Year 2015

VCE

Mathematical Methods

Trial Examination 1

Solutions

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Question 1

a. $y = \frac{\log_e(3x)}{2x}$ using the quotient rule

$u = \log_e(3x) \qquad v = 2x$

$\frac{du}{dx} = \frac{1}{x} \qquad \frac{dv}{dx} = 2$ M1

$\frac{dy}{dx} = \frac{2x \times \frac{1}{x} - 2\log_e(3x)}{(2x)^2} = \frac{2(1 - \log_e(3x))}{4x^2}$

$\frac{dy}{dx} = \frac{1}{2x^2}(1 - \log_e(3x))$ A1

b. $f(x) = x\sqrt{4x^2 + 9}$

$y = x\sqrt{4x^2 + 9}$ using the product and chain rules

$u = x \qquad v = \sqrt{4x^2 + 9} = (4x^2 + 9)^{\frac{1}{2}}$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{1}{2} \times 8x \times (4x^2 + 9)^{-\frac{1}{2}} = \frac{4x}{\sqrt{4x^2 + 9}}$

$f'(x) = u\frac{dv}{dx} + v\frac{du}{dx} = \frac{4x^2}{\sqrt{4x^2 + 9}} + \sqrt{4x^2 + 9}$ M1

$f'(-2) = \frac{16}{\sqrt{25}} + \sqrt{25} = \frac{16}{5} + 5$

$f'(-2) = \frac{41}{5}$ or $8\frac{1}{5}$ or 8.2 A1

Question 2

$3 \times 9^x - 28 \times 3^x + 9 = 0$

let $u = 3^x$ then $9^x = (3^2)^x = 3^{2x} = (3^x)^2 = u^2$

$3u^2 - 28u + 9 = 0$

$(3u - 1)(u - 9) = 0$ M1

$u = 3^x = \frac{1}{3} \qquad u = 3^x = 9$

$x = -1$ or $x = 2$ A1

Question 3

$kx + 8y = k - 4$
$3x + (k-2)y = 1$

$\Delta = \begin{vmatrix} k & 8 \\ 3 & k-2 \end{vmatrix} = k(k-2) - 24 = k^2 - 2k - 24$

$\Delta = (k-6)(k+4)$ M1

There is a unique solution when $\Delta \neq 0$ that is $k \in R \setminus \{-4, 6\}$ A1

When $k = -4$ the equations become $\begin{matrix} -4x + 8y = -8 \\ 3x - 6y = 3 \end{matrix}$ these lines are parallel with different y-intercepts, therefore there is no solution when $k = -4$

When $k = 6$ the equations become $\begin{matrix} 6x + 8y = 2 \\ 3x + 4y = 1 \end{matrix}$ these lines are both the same line, therefore we have an infinite number of solutions when $k = 6$ A1

Question 4

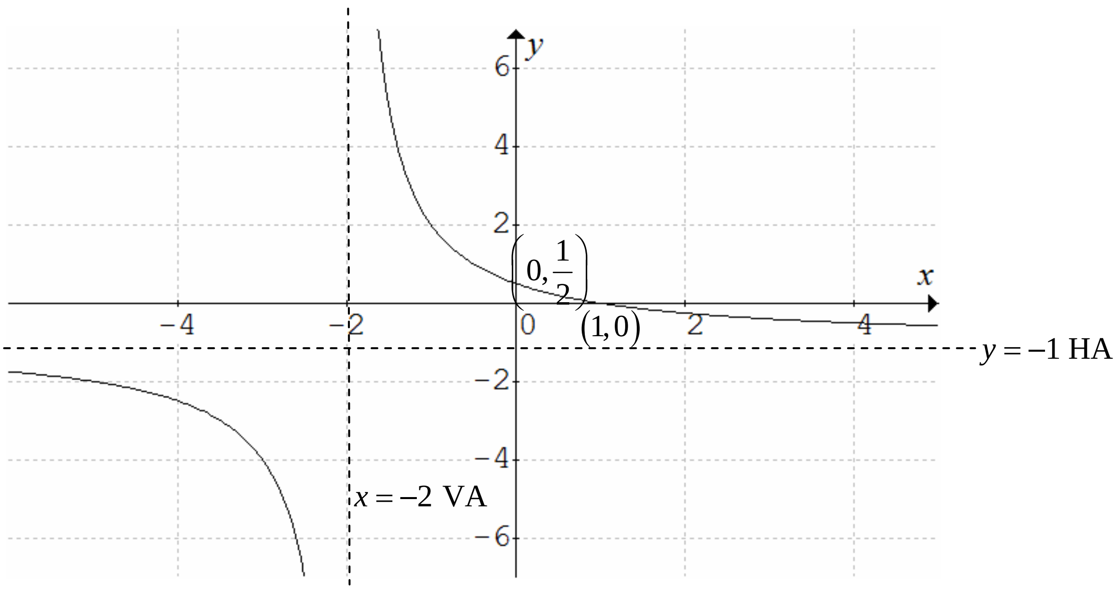
a. $y = \dfrac{3}{x+2} - 1$

crosses the x-axis when $y = 0 \Rightarrow x + 2 = 3 \quad x = 1 \quad (1,0)$

crosses the y-axis when $x = 0 \Rightarrow y = \dfrac{3}{2} - 1 \quad y = \dfrac{1}{2} \quad \left(0, \dfrac{1}{2}\right)$

