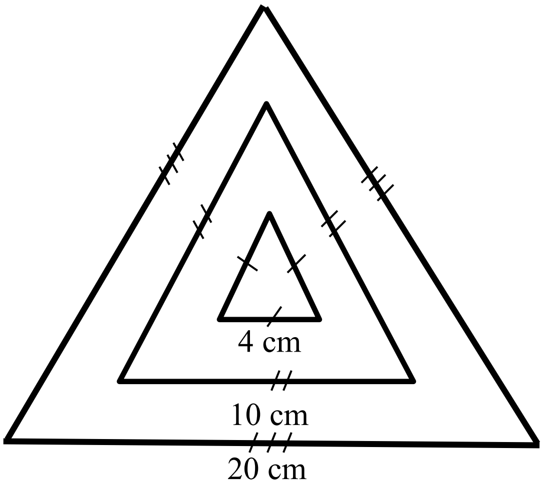
C. 0.5952

D. $\dfrac{\Pr(X < 135)}{\Pr\left(X > 130\right)}$

E. $2\Pr\left(X < 135\right)$

Question 22

A dart board has three equilateral triangle sections of lengths 4 cm, 10 cm and 20 cm as shown in the diagram below. The dart cannot land on the boundaries of any of the triangles.



If the dart lands in the centre triangle 10 points are scored, in the middle section 5 points and 1 point in the outer section. Let X represent the total score from two throws. All darts land on the board and have an equal likely chance of landing at any point on the board.

The probability of scoring 15 points would be

A. $\frac{21}{100}\times\frac{4}{100}\times 2$

B. $\frac{25}{100}\times\frac{4}{100}$

C. $\frac{25}{100}\times\frac{4}{100}\times 2$

D. $\frac{21}{100}\times\frac{4}{100}$

E. $\frac{75}{100}\times\frac{21}{100}\times 2$

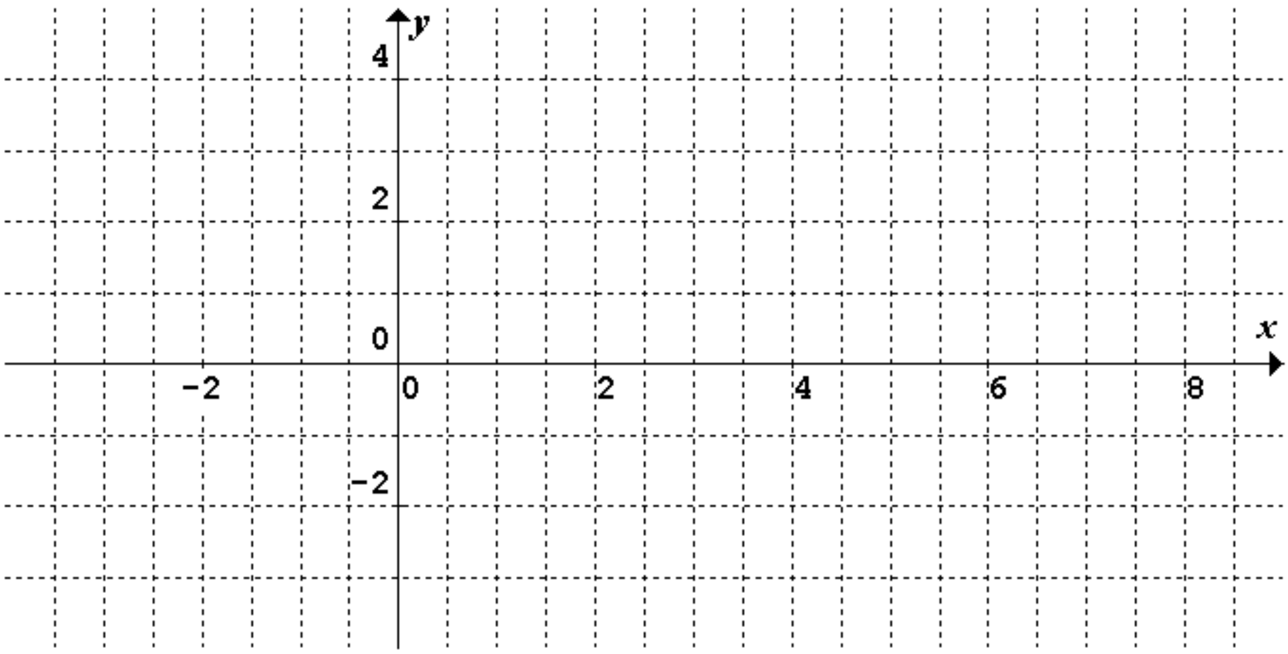
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (13 marks)

a. On the axes below sketch the graph of $f:[2,8) \to R, f(x)=\sqrt{x-2}$, labelling the endpoints with their coordinates. 3 marks



b. Find $f(4)$. 1 mark

c. Find $\left\{x: f(x)=\frac{1}{2}\right\}$. 1 mark

A tangent is drawn to the graph of $f:[2,8)\to R, f(x)=\sqrt{x-2}$ at $x = 4$.

