

**Year 2015**  
**VCE**  
**Mathematical Methods**  
**Trial Examination 2**



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**Victorian Certificate of Education  
2015**

**STUDENT NUMBER**

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**MATHEMATICAL METHODS CAS  
Trial Written Examination 2**

Reading time: 15 minutes

Total writing time: 2 hours

**QUESTION AND ANSWER BOOK**

**Structure of book**

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator ( memory DOES NOT need to be cleared ) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

- Question and answer booklet of 30 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

**Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions, and sign your name in the space provided.
- All written responses must be in English.

**At the end of the examination**

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**SECTION 1****Instructions for Section I**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is **correct** for the question.

A correct answer scores 1 mark, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No mark will be given if more than one answer is completed for any question.

**Question 1**

Consider the linear function  $f : [-a, 2a) \rightarrow R$ ,  $f(x) = a - x$  where  $a \in R \setminus \{0\}$ .

The range of the function is

- A.  $R$
- B.  $[-a, 2a)$
- C.  $(-a, 2a]$
- D.  $[-2a, a)$
- E.  $(-2a, a]$

**Question 2**

If  $a$  and  $b$  are real constants, then the function  $f : (-\infty, a] \rightarrow R$ ,  $f(x) = x^2 + 2bx$ , will have an inverse function if

- A.  $a = b$
- B.  $a + b < 0$
- C.  $a + b > 0$
- D.  $a > b$
- E.  $a < b$

**Question 3**

The solution of  $|k - x| < 2k$  is given by

- A.  $-k < x < k$
- B.  $k < x < 3k$
- C.  $-k < x < 3k$
- D.  $3k < x < -k$
- E.  $k < x < -3k$

**Question 4**

Let  $f$  be a function with domain  $R$ . The function has the following properties

$$f'(x) < 0 \text{ for } |x| < a, \quad f'(x) > 0 \text{ for } |x| > a, \quad f'(-a) = 0 \text{ and } f'(a) = 0.$$

Then the graph of  $f$  has

- A. a local minimum at  $x = -a$  and a local maximum at  $x = a$
- B. a local maximum at  $x = -a$  and a local minimum at  $x = a$
- C. local maximums at  $x = \pm a$
- D. local minimums at  $x = \pm a$
- E. stationary points of inflexion at  $x = \pm a$

**Question 5**

Given that  $\frac{d}{dx}[f(x)] = g(x)$  and  $h(x) = x^2$  then  $\frac{d}{dx}[f(h(x))]$  is equal to

- A.  $g'(x^2)$
- B.  $x^2 g'(x^2)$
- C.  $2x g'(x^2)$
- D.  $2x g(x^2)$
- E.  $2x g(x)$

**Question 6**

If  $f(x) = \sqrt{x} g(x)$  and  $g(4) = 8$ ,  $g'(4) = -1$ , then  $f'(4)$  is equal to

- A. 2
- B. 0
- C.  $-\frac{1}{4}$
- D. -2
- E. -4

**Question 7**

Consider the functions  $f : (-\pi, \pi) \rightarrow \mathbb{R}$ ,  $f(x) = \sin(x)$  and  $g(x) = |x|$ .

Which of the following is **false**?

- A.  $f(g(x)) = g(f(x))$
- B. The domain of  $g(f(x))$  is  $(-\pi, \pi)$  and the range of is  $[0, 1]$
- C.  $\lim_{x \rightarrow 0} f(g(x)) = 0$
- D.  $f(g(x))$  is continuous at  $x = 0$ .
- E.  $g(f(x))$  is differentiable at  $x = 0$ .