$x = -2$ is a vertical asymptote, $y = -1$ is a horizontal asymptote A1

correct graph, shape, asymptotes axial intercepts G1



b. The area is below the x-axis, the area is $A=-\int_1^4\left(\frac{3}{x+2}-1\right)dx$ A1

$A=\left[-3\log_e(x+2)+x\right]_1^4$

$A=-3\log_e(6)+4+3\log_e 3-1$

$A=3\log_e\left(\frac{1}{2}\right)+3$ or $3-3\log_e(2)$ or $3-\log_e(8)$ A1

c. $y=\frac{1}{x}$ into $y=\frac{3}{x+2}-1$

the translations must come last.

dilate by a factor of 3 parallel to the y-axis (or away from the x-axis)

translate 2 units to the left parallel to the x-axis (or away from the y-axis)

translate 1 unit down parallel to the y-axis (or away from the x-axis) A1

d. $f:\quad y=\frac{3}{x+2}-1$ swap x and y

$f^{-1}\quad x=\frac{3}{y+2}-1$

$x+1=\frac{3}{y+2}$

$y+2=\frac{3}{x+1}$ M1

$y=\frac{3}{x+1}-2$

but dom $f=R\setminus\{-2\}=$ ran f^{-1} and ran $f=R\setminus\{-1\}=$ dom f^{-1}

To state the function, we must state its domain

$f^{-1}:R\setminus\{-1\}\to R$, $f^{-1}(x)=\frac{3}{x+1}-2$ A1

Question 5

$f'(x)=4e^{-2x}-1$

$f(x)=\int\left(4e^{-2x}-1\right)dx=-2e^{-2x}-x+c$ M1

$f(0)=1\quad\Rightarrow 1=-2e^{-0}+c\ \Rightarrow\ c=3$

$f(x)=3-2e^{-2x}-x$ A1

Question 6

a. range $-2 \le y \le 6 \ = [-2,6]$, period $T = \dfrac{2\pi}{\frac{\pi}{6}} = 12$, amplitude 4 A1

b. $2 - 4\sin\left(\dfrac{\pi x}{6}\right) = 0 \ \Rightarrow \ 4\sin\left(\dfrac{\pi x}{6}\right) = 2$ or $\sin\left(\dfrac{\pi x}{6}\right) = \dfrac{1}{2}$

$\dfrac{\pi x}{6} = 2n\pi + \sin^{-1}\left(\dfrac{1}{2}\right)$ or $\dfrac{\pi x}{6} = (2n+1)\pi - \sin^{-1}\left(\dfrac{1}{2}\right)$

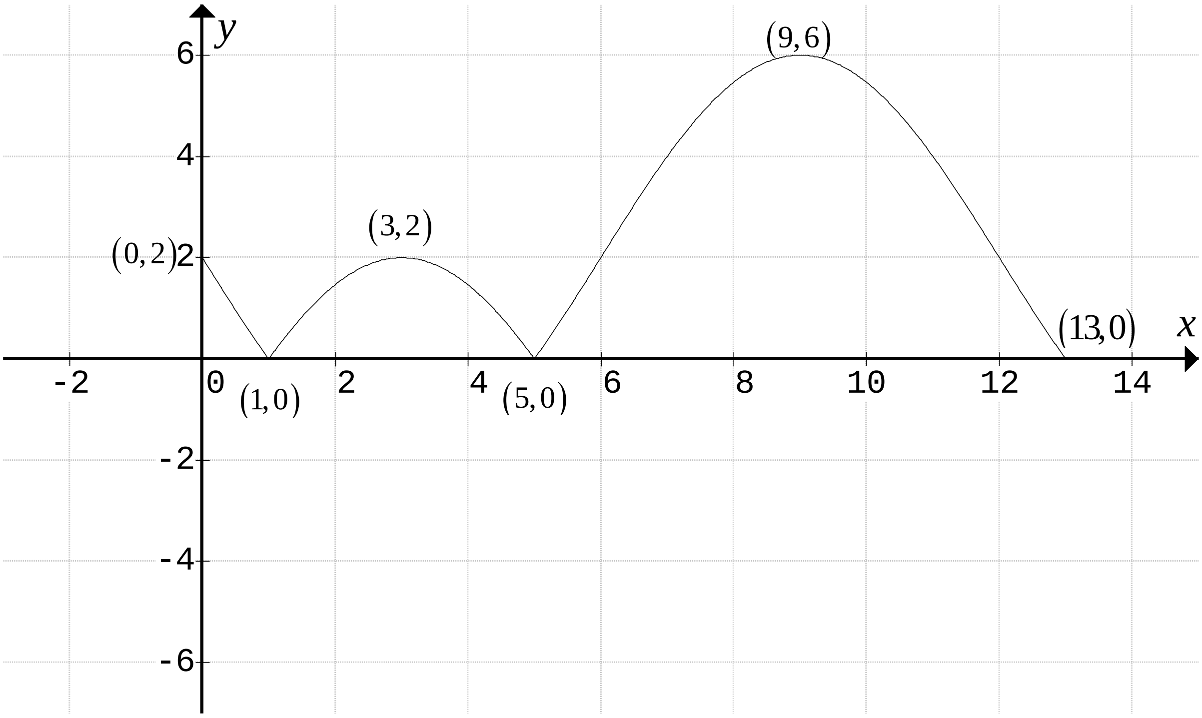
$\dfrac{\pi x}{6} = 2n\pi + \dfrac{\pi}{6}$ or $\dfrac{\pi x}{6} = (2n+1)\pi - \dfrac{\pi}{6}$ M1