d. Find the equation of this tangent. 1 mark

e. Write down an integral that will find the area bounded by the tangent found in **part d.**, the curve f, and the x-axis. 3 marks

f. Evaluate the integral in **part e**. 1 mark

g. Hence or otherwise, find the area bounded by the normal to the curve f at $x = 4$, the curve f, and the x-axis. 3 marks

Question 2 (15 marks)

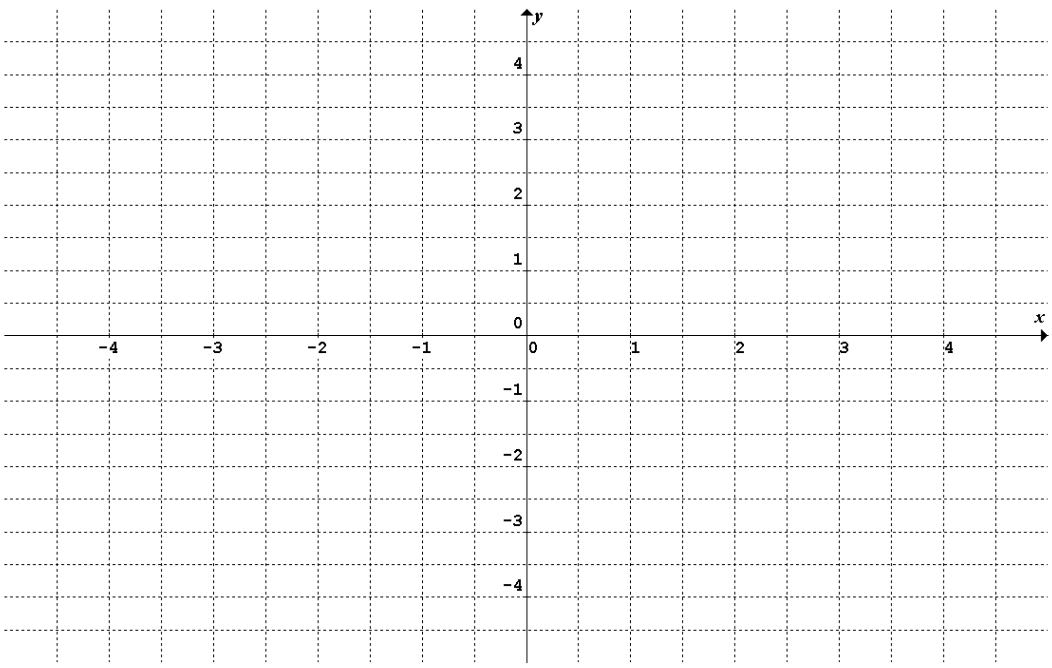
The function g is defined by $g:(-\infty,a)\to R, g(x)=-\log_e(5-2x)+1$, where a is a real constant.