**Question 8**

The inverse of the function  $f : \mathbb{R}^+ \rightarrow \mathbb{R}$ ,  $f(x) = x^2 - 4$  is

- A.  $f^{-1} : \mathbb{R} \setminus \{\pm 2\} \rightarrow \mathbb{R}$ ,  $f^{-1}(x) = \frac{1}{x^2 - 4}$
- B.  $f^{-1} : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f^{-1}(x) = x + 2$
- C.  $f^{-1} : (4, \infty) \rightarrow \mathbb{R}$ ,  $f^{-1}(x) = -\sqrt{x - 4}$
- D.  $f^{-1} : (-4, \infty) \rightarrow \mathbb{R}$ ,  $f^{-1}(x) = \sqrt{x + 4}$
- E.  $f^{-1} : (-4, \infty) \rightarrow \mathbb{R}$ ,  $f^{-1}(x) = -\sqrt{x + 4}$

**Question 9**

The point  $(3, -4)$  lies on the graph of the function  $y = f(x)$ . The graph of the function  $y = g(x)$  passes through the point  $(-5, 2)$ , Then

- A.  $g(x) = -f(-x - 2) - 2$
- B.  $g(x) = f(x + 2) - 2$
- C.  $g(x) = 2 - f(x - 2)$
- D.  $g(x) = 2 - f(2 - x)$
- E.  $g(x) = 2 - f(x + 2)$

**Question 10**

If  $\int_1^3 (4 - 3f(x)) dx = 2$ , then  $\int_3^1 f(x) dx$  is equal to

- A.  $-\frac{2}{3}$
- B.  $\frac{2}{3}$
- C.  $-2$
- D.  $2$
- E.  $1$

**Question 11**

The graph of the function  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^3 + bx^2 + c^2x$  crosses the  $x$ -axis only once and has two stationary points. If  $b$  and  $c$  are non-zero positive constants then

- A.  $0 < \frac{b}{c} < 2$
- B.  $\frac{b}{c} > 2$
- C.  $\frac{b}{c} > \sqrt{3}$
- D.  $0 < \frac{b}{c} < \sqrt{3}$
- E.  $\sqrt{3} < \frac{b}{c} < 2$

**Question 12**

A certain curve has its gradient given by  $6\cos\left(\frac{x}{3}\right)$ . If the curve crosses the  $x$ -axis at

$x = \frac{\pi}{2}$  then it crosses the  $y$ -axis at

- A.  $-9\sqrt{3}$
- B.  $-9$
- C.  $-3$
- D.  $-\sqrt{3}$
- E.  $\sqrt{3}$

**Question 13**

The area of the region enclosed by the graph of  $y = x(x+a)$ , where  $a > 0$ , the  $x$ -axis and the line  $x = a$  is

A.  $a^3$

B.  $\frac{a^3}{3}$

C.  $\frac{a^3}{6}$

D.  $\frac{5a^3}{6}$

E.  $\frac{2a^3}{3}$

**Question 14**

The transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ , which maps the curve with equation  $y = \frac{1}{x}$  to the

curve with equation  $y = 4 - \frac{2}{x-3}$ , has the rule

A.  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ -2 \end{bmatrix}$

B.  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

C.  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

D.  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

E.  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$



**Question 15**

The height  $h$  metres above the ground of a capsule on the London Eye, is given by

$h(t) = 68 - 67 \cos\left(\frac{\pi t}{15}\right)$  where  $t$  is the time in minutes and  $t \geq 0$ . The average height of a particular capsule in metres, between  $t = 3.75$  and  $t = 5$  is closest to

- A. 11.10
- B. 11.13
- C. 13.88
- D. 27.33
- E. 34.16

**Question 16**

If  $n \in \mathbb{Z}$ , then the graph of  $y = e^{-\frac{x}{3}} \cos\left(\frac{x}{3}\right)$

- A. crosses the  $x$ -axis at  $x = 3(4n - 1)\frac{\pi}{4}$  and has turning points at  $x = 3(2n - 1)\frac{\pi}{2}$
- B. crosses the  $x$ -axis at  $x = 3(4n - 1)\frac{\pi}{4}$  and has turning points at  $x = 3n\pi$
- C. crosses the  $x$ -axis at  $x = 3(2n - 1)\frac{\pi}{2}$  and has turning points at  $x = 3n\pi$
- D. crosses the  $x$ -axis at  $x = 3(2n - 1)\frac{\pi}{2}$  and has turning points at  $x = 3(4n - 1)\frac{\pi}{4}$
- E. crosses the  $x$ -axis at  $x = 3n\pi$  and has turning points at  $x = 3(4n - 1)\frac{\pi}{4}$

**Question 17**

Let  $A$  be the area bounded by the graph of  $y = \frac{1}{2}(x - 2)^2 + 1$ , the coordinate axes and  $x = 5$ .

