E

Free Exam
for 2006-15 VCE study design

Engage Education Foundation

Units 3 and 4 Maths Methods (CAS): Exam 1

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$\frac{dy}{dx} = 3x^2e^{-3x} - 3x^3e^{-3x}$ 1 mark for using product rule

$= x^2e^{-3x}(3 - 3x)$ 1 mark-answer

Question 1b

$\frac{dy}{dx} = \frac{3x^2+8x}{(3x+4)^2}$ 1 mark for correct dy/dx

at x=-1, $\frac{dy}{dx} = -5$ 1 mark-answer

Question 2a

$\frac{3x+2}{x+1} = \frac{3x+3-1}{x+1} = 3 - \frac{1}{x+1}$ or use long division 1 mark

$a = 3, b = -1$ 1 mark-answer

Question 2a

$\int \frac{3x+2}{x+1}dx = \int 3 - \frac{1}{x+1}dx = \int 3dx - \int \frac{1}{x+1}dx = 3x - \ln(|x+1|)$

1 mark for $3x$ and 1 mark for $-\ln(|x+1|)$

Question 3

$2\cos\left(3x + \frac{\pi}{6}\right) = -\sqrt{3}$

$\Rightarrow \cos\left(3x + \frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2}$

$\Rightarrow 3x + \frac{\pi}{6} = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}$ 1 mark

$\Rightarrow 3x = \frac{2\pi}{3}, \pi, \frac{8\pi}{3}, 3\pi$

$x = \frac{2\pi}{9}, \frac{\pi}{3}, \frac{8\pi}{9}, \pi$ 1 mark-(3 correct answers) 2 marks-(4 correct answers)

Question 4a

Full proof required.

Need, $p^2 + 1.6p + 0.2 = 1$ 1 mark

$\Rightarrow (p - 0.4)(p + 2) = 0$ 1 mark

$\Rightarrow p = 0.4 \; or \; p = -2$

$as \; 0 \leq p \leq 1, \Rightarrow p = 0.4$ 1 mark

Question 4b

$E(X) = 0.2 \times 0 + 0.4 \times 1 + 0.24 \times 2 + 0.16 \times 3$ 1 mark

$= 1.36$ 1 mark-answer

Question 4c

$\Pr = (0.2 + 0.4)^2 = 0.36$ 1 mark-answer

Question 5a

$x = 3 - e^{\frac{y-2}{3}}$ 1 mark

$3 - x = e^{\frac{y-2}{3}}$

$y - 2 = \ln(3 - x)$ 1 mark

$y = 3\ln(3 - x) + 2$

$\Rightarrow f^{-1}(x) = 3\ln(3 - x) + 2$ 1 mark

Question 5b

$Domain\ f^{-1}(x)\colon (-\infty, 3)$ 1 mark

Question 6

$x^3 - 13x + 12 = 0$

$(1)^3 - 13(1) + 12 = 1 - 12 + 13 = 0$

$use\ long\ division\ or\ another\ technique\ to\ get\ \frac{x^3 - 13x + 12}{(x-1)} = x^2 + x - 12$

$\Rightarrow (x-1)(x^2 + x - 12) = 0$

$\Rightarrow (x-1)(x+4)(x-3) = 0$

$\Rightarrow x = -4, 1, 3$ 1 mark for each correct solution

Question 7

$\ln(x+4) - 2\ln(x+1) + \ln(x-1) = \ln(\frac{(x+4)(x-1)}{(x+1)^2})$ 1 mark

$\Rightarrow \ln\left(\frac{(x+4)(x-1)}{(x+1)^2}\right) = 0$

$\Rightarrow \frac{(x+4)(x-1)}{(x+1)^2} = 1$ 1 mark

$\Rightarrow (x+4)(x-1) = (x+1)^2$

$\Rightarrow x^2 + 3x - 4 = x^2 + 2x + 1$

$\Rightarrow 3x - 4 = 2x + 1$

$\Rightarrow x = 5$ 1 mark

Question 8

$\sin(x) = \sqrt{3}\cos(x)$

$\Rightarrow \tan(x) = \sqrt{3}$

$\Rightarrow x = \frac{\pi}{3}, \frac{4\pi}{3}$ 1 mark

$Area = \int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} \sin(x) - \sqrt{3}\cos(x)\, dx$ 1 mark

$Area = \left[-\cos(x) - \sqrt{3}\sin(x)\right]_{x=\frac{\pi}{3}}^{x=\frac{4\pi}{3}}$ 1 mark

$Area = \left(\frac{1}{2} + \frac{3}{2}\right) - \left(-\frac{1}{2} - \frac{3}{2}\right)$

$Area = 4$ 2 marks

Question 9

Full proof required.

$$gradient = -\frac{1}{(-\frac{1}{3})} = 3$$

$\frac{dy}{dx} = -2(1 - x)$ 1 mark

$gradient = \frac{dy}{dx} \Rightarrow 3 = -2(1 - x)$ 1 mark

$\Rightarrow x = \frac{5}{2} \Rightarrow y = (1 - \frac{5}{2})^2 = \frac{9}{4}$ 1 mark

$\frac{9}{4} = -\frac{1}{3}\left(\frac{5}{2}\right) + c \Rightarrow c = \frac{37}{12}$ 1 mark

Question 10a

Full Proof required

$\int_{-6}^{6} k\left(1-\left|\frac{x}{6}\right|\right) dx = 1$ 1 mark

$$\int_{-6}^{6} k\left(1-\left|\frac{x}{6}\right|\right) dx = k\int_{0}^{-6}\left(1+\frac{x}{6}\right) dx + k\int_{0}^{6}(1-\frac{x}{6})dx = -k(-6+3)+k(6-3)$$

$\Rightarrow 3k+3k=1$

$\Rightarrow k=\frac{1}{6}$ 1 mark

Question 10b

$\frac{3}{4}=\frac{1}{6}\int_{-q}^{q}\left(1-\left|\frac{x}{6}\right|\right) dx$

$\frac{1}{6}\int_{-q}^{q}\left(1-\left|\frac{x}{6}\right|\right) dx = \frac{1}{3}\int_{0}^{q}\left(1-\frac{x}{6}\right) dx = \frac{1}{3}\left[x-\frac{x^2}{12}\right]_{x=0}^{x=q}$ 1 mark

$$\Rightarrow \frac{1}{3}\left(q-\frac{q^2}{12}\right)=\frac{3}{4}$$

$\Rightarrow 12q-q^2=27$

$\Rightarrow (q-9)(q-3)=0$

but $\boldsymbol{0 \leq q \leq 6, so\ q=3}$ 1 mark