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Free Exam
for 2006-15 VCE study design

Engage
Education
Foundation

Units 3 and 4 Maths Methods (CAS): Exam 1

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a
$\frac{dy}{dx} = \ln(x) + 1$ [1]

Question 1b
$\frac{dy}{dx} = 2(3x^2 - 2)(x^3 - 2x)$ [1]

At $x = 2$, $\frac{dy}{dx} = 80$ [1]

Question 2
$\int \cos(-2x + 3)dx = -\frac{1}{2}\sin(-2x + 3) + c$ [1]

One such antiderivative is $-\frac{1}{2}\sin(-2x + 3) + 1$ [1]

Question 3a
The inverse of $g(x)$ can be found by swapping x and y:

$x = 2e^{3y} + 1$ [1]

$e^{3y} = \frac{x-1}{2}$

$y = \frac{1}{3}\ln\frac{x-1}{2}$ [1]

Question 3b
$g(g^{-1}(x)) = 2e^{3\left(\frac{1}{3}\ln\frac{x-1}{2}\right)} + 1$ [1]

$= 2e^{\ln\frac{x-1}{2}} + 1$

$= x$ [1]

Question 4a
$E(X) = 0.1 + 2(0.25) + 3(0.4) = 1.8$ [1]

Question 4b
$\Pr = 0.25 \times 0.25 \times 0.25 = \frac{1}{64}$ [1]

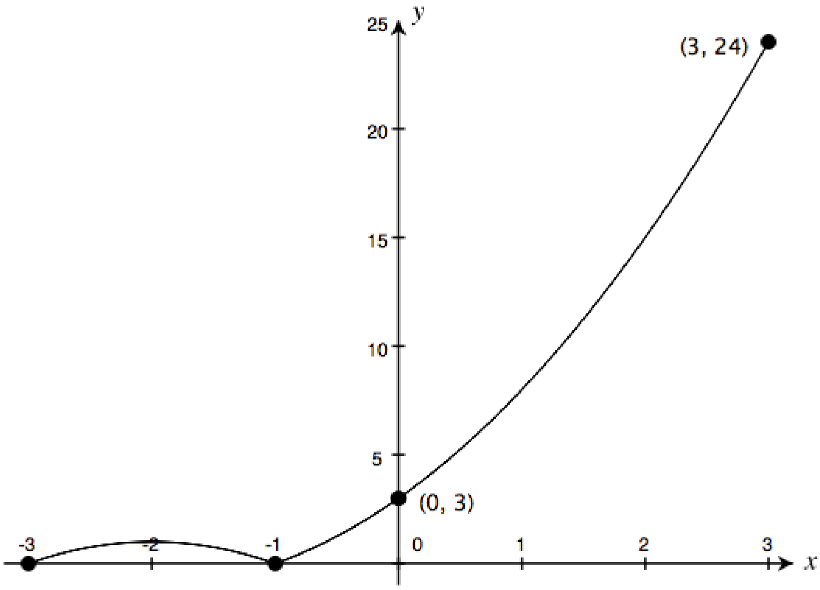
Question 4c
$\Pr(\text{3 calls over 2 day span})$

$= \Pr(\text{1 call then 2 calls}) + \Pr(\text{2 calls then 1 call}) + \Pr(\text{0 calls then 3 calls}) + \Pr(\text{3 calls then 0 calls})$ [1]

$= 0.1 \times 0.25 + 0.25 \times 0.1 + 0.25 \times 0.4 + 0.4 \times 0.25$ [1]

$= 0.25$ [1]

Question 5a



Graph should contain the following:

- End points are at $(-3, 0)$ and $(3, 24)$ [1]
- Axis-intercepts are at $(-3, 0)$, $(-1, 0)$ and $(0, 3)$ [1]
- Correct shape [1]

Question 5b
$(0, -1)$ [1]

Question 5c
The mapping is $(x, y) \rightarrow (-x, y)$ [reflection in y axis] $\rightarrow (-x, y + 2)$ [translation 2 units up]. This maps to (x', y'). So $x = -x'$ and $y = y' - 2$. So $y' - 2 = |-(-x')^2 - 4(-x') - 3|$.