The area is approximated using five equally spaced rectangles. Let  $L$  be the approximation using left rectangles and  $R$  be the approximation using right rectangles. Then

- A.  $L < A < R$
- B.  $L < A < E$
- C.  $A < L < R$
- D.  $R < A < L$
- E.  $R < L < A$

**Question 18**

A and B are independent events, with  $\Pr(A) = \sqrt{p}$  and  $\Pr(B) = \frac{p}{4}$ , where  $0 < p < 1$ .

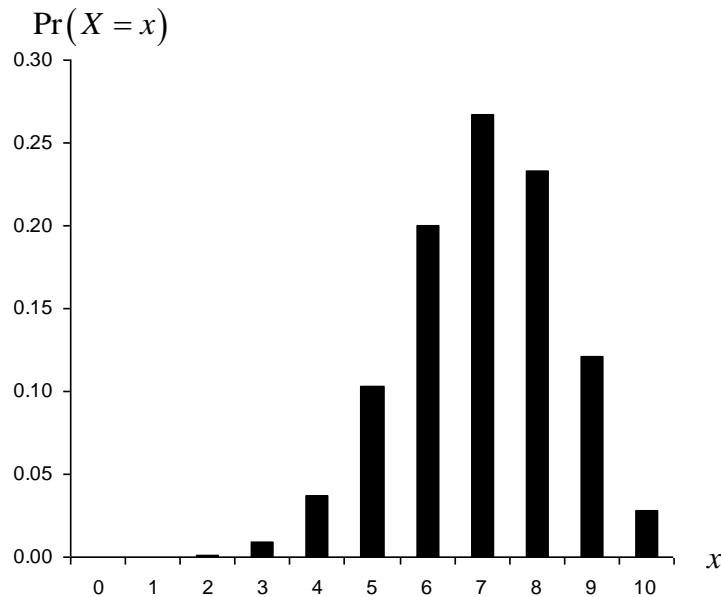
Then  $\Pr(A' \cap B')$  is equal to

- A.  $\frac{1}{4}p(3 - \sqrt{p})$
- B.  $\frac{1}{4}(1 - \sqrt{p})(p - 4)$
- C.  $\frac{1}{4}(\sqrt{p} - 1)(p - 4)$
- D.  $\frac{1}{4}p(1 - \sqrt{p})$
- E.  $\frac{1}{4}(4 - p\sqrt{p})$

**Question 19**

The probability function of a binomial random variable X is shown. If the number of trials is 10, and p is the probability of a success on any one trial, then the most likely value for p is

- A. 0.1
- B. 0.3
- C. 0.5
- D. 0.7
- E. 0.9



**Question 20**

A binomial random variable  $X$  with  $n$  trials, has a probability of  $p$  on any one trial, where  $0 < p < 1$ . If  $\Pr(X = 0) = A$ , then  $\Pr(X = 2)$  is equal to

- A.  $\frac{n(n-1)p^2A}{2(1-p)^2}$
- B.  $\frac{n(n-1)p^2A}{(1-p)^2}$
- C.  $\frac{1}{2}n(n-1)(1-p)^2 p^2A$
- D.  $A(np^2 + 1)$
- E.  $1 - A(np^2 + 1)$

**Question 21**

The random variable  $Z$  has the standard normal distribution, with  $\Pr(0 < Z < a) = A$  and  $\Pr(0 < Z < b) = B$ , where  $b > a$ . Then  $\Pr(Z < b | Z > a)$  is equal to

- A.  $\frac{B - A}{0.5 - A}$
- B.  $\frac{B - A}{1 - A}$
- C.  $\frac{0.5 + B}{0.5 - A}$
- D.  $\frac{A + B - 1}{0.5 - A}$
- E.  $\frac{A + B - 1}{1 - A}$

**Question 22**

A discrete random variable  $X$  has the following probability distribution.