$\dfrac{\pi x}{6} = \dfrac{\pi}{6}(12n+1)$ or $\dfrac{\pi x}{6} = \dfrac{\pi}{6}(12n+5)$

$x = 12n+1$ or $12n+5$ where $n \in Z$ A1

c. $n = 0$ or $n = 1 \ \Rightarrow \ x = 1, 5, 13$ A1

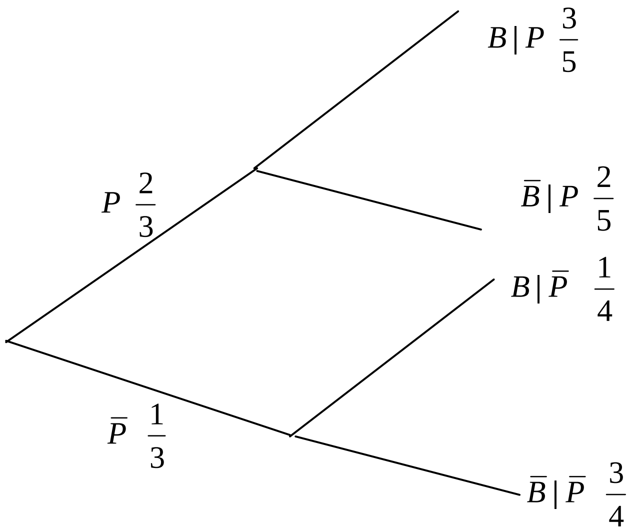
d. correct graph and coordinates G2



e. domain $(0,13)$ not including 1, 5 or $(0,1) \cup (1,5) \cup (5,13)$ A1

Question 7

a. P pleasant weather, B uses BBQ, using a tree diagram



$$\Pr\left(\bar{P} \mid \bar{B}\right) = \frac{\Pr\left(\bar{P} \cap \bar{B}\right)}{\Pr\left(\bar{B}\right)} = \frac{\frac{1}{3} \times \frac{3}{4}}{\frac{1}{3} \times \frac{3}{4} + \frac{2}{3} \times \frac{2}{5}} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{4}{15}} = \frac{1}{4} \times \frac{60}{31}$$ M1

$$\Pr\left(\bar{P} \mid \bar{B}\right) = \frac{15}{31}$$ A1

b. Since the probabilities sum to one. $\sum \Pr\left(X = x\right) = 1$

$\log_8\left(k-1\right) + \log_8\left(k-3\right) = 1$

$\log_8\left[\left(k-1\right)\left(k-3\right)\right] = 1$

$\left(k-1\right)\left(k-3\right) = 8$

$k^2 - 4k + 3 = 8$ M1

$k^2 - 4k - 5 = 0$

$\left(k-5\right)\left(k+1\right) = 0$

$k = 5$ or $k = -1$ but $k > 3$

only solution $k = 5$ A1

Question 8

a. using the product rule

$$\frac{d}{dx}\left(x\cos(2x)\right)=x\frac{d}{dx}\left(\cos(2x)\right)+\cos(2x)\frac{d}{dx}(x)$$
$$=-2x\sin(2x)+\cos(2x)$$ A1

b. $f:[0,\infty)\to R\ ,\ f(x)=x\sin(2x)$

The graph crosses the x-axis when $\sin(2x)=0$

$2x=0,\pi$ so that $x=0\ ,\ \frac{\pi}{2}$

The area is $A=\int_0^{\frac{\pi}{2}}x\sin(2x)\,dx$ A1

Since $\frac{d}{dx}\left(x\cos(2x)\right)=-2x\sin(2x)+\cos(2x)$ from a.

