# Mathematical Methods (CAS)

# 2015

# Trial Examination 1
# (1 hour)

## Instructions

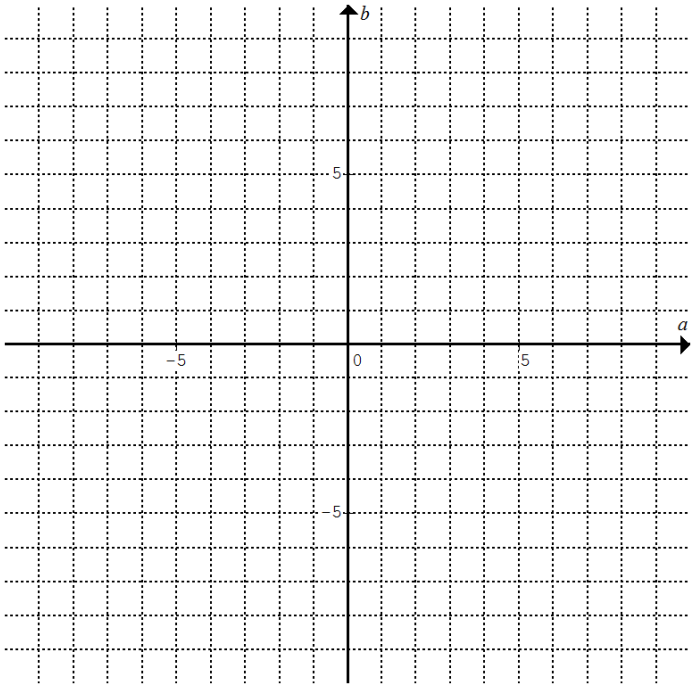
Answer **all** questions. Do **not** use CAS/calculators.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

### Question 1

Consider the parabolas $y = -x^2$ and $y = 2(x-a)^2 + b$ where $a, b \in R \setminus \{0\}$.
The two parabolas ***touch*** each other

a. Find a value for each of $a$ and $b$. 2 marks

b. Sketch the graph of $b$ versus $a$. 2 marks



**Question 2**

a. The graph of $y = \frac{1}{x+1} + 1$ is reflected in the line $y = x$, followed by a translation of 2 units in the negative $x$ - direction and then a translation of 2 units in the positive $y$ - direction.
Find the equation of the resulting graph. 2 marks

Given $h : D \to R$, $h(x) = \sqrt{\frac{1}{x+1} + 1}$ and $g : \left(a, \frac{5\pi}{6}\right) \to R$, $g(x) = 2\sin x$, where $D$ is the maximal domain of function $h$, and $a \in R$.

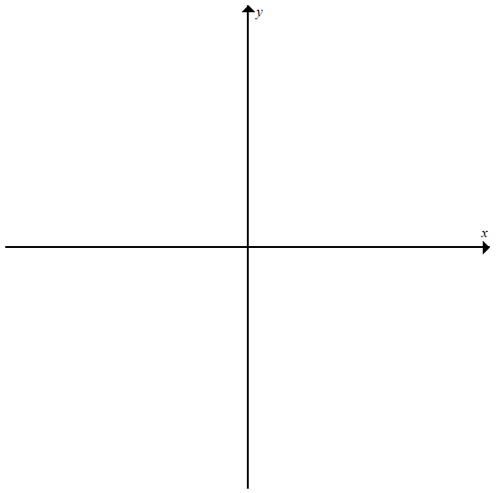
b. Find $D$, the maximal domain of function $h$. 2 marks