$X$	0	$A$
$\Pr(X = x)$	$1 - p$	$p$

If  $E(X) = \frac{4}{3}$  and  $\text{Var}(X) = \frac{8}{9}$  then

- A.  $A = 1$  and  $p = \frac{3}{4}$
- B.  $A = 2$  and  $p = \frac{2}{3}$
- C.  $A = 2$  and  $p = \frac{1}{3}$
- D.  $A = 3$  and  $p = \frac{4}{9}$
- E.  $A = 4$  and  $p = \frac{1}{3}$

**END OF SECTION 1**

**SECTION 2****Instructions for Section 2**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1** (7 marks)

At a busy intersection near a school, the rate  $R(t)$  of cars per hour that turn right during the morning peak is given by  $R(t) = 200\sqrt{t} \cos\left(\frac{t^2}{6}\right)$  for  $0 \leq t \leq 3$ . Where  $t = 0$  corresponds to 6 am and  $t = 3$  corresponds to 9:00 am.

a. Find  $R(3)$ , correct to one decimal place.

1 mark

b. Find the time when the rate at which the cars turn right is a maximum and find the maximum rate. Give both answers correct to one decimal place.

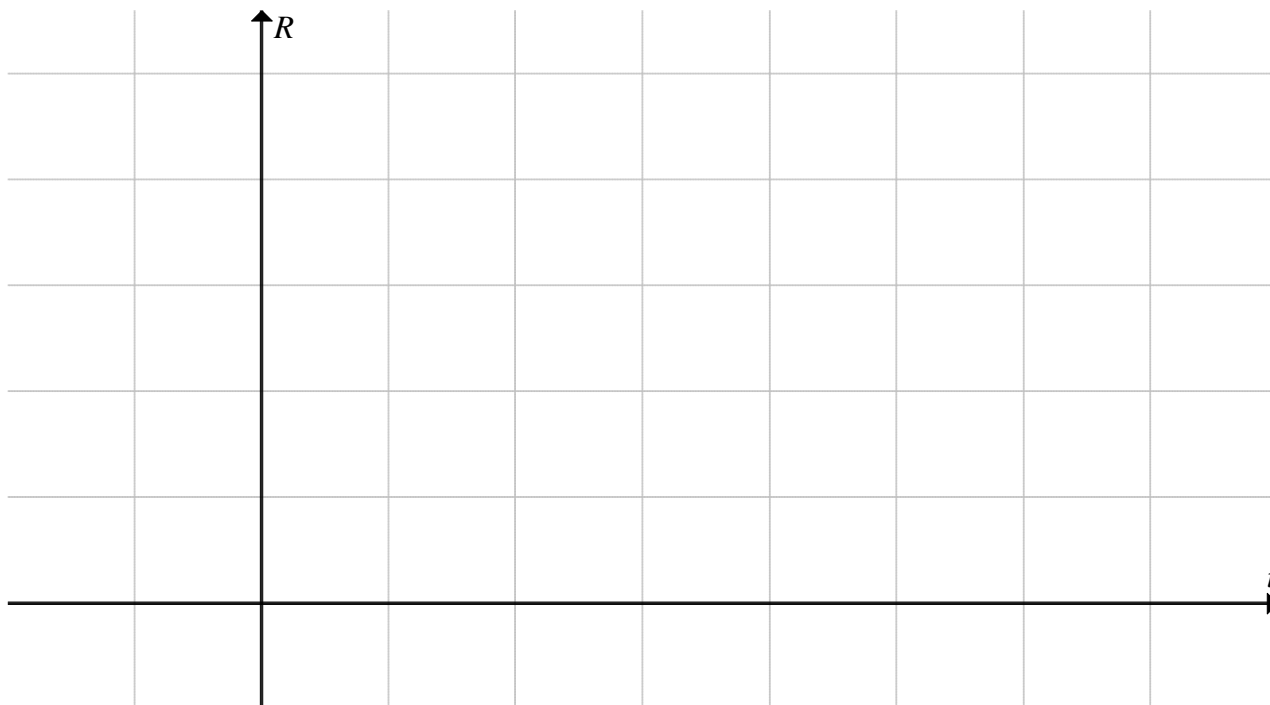
2 marks

c. Over the morning peak, find the total number of cars turning right.

