TEST 3

Functions, graphs and algebra
Technology-free end-of-year examination
Total marks: 25
Suggested writing time: 40 minutes

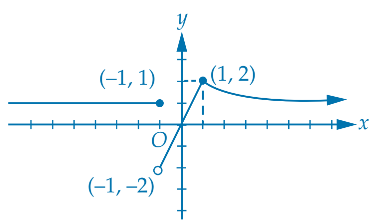
Specific instructions to students

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

QUESTION 1

a Sketch the graph of $y = \begin{cases} 1, & x \le -1 \\ 2x, & -1 < x < 1 \\ \frac{1}{x} + 1, & x \ge 1 \end{cases}$

labelling the coordinates of any axial intercepts and endpoints.



2 marks

b Evaluate $f(-2)$, $f(0)$ and $f(2)$.

$f(-2) = 1$

$f(0) = 0$

$f(2) = \frac{3}{2}$

2 marks

c Find the x value(s) for which $f(x) = 2$

$f(x) = 2$ only when $x = 1$

1 mark
(Total: 5 marks)

QUESTION 2

a A cubic function of the form $y = ax(x - b)(x - c)$ has x-intercepts at $x = 2$ and $x = 3$, and passes through the point (1, 1). Write down the equation of the function.

Incorporating the x-intercept information into $y = ax(x - b)(x - c)$, we get

$y = ax(x - 2)(x - 3)$

Substituting the point (1, 1) in the above equation, we have

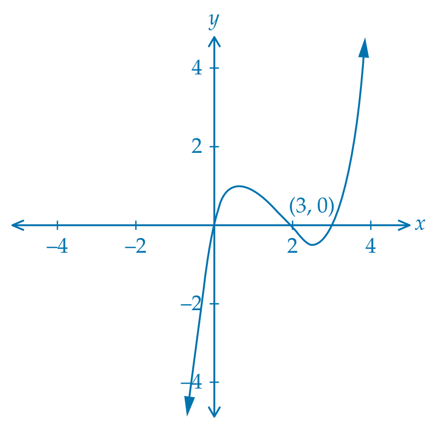
$1 = a(-1)(-2)$

$\therefore a = \frac{1}{2}$

The equation is $y = \frac{1}{2}x(x - 2)(x - 3)$.

2 marks

b Hence sketch the function, labelling axial intercepts.



2 marks
(Total: 4 marks)

QUESTION 3

An increasing exponential function of the form $y = ae^x + b$ has a horizontal asymptote at $y = 2$ and passes through the origin. Write down the equation of the function.

$y = ae^x + 2$ because of the horizontal asymptote at $y = 2$.

Substituting (0, 0) gives

$0 = ae^0 + 2$

$\therefore a = -2$

The equation is $y = -2e^x + 2$

2 marks

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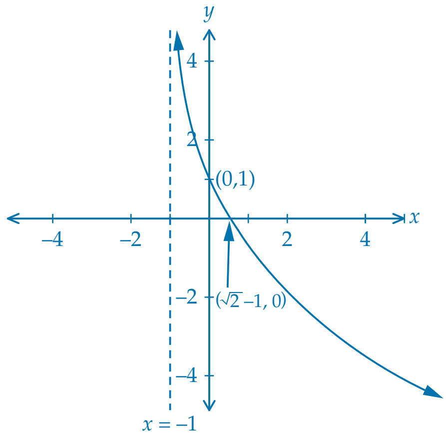
QUESTION 4

a Sketch the graph of $y = -2\log_2(x+1) + 1$, giving the equations of any asymptotes and labelling any axial intercepts.



2 marks

b State the domain and range of the graph.

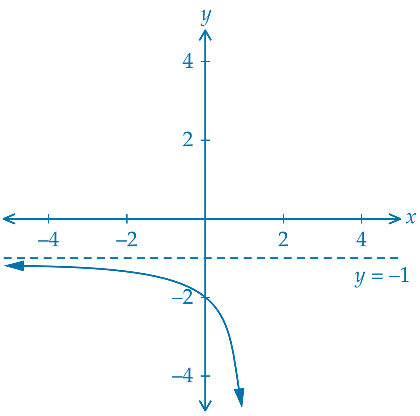
Domain = $(-1, \infty)$

Range = $\boldsymbol{R}$

2 marks
(Total: 4 marks)

QUESTION 5

a Sketch the graph of $y = -e^{2x} - 1$, labelling any intercepts with the axes and giving the equations of any asymptotes.



2 marks

b State the domain and range of the graph.

Domain = $\boldsymbol{R}$

Range = $(-\infty, -1)$

2 marks
(Total: 4 marks)

QUESTION 6

Let $P(x) = x^3 - 2x^2 - 4x + 8$. Use the factor theorem to solve the equation $P(x) = 0$

Try $P(2) = 8 - 8 - 8 + 8 = 0$

This means that $(x - 2)$ is a factor.

Use 2×2 grouping (or long division) to find a quadratic factor.

$$\begin{aligned} x^3 - 2x^2 - 4x + 8 &= x^2(x-2) - 4(x-2) \\ &= (x-2)(x^2-4) \\ &= (x-2)(x-2)(x+2) \\ &= (x-2)^2(x+2) \end{aligned}$$

$P(x) = 0$ at $x = 2$, $x = -2$

2 marks

QUESTION 7

Find the remainder when $Q(x) = x^3 + x^2 + 5x + 1$ is divided by $x + 2$.

$Q(x) = x^3 + x^2 + 5x + 1$

$$\begin{aligned} Q(-2) &= (-2)^3 + (-2)^2 + 5(-2) + 1 \\ &= -8 + 4 - 10 + 1 \\ &= -13 \end{aligned}$$

Remainder = -13

2 marks

QUESTION 8

Find the values of constants a and b if the polynomial $P(x) = x^3 + x^2 + ax + b$ is divisible by the factor $x - 2$, but when divided by $x + 3$ the remainder is 5.

$P(x) = x^3 + x^2 + ax + b$

$P(2) = 2^3 + 2^2 + 2a + b$

$\Rightarrow 12 + 2a + b = 0$

Also $P(-3) = (-3)^3 + (-3)^2 - 3a + b$

$\Rightarrow -18 - 3a + b = 5$

Solve simultaneously

$2a + b = -12$ and $-3a + b = 23$

This gives $a = -7$, $b = 2$

2 marks

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