SACRED HEART GIRLS' COLLEGE

OAKLEIGH

Mathematical Methods CAS 2014

Unit 3 SAC 1: TEST

Part A

Name: SOLUTIONS (SMITH)

Teacher (please circle): Ms Gates Mr Smith Ms Garkel

No CAS and no summary notes permitted

Part A: 3 short answer questions

Writing Time: 20 minutes

Marks: 15

SHORT ANSWER QUESTIONS

Instructions:
Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this test are **not** drawn to scale.

Question 1 (7 marks)

Consider the function $f: D \rightarrow R, f(x) = 3 - \sqrt{2x+4}$, where D is the maximal domain of f.

a) Find D. 1 mark

$2x + 4 \geq 0$ $D = [-2, \infty)$ 1

$x \geq -2$

b) Describe the transformations which when applied to the graph of $y = \sqrt{x}$, produce the graph of $y = f(x)$. 2 marks

$f(x) = 3 - \sqrt{2(x+2)} = -\sqrt{\frac{x}{\frac{1}{2}} + 4} + 3$

- DILATION of factor $\frac{1}{2}$ FROM y-axis
- REFLECTION in x-axis

either order 1

- TRANSLATION 2 UNITS left
- " 3 units up

either order 1

(DEDUCT 1 for incorrect order; give 1 for correct order with any 2 correct).

c) Find the rule for f^{-1}, the inverse of f. 2 marks

INVERSE,

$x = 3 - \sqrt{2y+4}$ 1

$(x-3)^2 = \left(-\sqrt{2y+4}\right)^2$

$(x-3)^2 = 2y + 4$

$y = \frac{1}{2}(x-3)^2 - 2$

$f^{-1}(x) = \frac{1}{2}(x-3)^2 - 2$ 1

also accept

$f^{-1}(x) = \frac{1}{2}(3-x)^2 - 2$

or

if $(x-3)^2$ expanded

$f^{-1}(x) = \frac{1}{2}(x^2 - 6x + 5)$

d) Show that the values of x for which $f(x) = x$ and hence the values of x for which $f(x) = f^{-1}(x)$ are $x = 4 \pm \sqrt{11}$. 2 marks

$f(x) = x$

$3 - \sqrt{2x+4} = x$

$3 - x = \sqrt{2x+4}$

$(3-x)^2 = 2x + 4$

$9 - 6x + x^2 = 2x + 4$

$x^2 - 8x + 5 = 0$ 1

$x = \frac{8 \pm \sqrt{64 - 4 \times 1 \times 5}}{2}$

$x = \frac{8 \pm \sqrt{44}}{2}$

$= \frac{8 \pm 2\sqrt{11}}{2}$ 1

$= 4 \pm \sqrt{11}$

Question 2 (4 marks)

For $f(x) = \sqrt{x+3} - 1$ and $g(x) = |x| + 2$

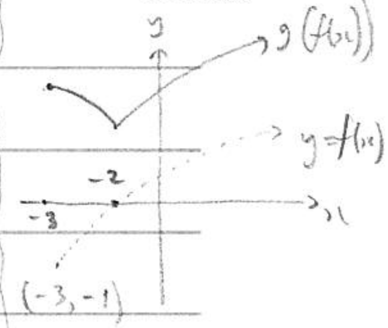
a) Find the rule for $g(f(x))$. 1 mark

$g(f(x)) = |\sqrt{x+3} - 1| + 2$ 1

b) Write the rule for $g(f(x))$ as a hybrid function and state the domain. 3 marks

$$g(f(x)) = \begin{cases} +(\sqrt{x+3} - 1) + 2, & x \geq -2 \\ -(\sqrt{x+3} - 1) + 2, & -3 \leq x < -2 \end{cases}$$

$$= \begin{cases} \sqrt{x+3} + 1, & x \geq -2 \\ -\sqrt{x+3} + 3, & -3 \leq x < 2 \end{cases}$$



1 for each rule, 1 for domain

(also, in this case, accepted dom = $[-3, \infty)$)

Question 3 (4 marks)

a) For the function $(x) = \frac{x+3}{x+2}$, write the function in the form $f(x) = a + \frac{b}{x+2}$. 2 marks

$$x+2 \overline{) \; x+3} \quad 1$$
$$\underline{-(x+2)}$$
$$1$$

$$f(x) = 1 + \frac{1}{x+2} \quad 1$$

OR

$$f(x) = \frac{x+2}{x+2} + \frac{1}{x+2}$$
$$= 1 + \frac{1}{x+2}$$

b) Hence, find $g(x)$ if $g(x) = f^{-1}(x)$. 2 marks

INVERSE

$$x = 1 + \frac{1}{y+2} \quad 1$$

$$x - 1 = \frac{1}{y+2}$$

$$(x-1)(y+2) = 1$$
$$y+2 = \frac{1}{x-1}$$

$$y = \frac{1}{x-1} - 2$$

$$g(x) \text{ or } f^{-1}(x) = \frac{1}{x-1} - 2 \quad 1$$

(in this case did not penalise for not giving answer in function notation with domain. Not just asked for 'the rule')

END OF QUESTION AND ANSWER BOOKLET