1 mark

- d. Sketch the graph of  $R(t)$  versus  $t$  on the axis below, clearly labelling the scale and showing critical points as coordinates correct to one decimal place.

1 mark



- e. Find the times correct to three decimal places, when the rate is 150 cars per hour and determine the average value of this rate over this time interval. Give your answer correct to one decimal place.

2 marks

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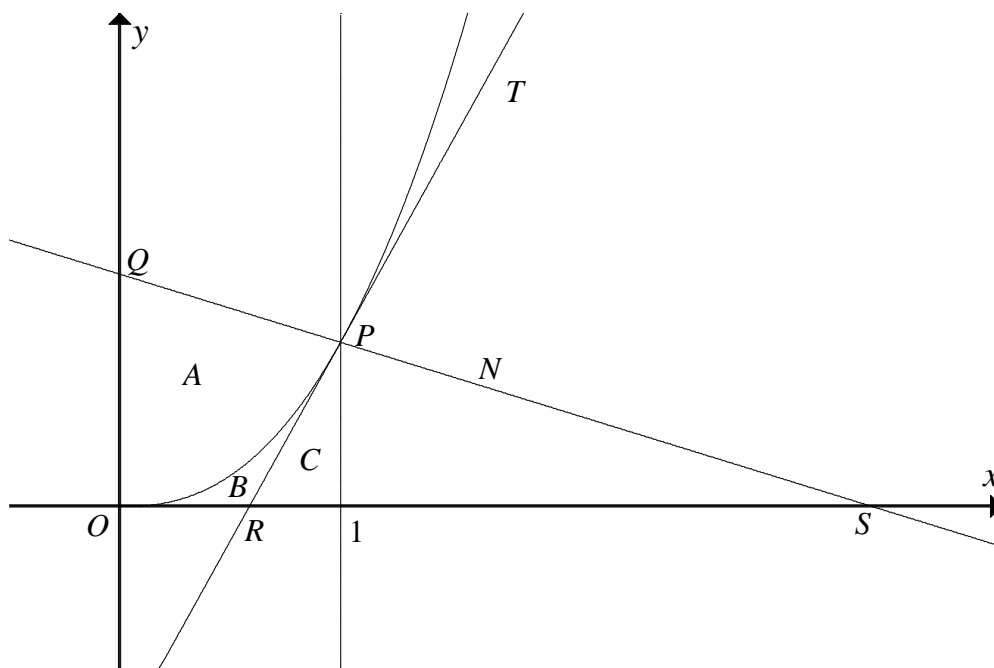
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**Question 2** (12 marks)

The diagram shows the point  $P$ , where  $x = 1$  on part of the graph of  $y = x^p$ , where  $p \in \mathbb{R}$  and  $p > 1$ .



a. Let  $T$  be the tangent to the curve  $y = x^p$  at the point  $P$ .

i. Find the equation of the tangent  $T$ .

1 mark

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ii. The tangent  $T$ , crosses the  $x$ -axis at the point  $R$ . Show that  $R$  has coordinates  $\left(\frac{p-1}{p}, 0\right)$ .

1 mark

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iii. Let  $C$  be the area bounded by the triangular region, the tangent  $T$ , the points  $R$  and  $P$ , the  $x$ -axis and the line through  $x = 1$ . Find the area  $C$  in terms of  $p$ .

1 mark

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iv. The area  $B$ , is the area bounded by the curve  $y = x^p$ , the origin  $O$ , the tangent  $T$  and the  $x$ -axis. Hence or otherwise, write down using a definite integral the area  $B$  in terms of  $p$ .

1 mark

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v. Find the area  $B$  in terms of  $p$ .

1 mark

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vi. Find the value of  $p$  which maximizes the area  $B$ .