c. Find the minimum value of $a$ such that $h \circ g$ is defined. 2 marks

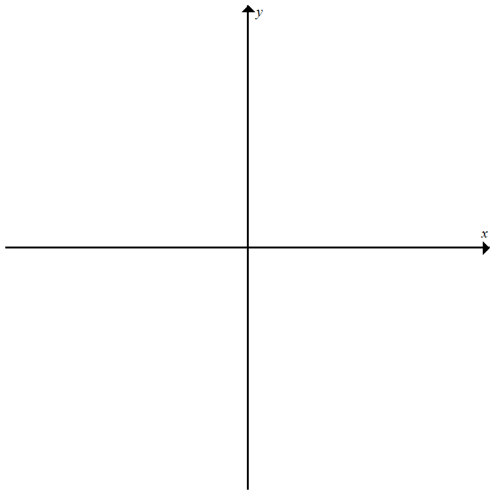
**Question 3**

Consider the function $f(x) = e^{-x} - \frac{x}{e}$, given $e \approx 2.7$.

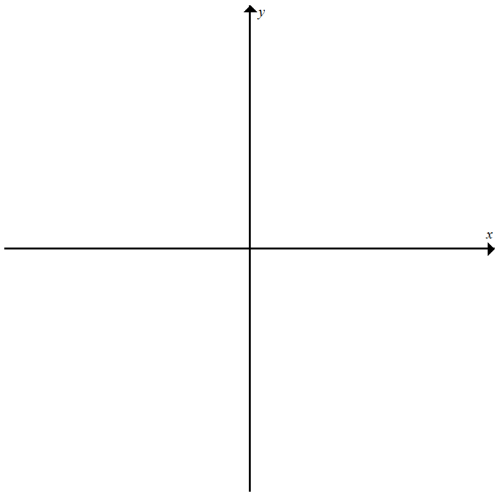
a. Use the method of addition of ordinates to sketch the graph of $y = f(x)$.
Show the coordinates of the axis-intercepts. 3 marks




b. Sketch the graph of $y = |f(x)|$. 1 mark




c. Sketch the graph of $y = |f(|x|)|$. 2 marks



**Question 4**

Consider the graphs of $y = \sqrt{x} - 1$ and $y = \sqrt{2x + 2} - 2$. $\frac{x}{a} + \frac{y}{b} = 1$ is common tangent to the two graphs.

a. Find the coordinates of the intersection(s) of the two curves. 3 marks

b. Find the value(s) of $a$ and the corresponding value(s) of $b$. 2 marks

**Question 5**

Consider $f(x) = 2\log_{10}(2x - 1) - \log_{10}(x + 1)$.

a. State the domain of the function. 1 mark

b. Write down the equation(s) of the asymptote(s). 1 mark

c. Find the coordinates of the $x$-intercept(s) of the graph of $y = f(x)$. 2 marks

**Question 6**

Use $f(a+h) \approx f(a)+hf'(a)$ to estimate the value of $\sin 46°$. Leave your answer in terms of $\pi$ and surds.

3 marks

**Question 7**

a. Given $y = e^x(\sin x - \cos x)$, show that $\frac{dy}{dx} = 2e^x \sin x$. 1 mark

b. Use the result in part a to find $\int_0^{\frac{\pi}{3}} e^x \sin x \, dx$. 2 marks

**Question 8**

a. Write down the equation of the inverse of $y = (x-1)^2 + 1$. Express $y$ in terms of $x$. 1 mark

b. Find the area of the region bounded by the line $x = 2$ and the inverse of $y = (x-1)^2 + 1$. 2 marks

**Question 9**

There are only two routes for a student to travel to school, Route A or Route B.
The two routes are equally likely to be chosen by the student to travel to school.
The student goes to school five days a week.

The probability that the student travels to school by Route A on $n$ days or more in a week is $\frac{13}{16}$, where $n \in \{1, 2, 3, 4, 5\}$.

Find the probability that the student travels to school by Route B on $n$ days or less in a week.

2 marks

**Question 10**

$$f(x)=\begin{cases}0, & x<1\\ |k(x-2)|+\frac{1}{5}, & 1\le x\le 5\\ 0, & x>5\end{cases}$$ is a probability density function of random variable $X$, where $x\in X$.

a. Show that $|k|=\frac{1}{25}$. 2 marks

b. Find the median of $X$. 2 marks

## End of exam 1