a. Find the greatest value of a for the maximal domain. 1 mark

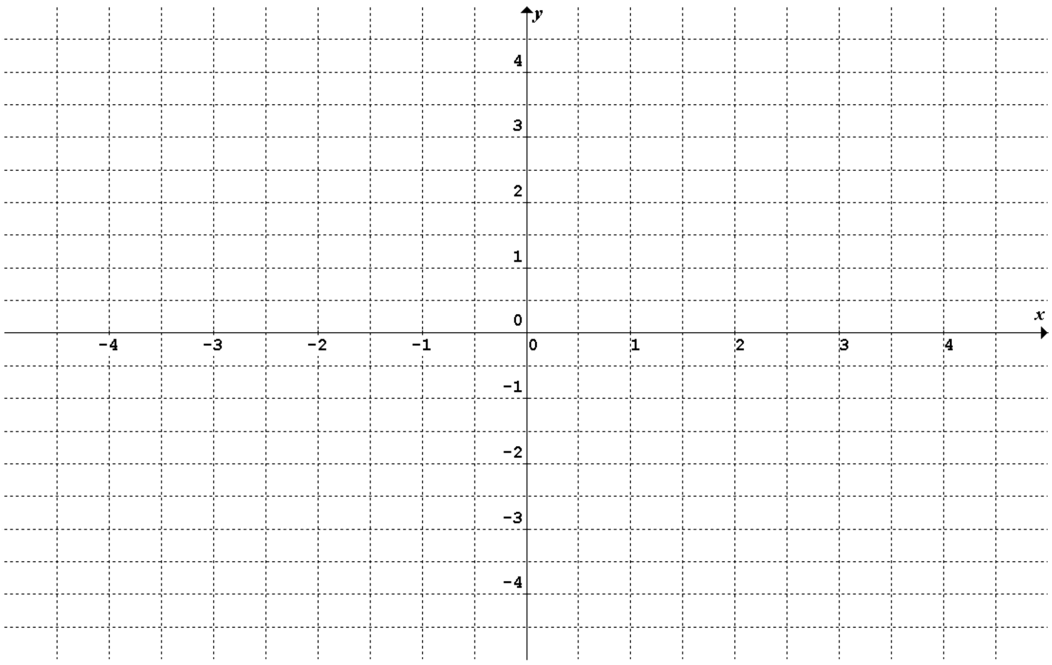
b. Find $\{x: g(x)=0\}$. 1 mark

c. Find $g(0)$. 1 mark

d. Hence sketch the graph of g on the axes below, labelling the asymptotes with their equations, and the axial intercepts with their coordinates. 3 marks



e. Sketch the graph of $y = g\left(|x|\right)$, labelling the axial intercepts with their coordinates and the asymptotes with their equations. 3 marks



f. State the domain for which $y = g\left(|x|\right)$ is differentiable. 1 mark

g. Hence find $g'\left(|x|\right)$ 3 marks

The graph of $g:\left(-\infty, a\right) \rightarrow R, g(x) = -\log_e\left(5-2x\right)+1$, for a maximal domain, can be transformed to the graph of $h(x) = \log_e\left(x\right)$.

h. State a set of transformations, in order, that would be needed to change $g(x)$ to $h(x)$. 2 marks

Question 3 (13 marks)

The side view of a water slide, S_1, in a swimming pool can be modelled by the function S where S is the height of the slide above the water, in metres, and x is the horizontal distance, in metres, of the slide from the side of the pool.

$$S:\left[0,\frac{5\pi}{4}\right]\to R,\ \text{where } S(x)=\tan\left(-\frac{1}{2}\left(x-\frac{\pi}{3}\right)\right)+6$$

The side view of another slide, B_1, in the same pool can be modelled by the function B where B is the height of the slide above the water, in metres, and x is the horizontal distance, in metres, of the slide from the side of the pool.

$$B:\left[0,\frac{\pi}{4}\right]\to R,\ \text{where } B(x)=3\cos\left(\frac{x}{2}\right)$$

S_1 is vertically above B_1

a. At what value of x is $|S'(x)|$ a minimum? 1 mark

b. What is the minimum value of S? Give your answer to the nearest cm. Interpret your result. 2 marks

c. Define a function, h which represents the vertical distance between the two slides? 2 marks

d. What is the minimum vertical distance between the two slides? Give your answer to the nearest cm. 2 marks

e. What is the horizontal distance between the slides when B_1 is furthest from the side of the pool? Give your answer to the nearest cm. 2 marks

Waves are generated across the pool at certain times during the day. The height of the waves, *w* m above the original water level, as they hit the side of the slides is given by the rule

$w(t) = 2\sin\left(\frac{\pi}{3}t\right) + 2$, where *t* is the time in seconds and $0 \le t \le 300$.

f. How many waves are generated during this time? 2 marks

g. For what proportion of time will B_1 be completely under water? 2 marks

Question 4 (17 marks)

If Jeremy passes a maths test the probability he will pass the next maths test is 0.6. If he fails a maths test the probability he will pass the next maths test is 0.3. Jeremy failed the first maths test.

a. What is the probability Jeremy will pass the next three tests? 1 mark

b. What is the probability Jeremy will pass only one of the next three tests? 2 marks

c. Given that Jeremy passed at least one of the next three tests, what is the probability he passed all three tests? 2 marks

d. What is the probability Jeremy will pass the sixth test? Give your answer correct to three decimal places. 2 marks

e. In the long term is Jeremy more likely to pass or fail a test. Explain using long term values. 2 marks

The ATAR scores of applicants for a particular university were normally distributed. The university decided to accept the top 15% of applicants. Jeremy was accepted into the university with an ATAR score of 86. Only 10% applicants had a better score than him. His friend Jody was offered a scholarship. Her ATAR score was 95 and only 1% of applicants had a better score than hers.

f. What was the mean and standard deviation of the ATAR scores? Give your answers correct to two decimal places. 4 marks

g. What was the lowest ATAR score for entrance into the university? Give your answer correct to two decimal places. 1 mark

Jody wants to join one of the university's sports clubs. The probability she will be selected for soccer and not rugby is p. The probability she will selected for rugby is p^3 and the probability she will be selected for neither sport is p^2. Being selected for soccer is independent of being selected for rugby.

h. What is the probability Jody will be selected for both sports? Give your answer correct to two decimal places. 3 marks

END OF QUESTION AND ANSWER BOOK