2 marks

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**b.** Let  $N$  be the equation of the normal to the curve  $y = x^p$  at the point  $P$ .

**i.** Show that the equation of the normal  $N$  is given by  $y = \frac{p+1}{p} - \frac{x}{p}$ .

1 mark

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**ii.** The normal  $N$ , crosses the  $y$ -axis at  $Q$  and the  $x$ -axis at  $S$ . Determine the coordinates of the points  $Q$  and  $S$ .

1 mark

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**iii.** The area  $A$ , is the area bounded by the  $y$ -axis, the curve  $y = x^p$  and the normal  $N$ . Write down in terms of  $p$  a definite integral for the area  $A$ .

1 mark

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**iv.** Find the value of  $p$  which optimizes the area  $A$ . Is this area a maximum or a minimum?

2 marks

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**Question 3** (18 marks)

- a.** A packet of pens, contains 8 blue, 7 red and 5 black pens. Three pens are drawn without replacement from the packet. Find the probability that there is one pen of each colour.

1 mark

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- b.** Every day Lilly does a cross word puzzle. She either uses a red pen or a blue pen to do the cross word puzzle. If on one day she uses a blue pen, the probability that she uses a red pen the next day is 0.6. If on one day she uses a red pen, the probability that she uses a red pen the next day is 0.3. On Monday she did the cross word puzzle using a red pen.

- i.** Find the probability, correct to three decimal places that from Monday to Thursday, Lilly uses a red pen exactly three times.

2 marks

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- ii.** Find the probability, correct to three decimal places that on Thursday, Lilly uses a red pen.

2 marks

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- iii.** In the long run, is Lilly more likely to use a red or a blue pen?

1 mark

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c. The time  $T$  in minutes, for Lilly to complete the crossword puzzle was found to satisfy a

probability density function, defined by 
$$T(t) = \begin{cases} a \cos\left(\frac{\pi t}{80}\right) & \text{for } 0 \leq t \leq 40 \\ 0 & \text{elsewhere} \end{cases}$$

i. Show that  $a = \frac{\pi}{80}$ .

2 marks

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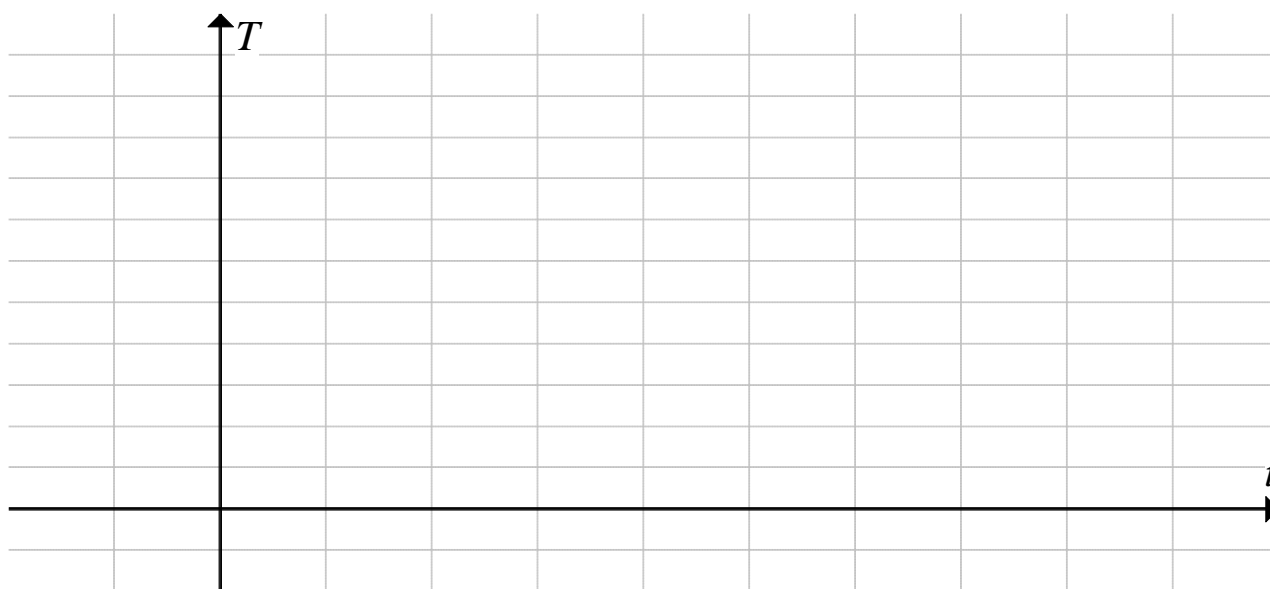
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ii. Sketch the graph of  $T(t)$  on the axes below, clearly labelling the scale.