$$\int\left(-2x\sin(2x)+\cos(2x)\right)dx=x\cos(2x)$$
$$-2\int x\sin(2x)\,dx+\int\cos(2x)\,dx=x\cos(2x)$$
$$2\int x\sin(2x)\,dx=\int\cos(2x)\,dx-x\cos(2x)$$ M1
$$=\frac{1}{2}\sin(2x)-x\cos(2x)$$
$$\int x\sin(2x)\,dx==\frac{1}{4}\sin(2x)-\frac{1}{2}x\cos(2x)+c$$

$$A=\left[\frac{1}{4}\sin(2x)-\frac{1}{2}x\cos(2x)\right]_0^{\frac{\pi}{2}}$$
$$A=\frac{1}{4}\sin(\pi)-\frac{1}{2}\times\frac{\pi}{2}\cos(\pi)-\frac{1}{4}\sin(0)-0$$
$$A=\frac{\pi}{4}\ \text{units}^2$$ A1

Question 9

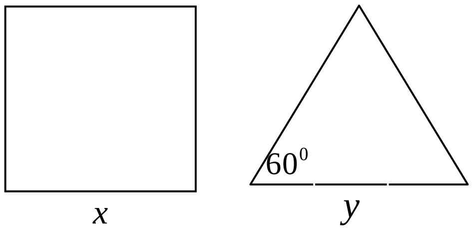
The normal $4y-x+c=0 \quad\Rightarrow\quad y=\frac{x}{4}-\frac{c}{4} \quad\Rightarrow\quad m_N=\frac{1}{4} \quad m_T=-4$ M1

$y=2e^{-2x}+1 \quad\Rightarrow\quad \frac{dy}{dx}=-4e^{-2x}=-4 \quad\Rightarrow\quad x=0$ so $y=3 \quad P(0,3)$ M1

$c=x-4y=-12$ A1

Question 10

a.



$(1)\quad 4x+3y=36$ the total length of string

$A=x^2+\frac{1}{2}y^2\sin\left(60^0\right)$ total area of square and equilateral triangle

$(2)\quad A=x^2+\frac{\sqrt{3}y^2}{4}$ A1

$(1)\quad \Rightarrow\ 3y=36-4x=4(9-x)$ substitute $y=\frac{4}{3}(9-x)$ into (2)

But since the lengths cannot be negative, we require $0\le x\le 9$ A1

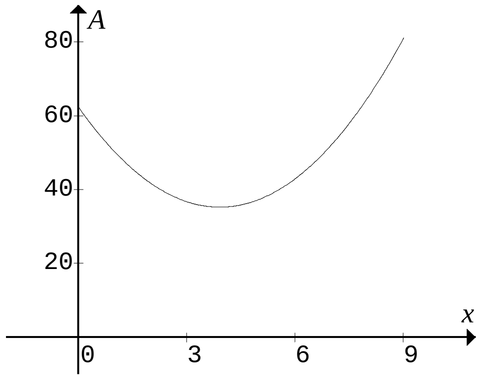
$$A(x)=x^2+\frac{\sqrt{3}}{4}\left(\frac{4}{3}(9-x)\right)^2=x^2+\frac{4\sqrt{3}}{9}\left(81-18x+x^2\right)$$

$$A(x)=\left(\frac{4\sqrt{3}}{9}+1\right)x^2-8\sqrt{3}\,x+36\sqrt{3}\quad\text{for}\quad 0\le x\le 9$$

b. For a maximum or a minimum

we require $\frac{dA}{dx}=2x\left(\frac{4\sqrt{3}}{9}+1\right)-8\sqrt{3}=0$

$$\Rightarrow x=\frac{4\sqrt{3}}{\frac{4\sqrt{3}}{9}+1}=\frac{36\sqrt{3}}{4\sqrt{3}+9}=\frac{36}{4+3\sqrt{3}}$$



Now $1.5<\sqrt{3}<2$ so that $3<\frac{54}{15}<\frac{36\sqrt{3}}{4\sqrt{3}+9}<\frac{72}{17}<5$

To find the maximum value investigate the end-points of the function

$$A(x)=x^2+\frac{\sqrt{3}}{4}\left(\frac{4}{3}(9-x)\right)^2$$

$A(0)=\frac{\sqrt{3}}{4}\times\left(\frac{4\times 9}{3}\right)^2=36\sqrt{3}$ and $A(9)=81$ A1

The minimum value of the area occurs when $x=\frac{36\sqrt{3}}{4\sqrt{3}+9}$

The maximum value of the area occurs when $x=9$.

That is when only the square is formed. A1

END OF SUGGESTED SOLUTIONS