Image of $y = |-x^2 + 4x - 3| + 2$ [1]

Question 6
$\sin\left(2x - \frac{\pi}{2}\right) = -\frac{\sqrt{2}}{2}$ [1]

$2x - \frac{\pi}{2} = -\frac{\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{13\pi}{4}$ [1]

$x = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$ [1]

Question 7

$\ln\left(\frac{(x-3)(2x-1)}{(x+1)^2}\right) = 0$ [1]

$(x-3)(2x-1) = (x+1)^2$

$x^2 - 9x + 2 = 0$ [1]

Reject $x = \frac{9}{2} - \frac{\sqrt{73}}{2}$ since $x > 1$, so:

$x = \frac{9}{2} + \frac{\sqrt{73}}{2}$ [1]

Question 8a

$\Pr(X > 161 | X > 150) = \frac{\Pr(X > 161)}{\Pr(X > 150)}$ [1]

Now $P(X > 161) = P(Z > (\frac{(161-150)}{11}) = P(Z > 1) = P(Z < -1)$

and $P(X < 139) = P(Z < \frac{((139-150))}{11} = P(Z < -1) = q.$

So $P(X > 161) = q$. Since $P(X > 150) = P(Z > 0) = 0.5$,

$\frac{\Pr(X > 161)}{\Pr(X > 150)} = \frac{q}{0.5} = 2q$ [1]

Question 8b

$\Pr(X \leq k) = \int_2^k \frac{x-2}{8}\, dx$ [1]

$= \frac{1}{8}\left(\frac{k^2}{2} - 2k + 2\right) = \frac{1}{2}$

$k^2 - 4k - 4 = 0$ [1]

Reject $k = 2 - 2\sqrt{2}$ since $2 < k < 6$

$k = 2 + 2\sqrt{2}$ [1]

Question 9a

$f'(x) = 2x\ln(x) + x$ [1]

Question 9b

$x\ln(x) = \frac{f'(x) - x}{2}$ [1]

$\int_1^e (x \ln x)dx = \int_1^e \frac{f'(x) - x}{2} dx = \int_1^e (\frac{f'(x)}{2} - \frac{x}{2})dx = \left[\frac{f(x)}{2} - \frac{x^2}{4}\right]_1^e = \left[\frac{1}{2}x^2\ln(x) - \frac{x^2}{4}\right]_1^e$ [1]

$= \frac{e^2+1}{4}$ [1]

Question 10

Area below curve: first we must find x-intercepts.

$x^3 - kx = 0$

$x = 0, \pm\sqrt{k}$ [1]

So the area underneath the x-axis for $x > 0$ is:

$-\int_0^{\sqrt{k}} (x^3 - kx)dx = -\left[\frac{x^4}{4} - \frac{kx^2}{2}\right]_0^{\sqrt{k}} = \frac{k^2}{4}$ [1]

Area above the curve: first we must find m, the point of intersection

$f(x) = g(x)$

$x^3 - 2kx = 0$

$x = 0, \pm\sqrt{2k}$ [1]. As $m > 0$, $m = \sqrt{2k}$

To find the area above the x-axis, we find the total area between $g(x)$ and $f(x)$, then subtract the area under the x-axis as found earlier:

$$\begin{aligned} Area\ between\ g(x)\ and\ f(x) &= \int_0^m \big(kx - (x^3 - kx)\big)dx \\ &= \int_0^m (2kx - x^3)dx \\ &= \int_0^{\sqrt{2k}} (2kx - x^3)dx \\ &= \left[kx^2 - \frac{x^4}{4}\right]_0^{\sqrt{2k}} = k(2k) - \frac{4k^2}{4} = k^2 \end{aligned}$$

Therefore the area bounded by f and g is four times the size of the area bounded by the g and the x-axis.