1 mark



**iii.** Find the probability correct to four decimal places that Lilly completes the crossword puzzle in under half an hour.

1 mark

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**iv.** Find the mean time, that Lilly takes to complete the crossword puzzle.

1 mark

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**v.** Find the median time correct to two decimal places, that Lilly takes to complete the crossword puzzle.

2 marks

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- d.** The linear distance that a certain type of pen can write for, before it becomes unusable is found to be normally distributed. It is found that 9.1% of these pens write for a linear distance of more than 2600 metres, while 32.8% of these pens write for a distance of less than 1800 metres. Find the mean and standard deviation correct to the nearest metre, of the linear distance that this brand of pen can write for.

4 marks

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- e.** Another brand of pen writes for a linear distance which is also normally distributed. This brand is found to have a mean of 2500 metres with a standard deviation of 300 metres. Find the probability, correct to four decimal places that at least one of three of these brands of pens writes for a linear distance of at least 3000 metres.

1 mark

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**Question 4** (10 marks)

When an above ground plastic swimming pool is filled with water to a depth of  $h$  metres, where  $0 \leq h \leq 1$ , the volume  $V$  of water in cubic metres is given by

$$V(h) = \frac{\pi h}{8}(9h + 32)$$

- a.** The pool is filled to its maximum depth of one metre.
- i.** When the depth of water changes from 1.0 metre to 0.9 metres, find using a linear approximation, the change in the volume.

2 marks

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- ii.** If the volume of water in the pool decreases at a constant rate of  $k$  cubic metres per minute, the pool is empty in four hours. Find the value of  $k$ .

1 mark

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- b.** The pool is refilled. Water is poured in at a rate of  $\sin\left(\frac{\sqrt{t}}{4}\right)$  cubic metres per minute,

where  $t$  is the time when refilling starts. Find the time correct to two decimal places, required to refill the pool to a depth of one metre.

2 marks

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- c. When the pool is refilled to a height of one metre, no more water is poured in. However the water continues to leak out. The volume of water in the pool now decreases at a rate equal to  $c\sqrt{h}$  cubic metres per minute, where  $c$  is a constant.
- i. Express  $\frac{dh}{dt}$  in terms of  $h$ , where  $t$  is the time in minutes after the pool is refilled.

2 marks

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- ii. This time the pool is empty in three hours. Find the value of  $c$ .

3 marks

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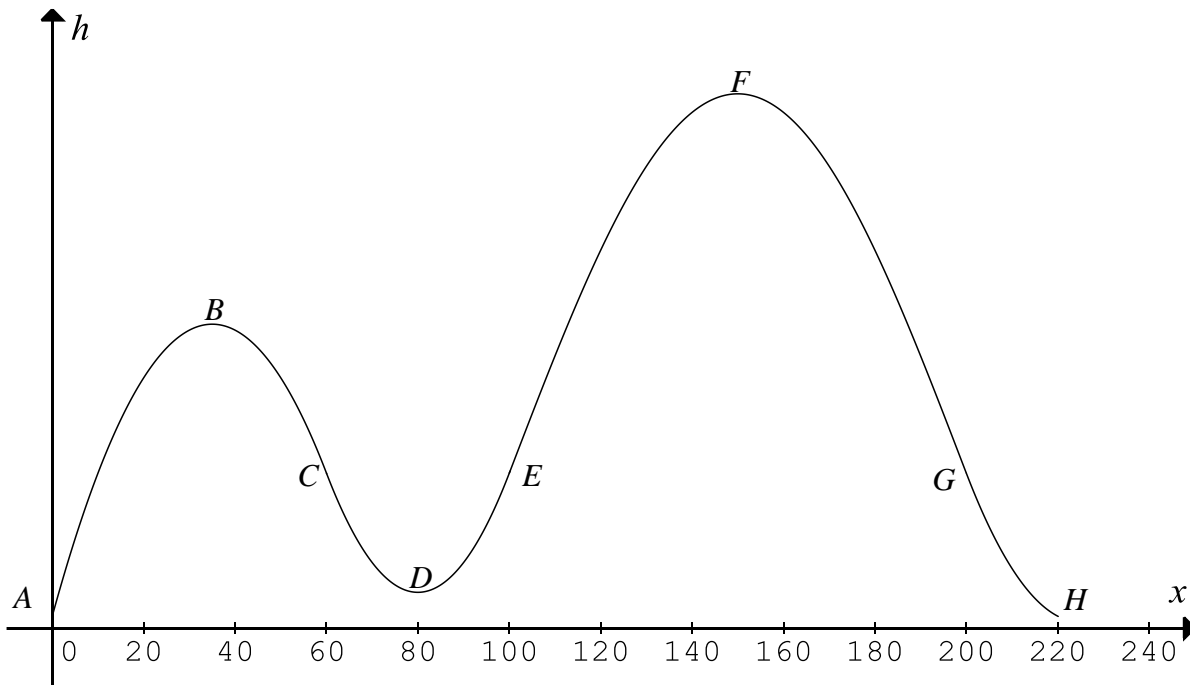
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**Question 5** (11 marks)

The diagram below, shows a design for a new roller coaster ride. The ride starts at the point  $A$  and passes through the points  $BCDEFG$  and finishes at the point  $H$ . The distance  $x$  in metres represents the horizontal distance from the start of the ride and  $y$  is the vertical height in metres above ground level. When riders finish and exit the ride at the point  $H$ , when they are 220 metres horizontally from the start of the ride. The exit for the ride is at the same vertical height as at the start of the ride.



The designers of the ride propose that the ride be comprised of a hybrid function consisting of four sections. Each curve is defined by its horizontal distance from the start of the ride. The section  $ABC$  is defined for  $x \in [0, 60]$ ,  $CDE$  is defined for  $x \in [60, 100]$ ,  $EFG$  is defined for  $x \in [100, 200]$  and  $GH$  is defined for  $x \in [200, 220]$ .

The curves are defined by the graph of  $y = f(x)$ , where

$$y = f(x) = \begin{cases} ax^2 + bx + c & \text{for } 0 \leq x \leq 60 \\ \frac{x^2}{40} - 4x + 163 & \text{for } 60 \leq x \leq 100 \\ R \sin\left(\frac{\pi(x-100)}{n}\right) + k & \text{for } 100 \leq x \leq 200 \\ \frac{x^2}{50} - 9x + 1013 & \text{for } 200 \leq x \leq 220 \end{cases}$$









# MATHEMATICAL METHODS CAS

## Written examination 2

### FORMULA SHEET

#### Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Mathematical Methods and CAS Formulas

### Mensuration

area of a trapezium:  $\frac{1}{2}(a+b)h$

Volume of a pyramid:  $\frac{1}{3}Ah$

curved surface area of a cylinder:  $2\pi rh$

volume of a sphere:  $\frac{4}{3}\pi r^3$

volume of a cylinder:  $\pi r^2 h$

area of triangle:  $\frac{1}{2}bc \sin(A)$

volume of a cone:  $\frac{1}{3}\pi r^2 h$

### Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$$

product rule:  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule:  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

approximation:  $f(x+h) \approx f(x) + h f'(x)$

### Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

**Transition Matrices**  $S_n = T^n \times S_0$

mean:  $\mu = E(X)$

variance:  $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

# ANSWER SHEET

STUDENT NUMBER

Figures  
Words


Letter

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SIGNATURE \_\_\_\_\_

## SECTION 1

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E